Subsystem decompositions of quantum circuits and processes with indefinite causal order

Julian Wechs

joint work with Cyril Branciard and Ognyan Oreshkov

CANA Seminar — Marseille

10 October 2023

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Introduction

usual understanding of causality: events are embedded into a **causal order**

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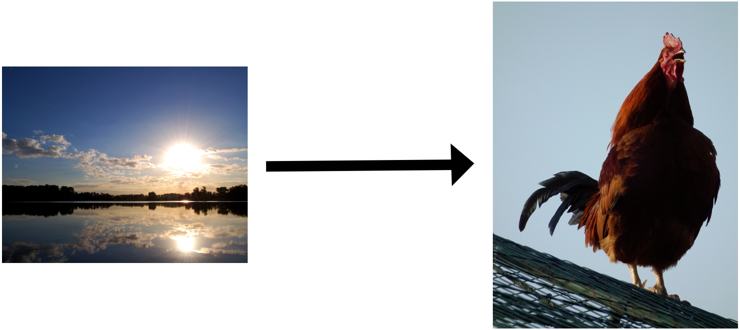

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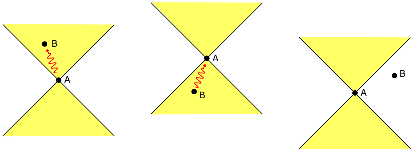

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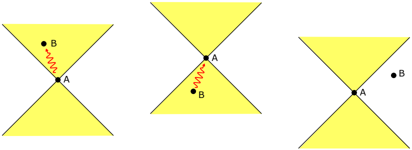

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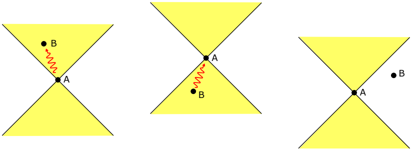

- in recent years: increasing interest in **quantum causal relations**

- abstract framework for quantum causal relations: **process matrix formalism**[1]

  $\hookrightarrow$ allows for processes that are not compatible with a well-defined causal order!

[1] O.Oreshkov, F.Costa, Č.Brukner, Nat. Commun. 3, 1092 (2012)

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- relevant from a fundamental point of view (**quantum foundations, quantum gravity**)

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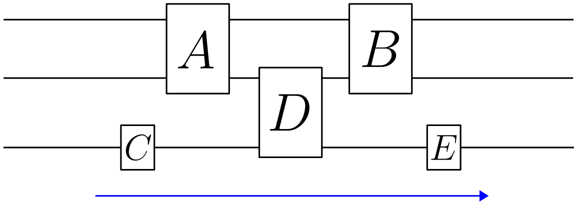
- relevant for **quantum information theory**

  $\hookrightarrow$ goes beyond the standard paradigm of **quantum circuits**



$\hookrightarrow$ new possibilities for quantum computing?

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- central open question: physical realisability of indefinite causal order?

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- some processes with indefinite causal order are believed to have a physical realisation in standard quantum theory

  $\hookrightarrow$ optical laboratory experiments[1,2,3,4,5,6,7]

  $\hookrightarrow$ controversy: Genuine "realisations" or "simulations" of indefinite causal order?

[1] L. M. Procopio et al., Nat. Commun. 6, 7913 (2015)

[2] G. Rubino et al., Sci. Adv.3, e1602589 (2017)

[3] K. Goswami et al., Phys. Rev. Lett. 121, 090503 (2018)

[4] K. Wei et al., Phys. Rev. Lett. 122, 120504 (2019)

[5] Y. Guo et al., Phys. Rev. Lett. 124, 030502 (2020)

[6] K. Goswami et al., Phys. Rev. Research 2, 033292 (2020)

[7] M. M. Taddei et al., PRX Quantum 2, 010320 (2021)

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  $\Rightarrow$ In which precise sense does indefinite causal order exist within standard quantum theory?

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Rigorous approach:

Standard quantum description	$\Longleftrightarrow$ **Change of subsystems in quantum circuits**	Description as indefinite causal order process

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Standard quantum description $\Longleftrightarrow$ **Change of subsystems in quantum circuits** $\Longleftrightarrow$ Description as indefinite causal order process

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- application to processes with indefinite causal order

$\hookrightarrow$ J. Wechs, C. Branciard, O. Oreshkov, Existence of processes violating causal inequalities on time-delocalised subsystems, Nat. Commun. 14, 1471 (2023)

$\hookrightarrow$ J. Wechs, O. Oreshkov, Subsystem decompositions of quantum circuits and processes with indefinite causal order, in preparation (2023)

Indefinite causal order: The process matrix framework[1]

[1] O.Oreshkov, F.Costa, Č.Brukner, Nat. Commun. 3, 1092 (2012)

The process matrix framework: General idea[1]

- consider separate parties (Alice, Bob, . . .)

