Towards a classification of transitivity classes for Hom shifts

S.Gangloff*, joint work with **B.Hellouin**** and **P.Oprocha***

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Motivations

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0	0	0	0	0
0	0	0	1	0
0	0	0	0	1
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oops

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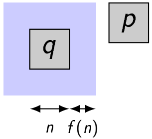
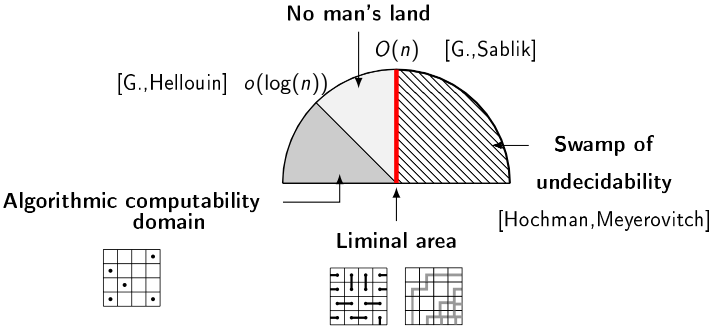
Computability : $x \in \mathbb{R}$ is computable when there is an algorithm which approximates x with elements of $\mathbb{Q}$ with arbitrary precision.

A computational 'transition' :

f-Block gluing :



Worldmap :



The question of intermediate gap functions

Question[G.,Sablik, also related by M.Hochman] : does there exist some f-block gluing bidimensional SFT with undecidable language and $\log(n) = o(f(n))$ and $f(n) = o(n)$?

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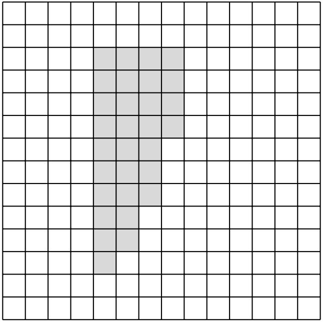

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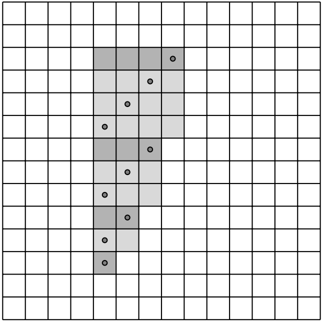

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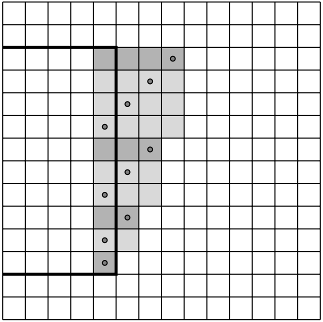

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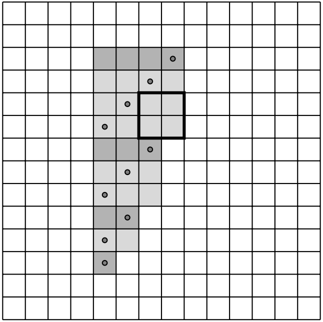

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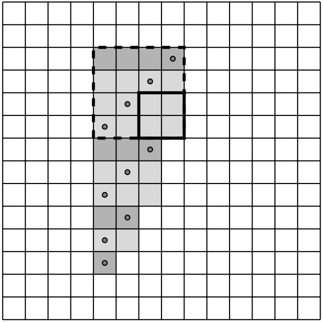

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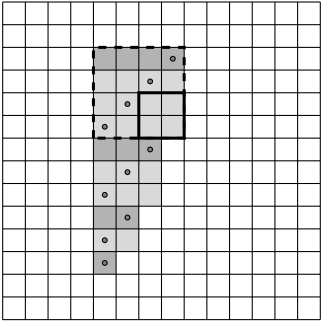

Problem : it is actually linear block gluing.

Homshifts

Homshift : SFT X_G whose forbidden patterns are :

$$\begin{array}{|c|}\hline \mathrm{a} \\ \hline \mathrm{b} \\ \hline\end{array}, \quad \begin{array}{|c|c|}\hline \mathrm{a} & \mathrm{b} \\ \hline\end{array},$$

where (a, b) not an edge in G (non-oriented simple graph).

