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ZX-calculus

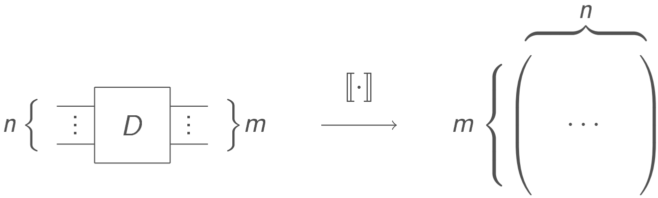
A Swiss army katana for quantum computing

Séminaire CANA

Chapter 1:

Graphical Languages

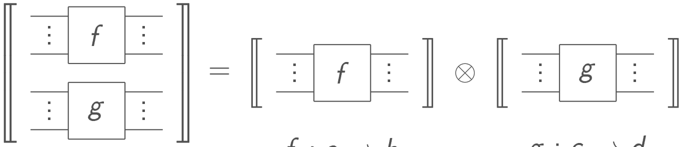
From diagrams to matrices



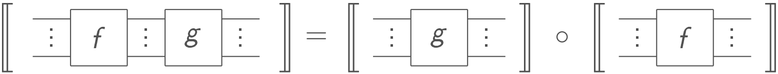
$D : n \to m$

$[\![D]\!] \in \mathcal{M}_{2^m \times 2^n}(\mathbb{C})$

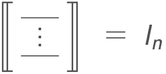
Compositionality



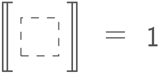
$f \otimes g : a + c \to b + d$ $\quad f : a \to b$ $\quad g : c \to d$



$g \circ f : a \to d$ $\quad f : a \to b$ $\quad g : b \to c$

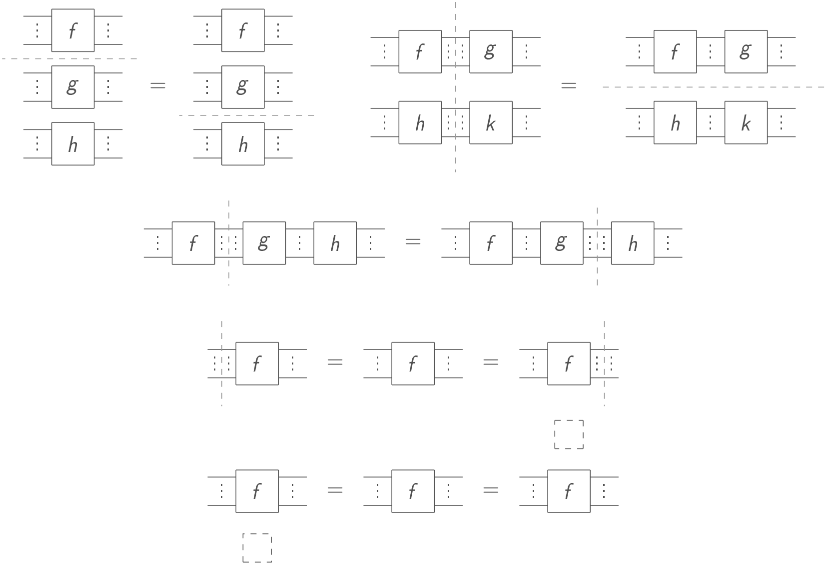


$I_n : n \to n$

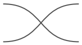


$I_0 : 0 \to 0$

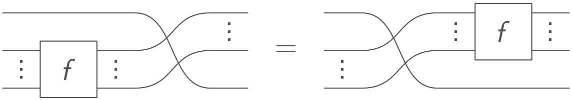
Picturing tautologies



Swaps

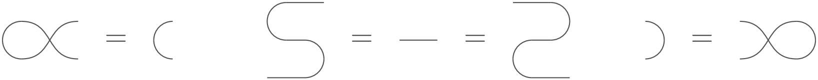


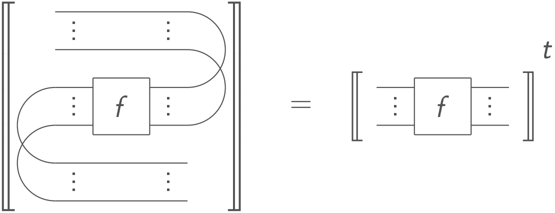
$\sigma_2 : 2 \to 2$



Cups and caps

$\eta : 0 \to 2$ $\qquad\qquad$ $\epsilon : 2 \to 0$



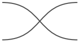


Chapter 2:

Vanilla ZX-Calculus

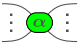
The generators of ZX-calculus

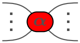
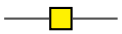
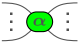
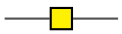
- The wires:

$[\![$ $]\!] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = |x\rangle \mapsto |x\rangle$ $[\![$  $]\!] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = |xy\rangle \mapsto |yx\rangle$

$[\![$  $]\!] = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = |00\rangle + |11\rangle$ $[\![$  $]\!] = \begin{pmatrix} 1 & 0 & 0 & 1 \end{pmatrix} = \langle 00| + \langle 11|$

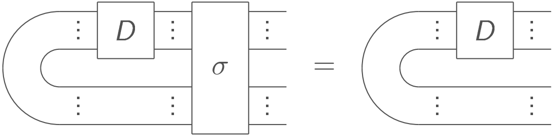
- The generators:

$[\![$  $]\!] = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ $[\![$  $]\!] = |\overrightarrow{0}\rangle\langle\overrightarrow{0}| + e^{i\alpha}|\overrightarrow{1}\rangle\langle\overrightarrow{1}|$

$[\![$  $]\!] = [\![$  $]\!]^{\otimes m} \circ [\![$  $]\!] \circ [\![$  $]\!]^{\otimes n}$

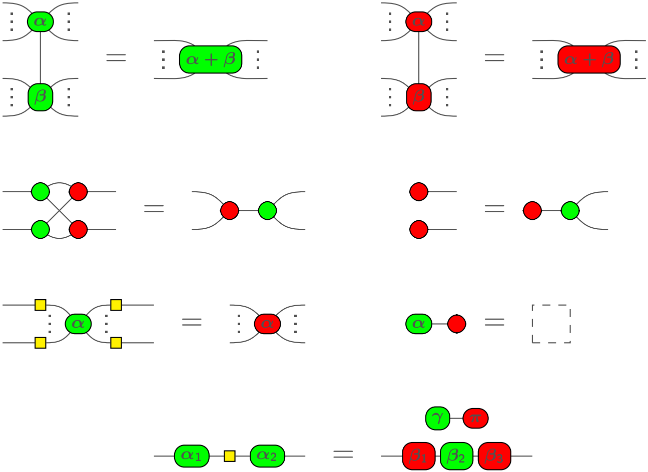
Flexsymmetry

A diagram $D : n \to m$ is said **flexsymmetric** if for all permutation $\sigma \in \mathfrak{S}_{n+m}$:

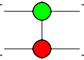


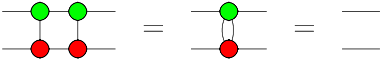
The permutation σ is obtained by composing swaps.

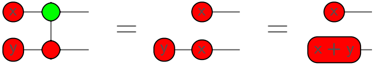
The equations of ZX-calculus

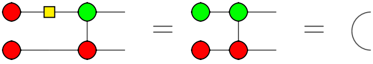


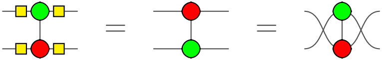
A tell of CNots

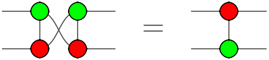
$$\llbracket$$  $$\rrbracket = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = |xy\rangle \mapsto |x\rangle|x \oplus y\rangle$$











Chapter 3:

Discard ZX-Calculus

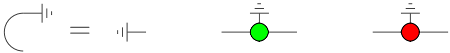
Mixed states

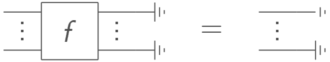
We extend our semantic to consider probabilistic mixtures of qubits.

- Density matrix ρ of the form $\rho = rr^\dagger$.
- $|x\rangle \mapsto |x\rangle\langle x|$.
- Given $D : n \to m$ we have $[\![D]\!]^{\ground} : \mathcal{M}_{2^n\times 2^n}(\mathbb{C}) \to \mathcal{M}_{2^m\times 2^m}(\mathbb{C})$ completely positive.
- Given $V : \mathbb{C}^{2^n} \to \mathbb{C}^{2^m}$ we have $\rho \mapsto V\rho V^\dagger$.