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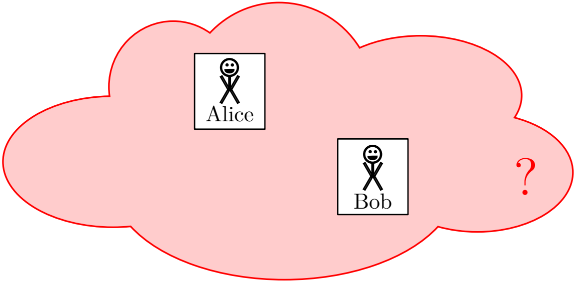
The process matrix framework: General idea[1]

- consider separate parties (Alice, Bob, . . .)
- **locally** described by quantum theory, but no a priori **global** causal order



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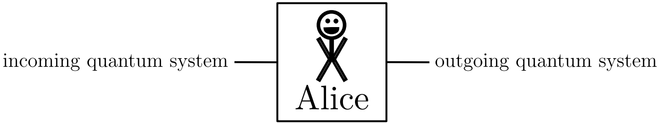
The process matrix framework: Local quantum theory



quantum operation
$\Downarrow$
measurement outcome

- Alice receives an incoming quantum system
- performs a quantum operation (quantum channel, quantum measurement, ...)

  $\hookrightarrow$ obtains a (probabilistic) measurement outcome
- sends out an outgoing quantum system

The process matrix framework: Local quantum theory

Formally:

- incoming and outgoing quantum systems A_I (associated to Hilbert space $\mathcal{H}^{A_I}$) and A_O (associated to Hilbert space $\mathcal{H}^{A_O}$)

The process matrix framework: Local quantum theory

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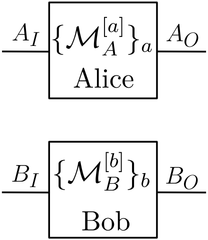
- incoming and outgoing quantum systems A_I (associated to Hilbert space $\mathcal{H}^{A_I}$) and A_O (associated to Hilbert space $\mathcal{H}^{A_O}$)
- **quantum instrument** $\{\mathcal{M}_A^{[a]}\}_a, \quad a = 1, \ldots, N$

  $\hookrightarrow$ probability associated to outcomes: $p(a) = \mathrm{Tr}(\mathcal{M}^{[a]}(\rho^{A_I}))$

  $\hookrightarrow$ corresponding output state: $\mathcal{M}^{[a]}(\rho^{A_I})/p(a) \in \mathcal{L}(\mathcal{H}^{A_O})$

$$[\mathcal{M}_A^{[a]} : \mathcal{L}(\mathcal{H}^{A_I}) \to \mathcal{L}(\mathcal{H}^{A_O}) \text{ completely positive,}$$

$$\mathrm{Tr}\left(\sum_a \mathcal{M}_A^{[a]}(\rho^{A_I})\right) = \mathrm{Tr}(\rho^{A_I}) \quad \forall \rho^{A_I} \in \mathcal{L}(\mathcal{H}^{A_I})]$$



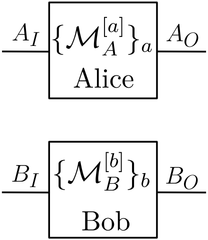
Most general correlations: obtained by "**generalised Born's rule**"

$$P(a,b) = \mathrm{Tr}\left[M_A^{[a]} \otimes M_B^{[b]} \cdot W\right]$$

[$\hookrightarrow M_A^{[a]} \in \mathcal{L}(\mathcal{H}^{A_I} \otimes \mathcal{H}^{A_O})$: Choi representation[1]]

[1] M.D.Choi, Linear Algebra Appl. 10, 285 (1975)

The process matrix



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$W \in \mathcal{L}(\mathcal{H}^{A_I} \otimes \mathcal{H}^{A_O} \otimes \mathcal{H}^{B_I} \otimes \mathcal{H}^{B_O})$: **process matrix**

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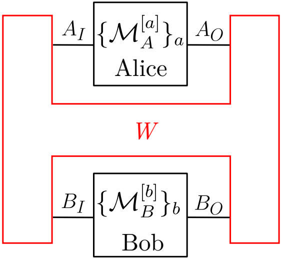

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$\hookrightarrow$ "physical resource" or "environment" that relates the parties