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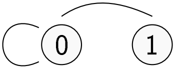

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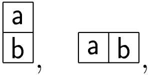

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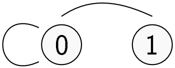

Interest : symmetries break down undecidability phenomena ; in general : the language is decidable, the entropy is computable (Friedland).

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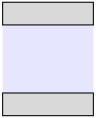

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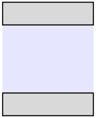

2. Gap functions $\rightarrow$ Classes for the equivalence $f \sim g$ defined by for all n :

$$c + kf(n) \leq g(n) \leq c' + k'f(n).$$

Expected result :

Theorem : *The transitivity classes for bidimensional Homshifts are* $\Theta(1), \Theta(\log(n))$ *and* $\Theta(n)$*.*

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Builds on tools developped by B.Marcus and N.Chandgotia.

For c vertex, the **universal cover** $\mathcal{U}_c(G)$ of G is the graph s.t. : **i)** vertices : $ca_1...a_k$, $k \geq 0$ without back-tracking (aba) ; **ii)** edges : $(ca_1...a_{k+1}, ca_1...a_k)$.

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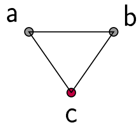

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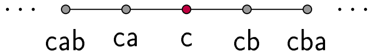
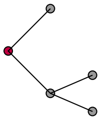
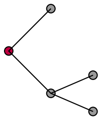
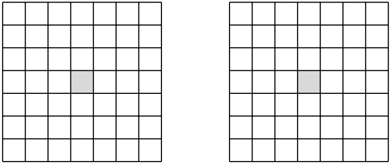






When G is square free, every pair (c, z), $z \in \mathbb{Z}^2$ defines a 'natural' function from X_G to $X_{\mathcal{U}_c(G)}$:

$y \in X_{\mathcal{U}_c(G)}$ $\qquad$ $x \in X_G$



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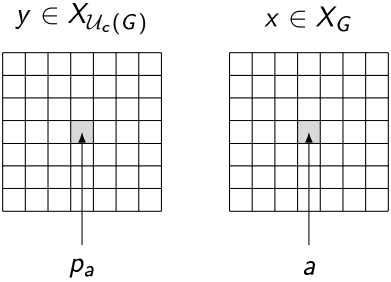

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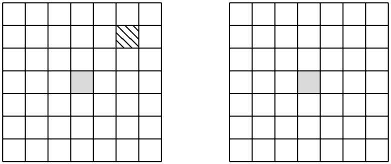

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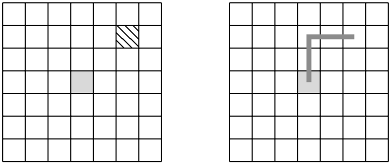

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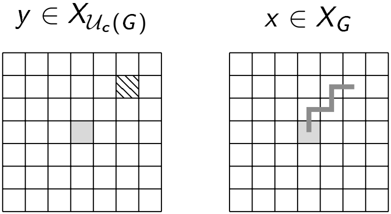

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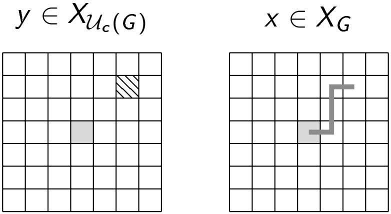

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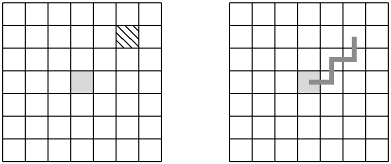

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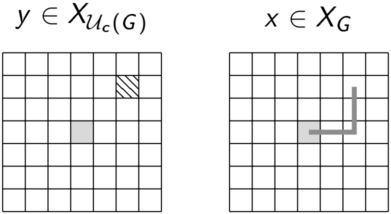

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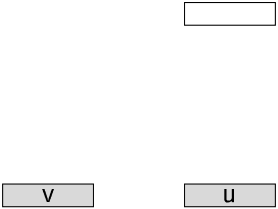

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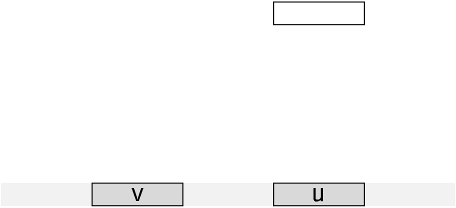