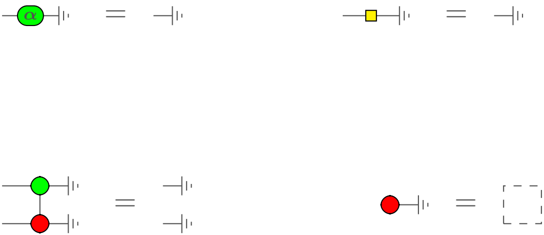
Discard map

$$[\![\, -\!\!\!\!\!\!\!\; \text{⏚} \,]\!] = \rho \mapsto Tr(\rho)$$

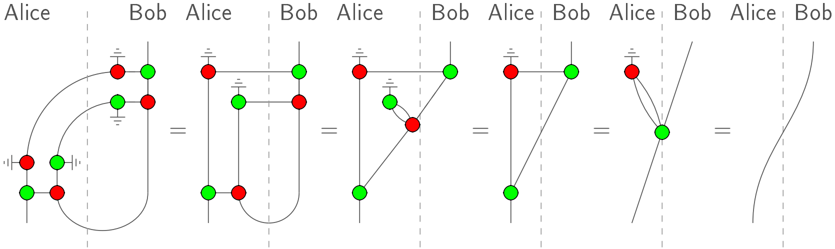




Equations of discard ZX-calculus



Quantum teleportation



Chapter 4:

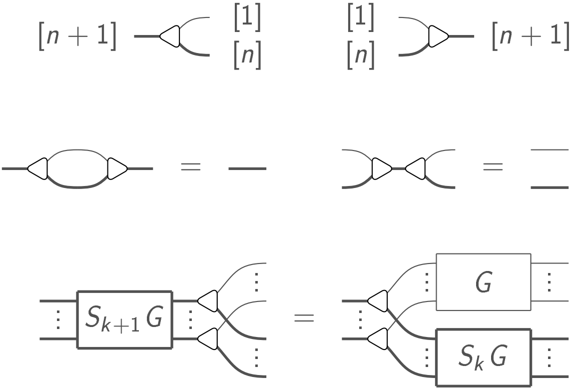
Scalable ZX-Calculus

Register types

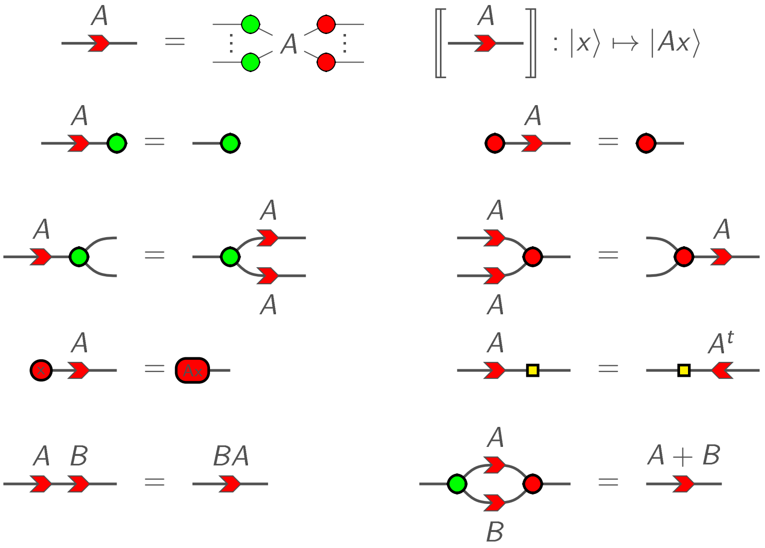
We can now gather qubits into registers using the operator $[\cdot]$.

- $[n]$ is the type of a register of size n.
- $[0] = 0$.
- $[1] + [1] \neq [2]$, however those two types are isomorphic.
- Given a gate $G : n \to m$ we can form a scaled gate $S_k G : [k]^{\otimes n} \to [k]^{\otimes m}$.

Divide and gather



Matrices

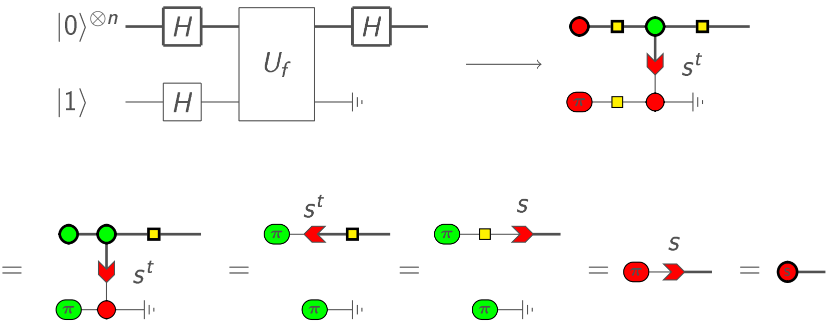


Bernstein-Vazirani algorithm

Given $U_f : |x\rangle|y\rangle \mapsto |x\rangle|f(y)\rangle$ with:

$$f : \{0,1\}^n \to \{0,1\} \qquad f : y \mapsto s^t \cdot y \qquad s \in \{0,1\}^n$$

we want to find s.