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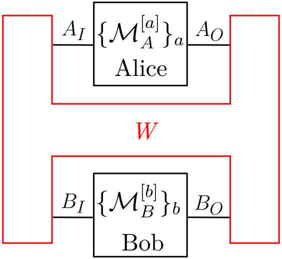

$$P(a,b) = \mathrm{Tr}\left[M_A^{[a]} \otimes M_B^{[b]} \cdot W\right]$$

Only constraint: valid probabilities $\Leftrightarrow$ process matrices must be:

- positive semidefinite: $W \geq 0$
- in the linear subspace of valid process matrices
  $W \in \mathcal{L}_V \subset \mathcal{L}(\mathcal{H}^{A_I} \otimes \mathcal{H}^{A_O} \otimes \mathcal{H}^{B_I} \otimes \mathcal{H}^{B_O})$
- normalised: $\mathrm{Tr}\, W = d_{A_O} d_{B_O}$

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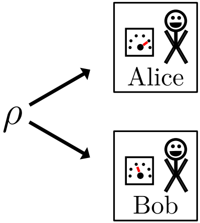
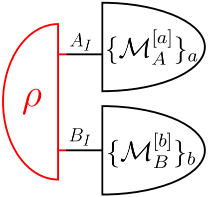
Examples for process matrices

$$P(a,b) = \mathrm{Tr}\left[M_A^{[a]} \otimes M_B^{[b]} \cdot W\right]$$

- **state**: no signaling between the parties





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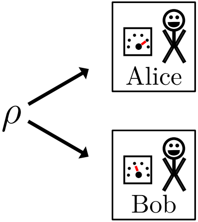
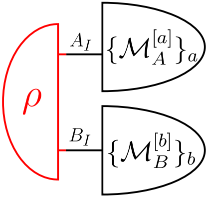
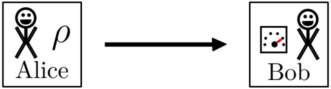
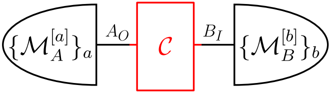




- **channel**: one-way signaling from A to B





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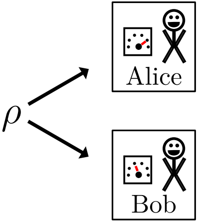

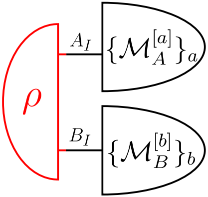

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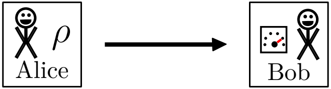
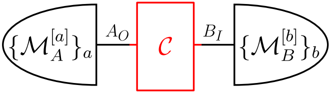
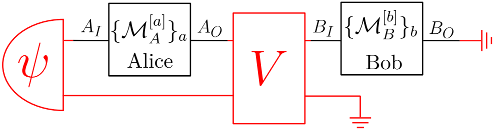




- more general possibilities...

Causally separable process matrices

- process matrices that do not allow Bob to signal to Alice $\equiv$ standard quantum circuits with A before B [1,2]



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[2] G. Gutoski, J. Watrous, Proceedings of 39th ACM STOC, 565-574 (2007)

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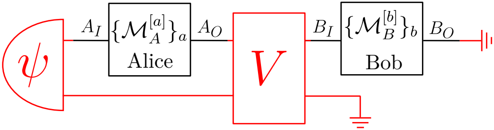

- process matrices that do not allow Alice to signal to Bob $\equiv$ standard quantum circuits with B before A

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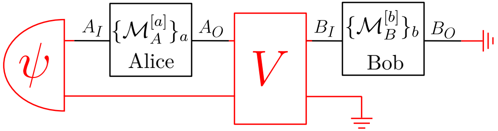

- process matrices that do not allow Alice to signal to Bob
  $\equiv$ standard quantum circuits with B before A
- probabilistic mixtures:

$$W^{\mathsf{sep}} = q \cdot W^{A \prec B} + (1-q) \cdot W^{B \prec A}, \quad q \in [0,1]$$

$\equiv$ **causally separable** process matrices[3]

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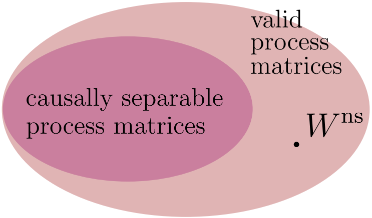
Causally nonseparable process matrices

- there are valid process matrices that are not causally separable![1,2]



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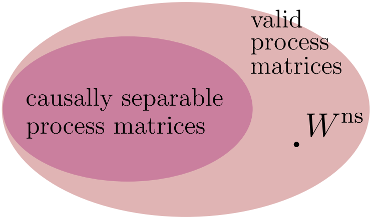