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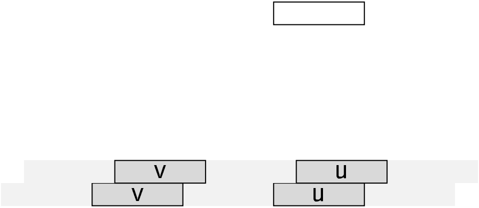

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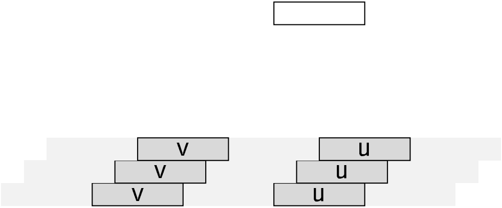

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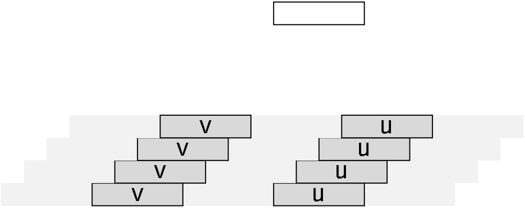

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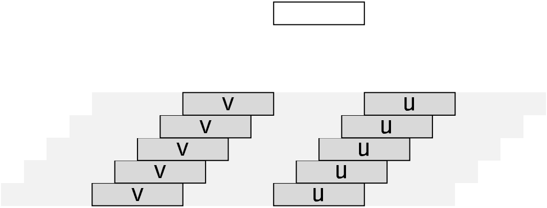

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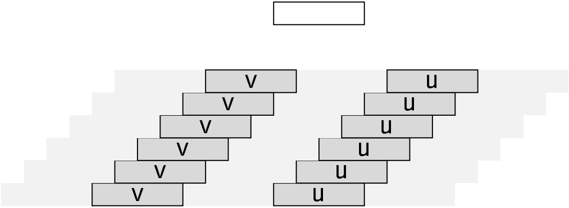

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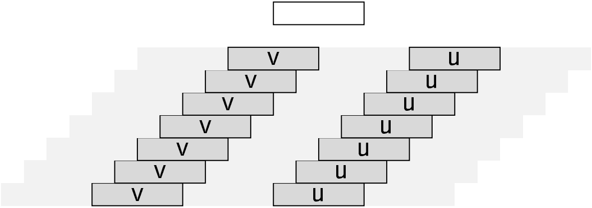

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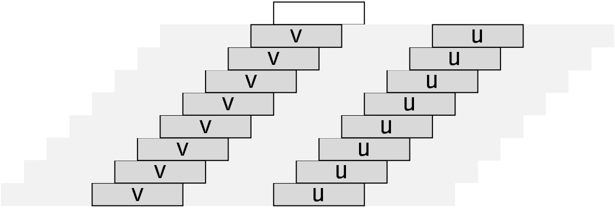

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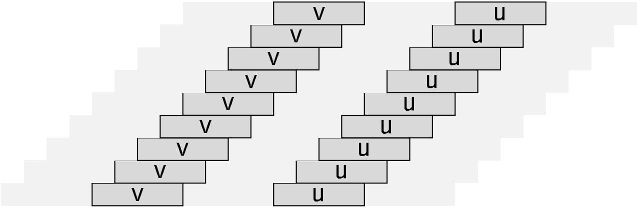

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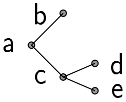
Proof : **1.** The universal cover is a finite graph. This implies that G is a finite tree.



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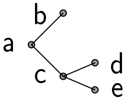

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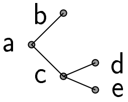

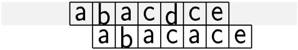

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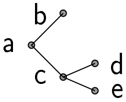

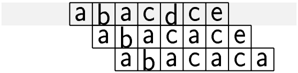

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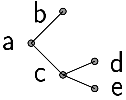

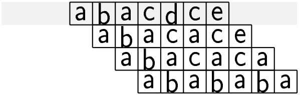

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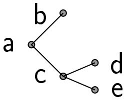

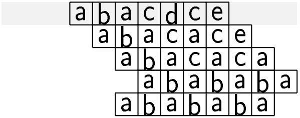

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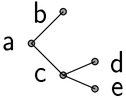