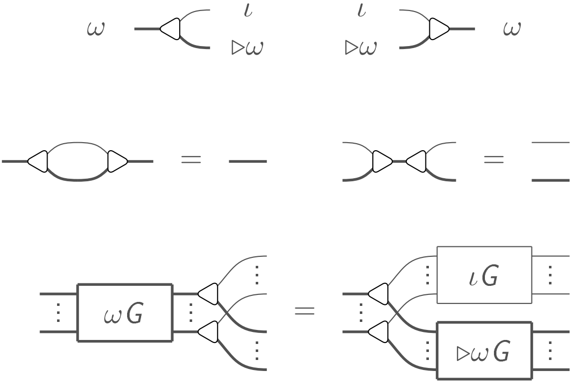
Chapter 5:

Stream ZX-Calculus

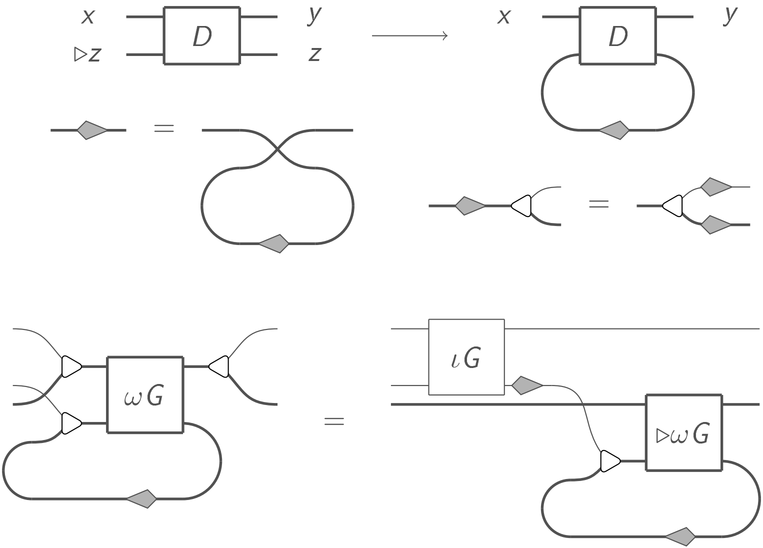
Stream types

- ι is the type of one qubit at tick 1.
- ιn is the type of n qubits at tick 1.
- Given a gate $G : n \to m$ we can form $\iota G : \iota n \to \iota m$.
- ω is the type of a stream of qubits.
- Given a gate $G : n \to m$ we can form $\omega G : \omega n \to \omega m$.
- $\triangleright^k \iota$ is the type of one qubit at tick k.
- $\triangleright^k \omega$ is the type of a stream of qubits delayed by k ticks.
- Given a diagram $D : x \to y$ we can form $\triangleright D : \triangleright x \to \triangleright y$.

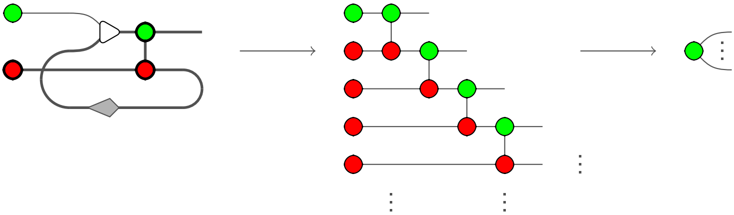
Initialization and derivation



Delayed trace

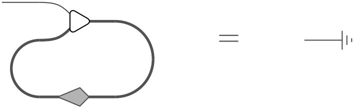


Cascades of CNots



Infinite storing

$$\frac{\Gamma, \triangleright S = \triangleright T \vdash S = T}{\Gamma \vdash S = T} \ \triangleright$$



Future?

- Iterating the construction we managed to simulate a quantum Turing machine. Work in progress.
- Toward continuous variables and infinite dimensional quantum mechanics.
- A unified theory of graphical languages (Props).
- Graphical languages as a Rosetta stone between symbolic dynamic and condensed matter physics?