- some causally nonseparable process matrices can generate correlations $P(a, b|x, y)$ that **violate causal inequalities**[1,3]

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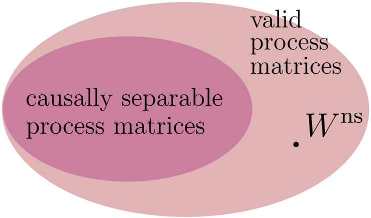

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[Analogy: causal nonseparability ⇔ entanglement
causal inequalities ⇔ Bell inequalities]

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[2] J. Wechs, A. Abbott, C. Branciard, New J. Phys. 21, 013027 (2019)
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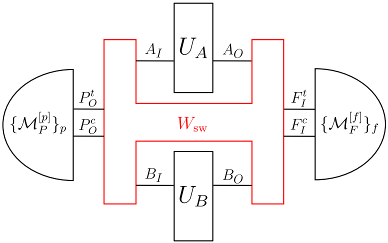
Physical realisability of indefinite causal order?

Quantum switch[1]: fourpartite causally nonseparable process matrix[2,3] (Alice + Bob + initial party + final party)



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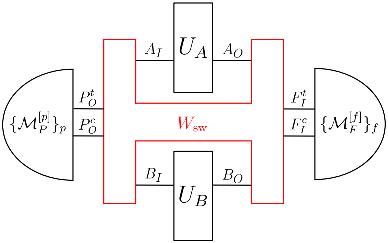

- interpretation: **quantum control** of causal order

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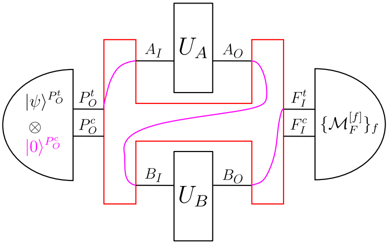

- interpretation: **quantum control** of causal order
- initial party initialises a “target” qubit and a “control” qubit

$\hookrightarrow$ control qubit in state $|0\rangle$: Alice acts on target qubit before Bob

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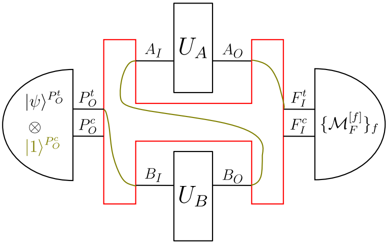

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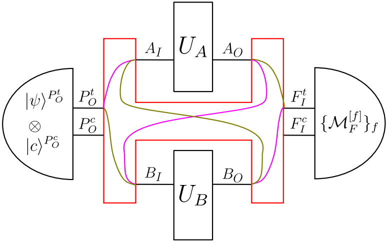

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$\hookrightarrow$ control qubit in a **superposition state** $|c\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}}$: no well-defined causal order

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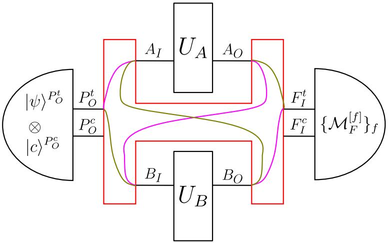
The quantum switch



- information processing advantages for the switch have been identified (e.g. in query complexity[1,2], communication complexity[3])

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[2] M.Araújo, F.Costa, Č.Brukner, Phys. Rev. Lett. 113, 250402 (2014)

[3] P.A.Guérin, A.Feix, M.Araújo, Č.Brukner, Phys. Rev. Lett. 117, 100502 (2016)

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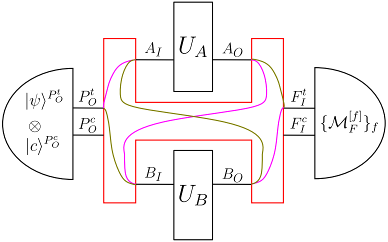

- information processing advantages for the switch have been identified (e.g. in query complexity[1,2], communication complexity[3])
- the quantum switch cannot violate a causal inequality[4,5,6]

[1] G. Chiribella, Phys. Rev. A 86, 040301 (2012)

[2] M.Araújo, F.Costa, Č.Brukner, Phys. Rev. Lett. 113, 250402 (2014)

[3] P.A.Guérin, A.Feix, M.Araújo, Č.Brukner, Phys. Rev. Lett. 117, 100502 (2016)

[4] M.Araujo et al., New J. Phys. 17, 102001 (2015)

[5] O. Oreshkov, C. Giarmatzi, New J. Phys. 18, 093020 (2016)

[6] J. Wechs, H. Dourdent, A. Abbott, C. Branciard, PRX Quantum 2, 030335 (2021)

Physical realisability of indefinite causal order?