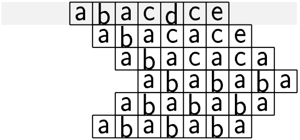

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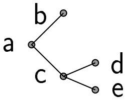

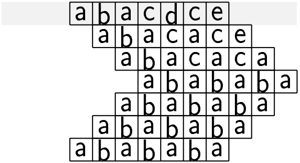

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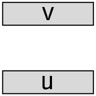

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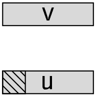

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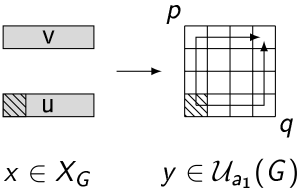

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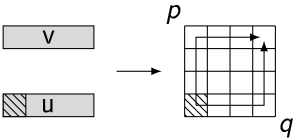

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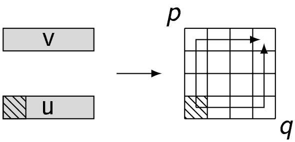

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The paths p and q have to be equal in the universal cover, which is impossible.

Our results

Pavlov and Schraudner's conjecture

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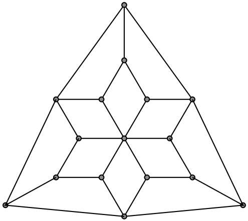
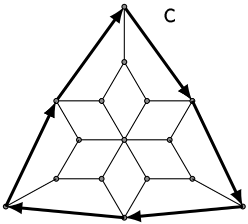
Counterexample[S.Gangloff,B.Hellouin,P.Oprocha] : The following graph K provides a counter-example :



.

Indeed, we proved that X_K is $\Theta(\log(n))$-transitive.

Proof : **1.** X_K is at least $\log(n)$-transitive.



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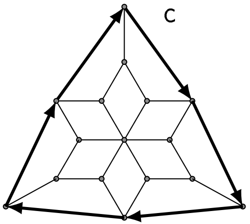

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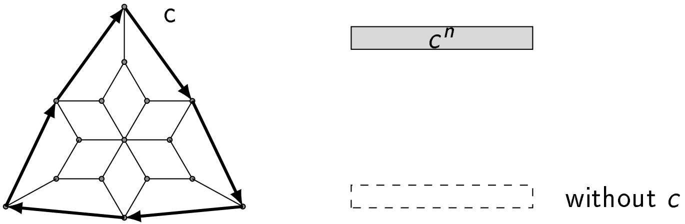

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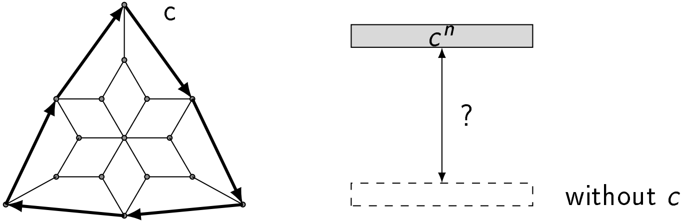

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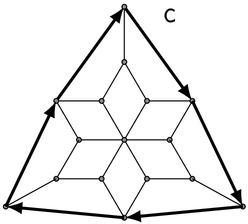

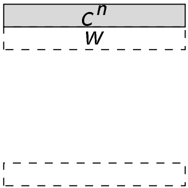

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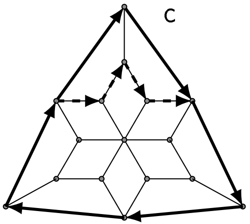

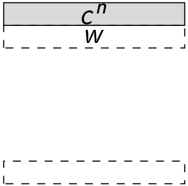

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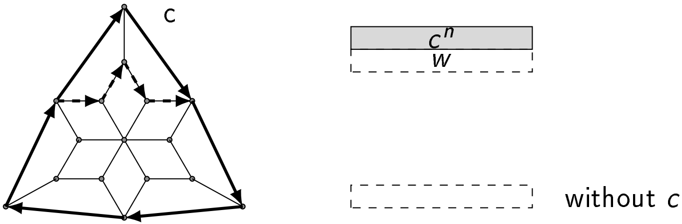

The shift is forced on the remainder of w.

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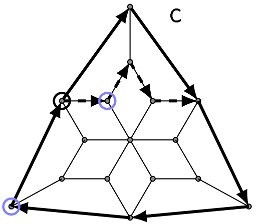

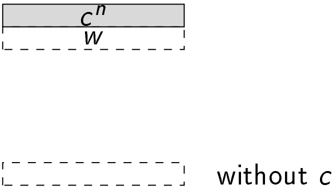

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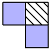

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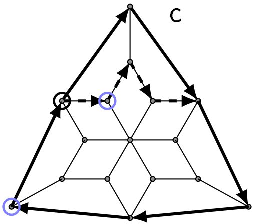

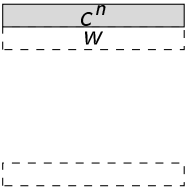

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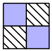

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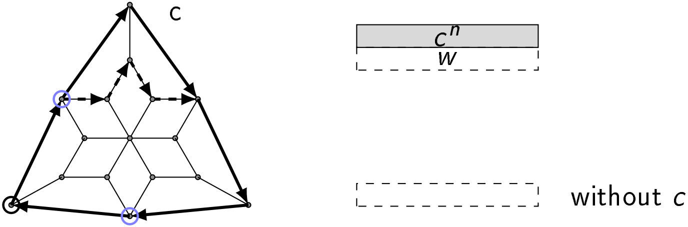

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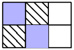

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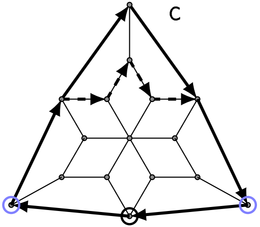

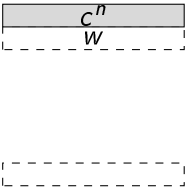

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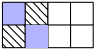

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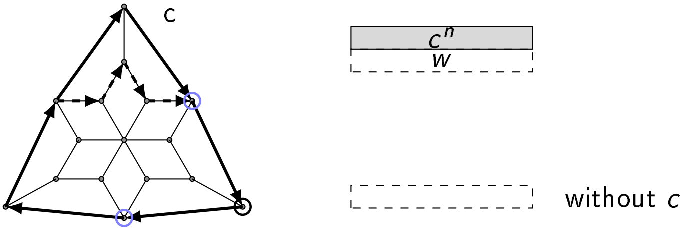

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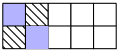

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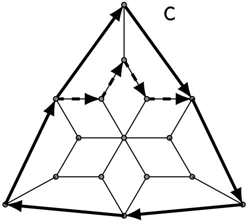

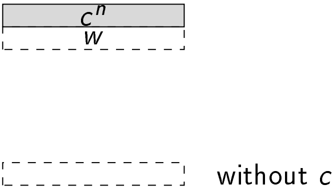

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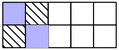

For $\mu_c(w)$ maximal size of a c-block in w : $\mu_c(w) \geq \frac{1}{2}\mu_c(c^n) - 3$.

Proof : **2.** X_K is at most $\log(n)$-transitive.

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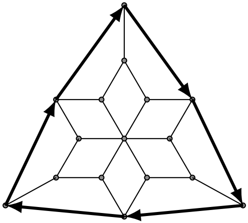

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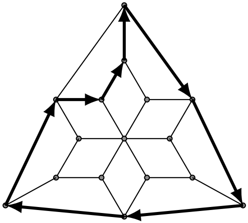

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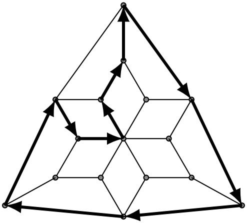

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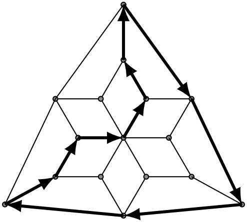

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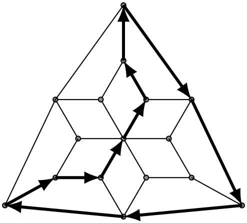

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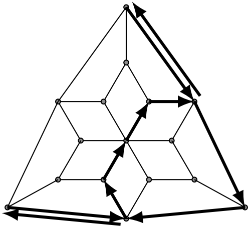

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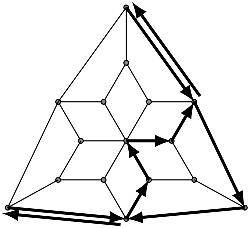

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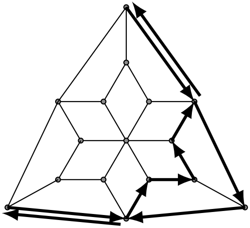

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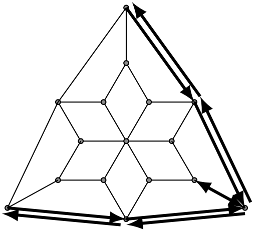

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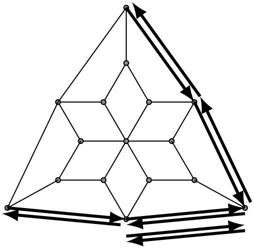

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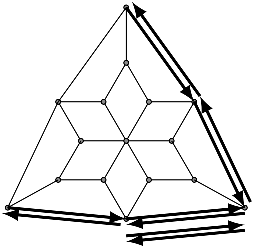
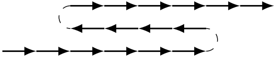
i) Procedure to smash down a simple cycle in K :



Expansion of backtracking parts :



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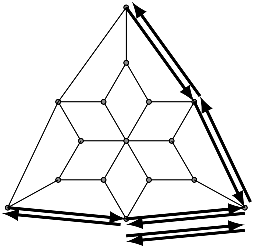

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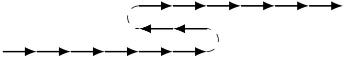

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ii) How to smash down an iterate of a cycle :

$$c \quad c \quad c \cdots c \cdots c \quad c \quad c$$

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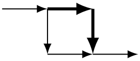

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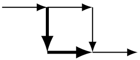

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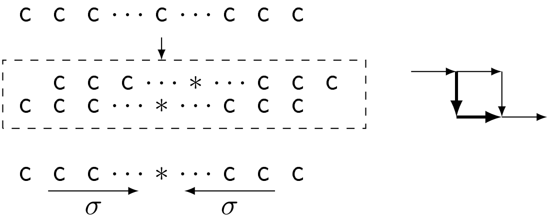

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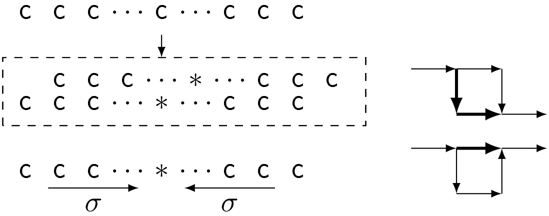

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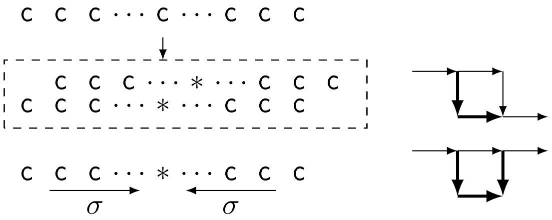

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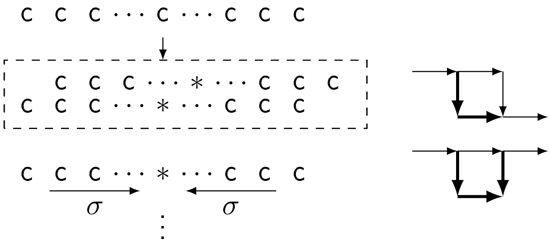

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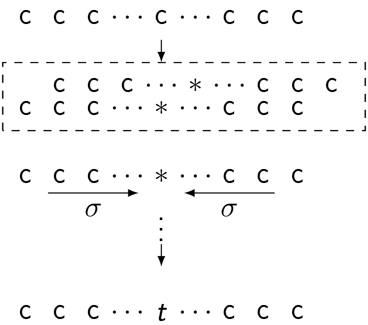

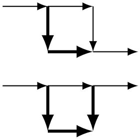

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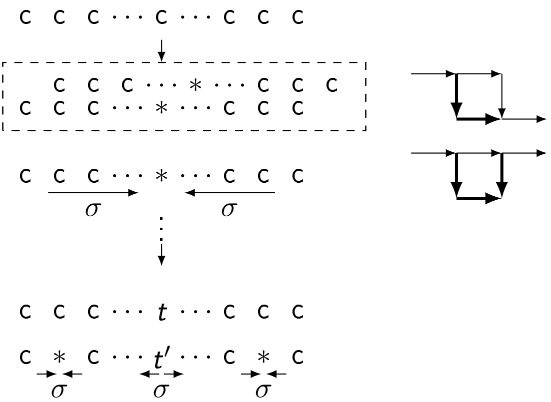

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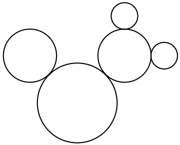

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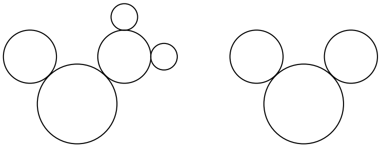

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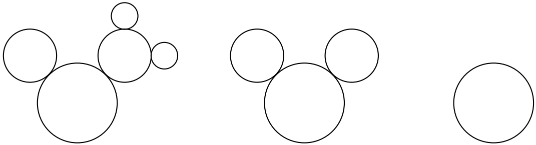

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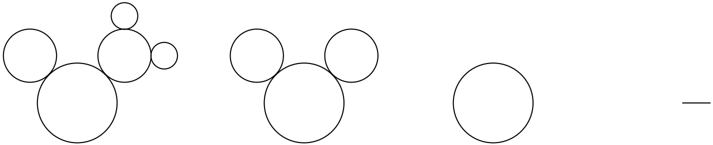

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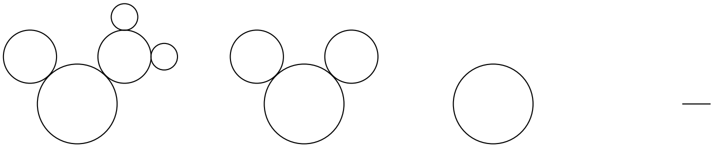
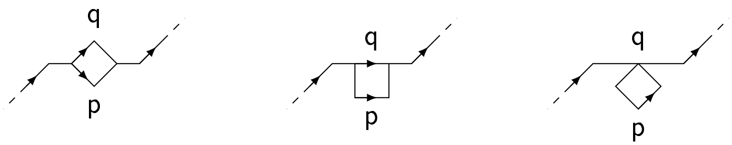


iv) Every path of even length can be transformed into a cycle in a bounded number of steps.

Quaternary cover :

Square equivalence for non-backtracking paths :



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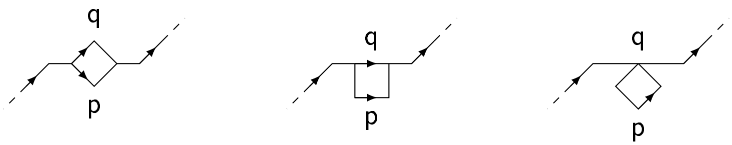
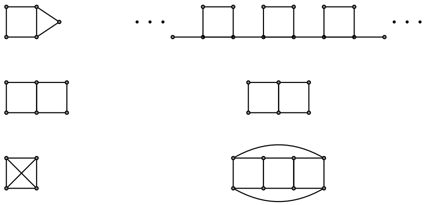


Quaternary cover : quotient of the universal cover by square equivalence.

Some examples of quaternary cover



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Lemma : the quaternary cover of a graph is always square-dismantlable.

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Theorem[S.Gangloff,B.Hellouin,P.Oprocha] : Whenever the graph G is *square dismantlable*, X_G is $O(\log(n))$-transitive.

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Furthermore :

Theorem[S.Gangloff,B.Hellouin,P.Oprocha] : Whenever the quaternary cover of G is infinite, X_G is $\Theta(n)$-transitive.

Further research

Middle term goal : Prove a similar result for the class of bidimensional SFT, or tools to produce examples between $\Theta(\log(n))$ and $\Theta(n)$.

Long term goal : What happens to the computability of entropy between $\Theta(\log(n))$ and $\Theta(n)$ for bidimensional SFT ?

Some natural short-term questions :

1. Is there an algorithm which decides, provided G, if its quaternary cover is finite or infinite ?
2. What happens when G is oriented ?
3. For shifts of finite type corresponding to graphs G_1, G_2 isomorphic ?