In what physical situations does indefinite causal order occur?

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- possible scenarios at the interface of quantum theory and gravity?

  $\hookrightarrow$ "gravitational quantum switch"[1]

[1] M.Zych, F.Costa, I.Pikovski, Č.Brukner, Nat. Commun. 10, 3772 (2019)

Physical realisability of indefinite causal order?

In what physical situations does indefinite causal order occur?

- possible scenarios at the interface of quantum theory and gravity?

  $\hookrightarrow$ “gravitational quantum switch”[1]

- optical laboratory experiments[2,3,4,5,6,7,8]?

[1] M.Zych, F.Costa, I.Pikovski, Č.Brukner, Nat. Commun. 10, 3772 (2019)

[2] L. M. Procopio et al., Nat. Commun. 6, 7913 (2015)

[3] G. Rubino et al., Sci. Adv.3, e1602589 (2017)

[4] K. Goswami et al., Phys. Rev. Lett. 121, 090503 (2018)

[5] K. Wei et al., Phys. Rev. Lett. 122, 120504 (2019)

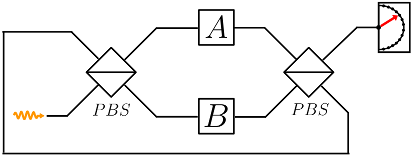
[6] Y. Guo et al., Phys. Rev. Lett. 124, 030502 (2020)

[7] K. Goswami et al., Phys. Rev. Research 2, 033292 (2020)

[8] M. M. Taddei et al., PRX Quantum 2, 010320 (2021)

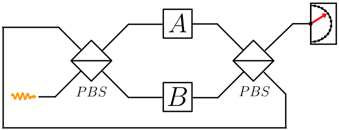
Optical experiments for the quantum switch

$\hookrightarrow$ interferometric experiments:

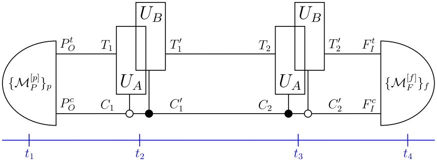


- control qubit: photon polarisation
- target qubit: another degree of freedom of the photon (e.g. orbital angular momentum)
- photon sent through an interferometer with polarising beam splitters (PBS) along two possible paths

Optical experiments: “Realisations” or “simulations”?



$\hookrightarrow$ temporal perspective: coherently controlled application of U_A and U_B at two possible times



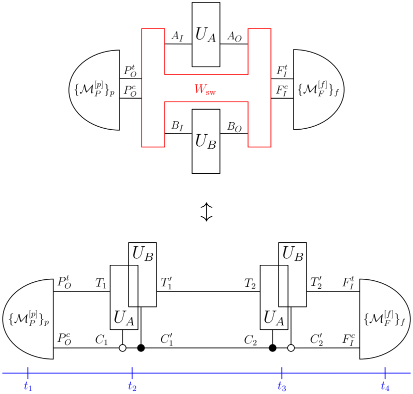
$\hookrightarrow$ debate in the community: Are such experiments genuine “realisations” or “simulations” of the quantum switch (see e.g.[1,2,3])?

[1] O. Oreshkov, Quantum 3, 206 (2019)

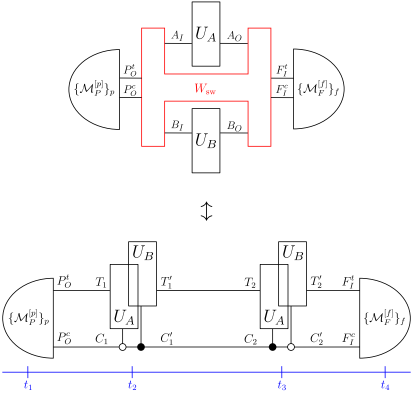
[2] N. Paunkovic, M. Vojinovic, Quantum 4, 275 (2020)

[3] V. Vilasini, R. Renner, arXiv:2203.11245 [quant-ph]

Link between temporal, standard quantum description and abstract process matrix framework?



Link between temporal, standard quantum description and abstract process matrix framework?



$\hookrightarrow$ related by a **change of subsystems**! (cf.[1,2])

[1] O. Oreshkov, Quantum 3, 206 (2019)

[2] J. Wechs, C. Branciard, O. Oreshkov, Nat. Commun. 14, 1471 (2023)

Remainder of this talk

$\hookrightarrow$ general framework to describe transformations between different subsystem decompositions of quantum circuits[1]

$\hookrightarrow$ application to processes with indefinite causal order[1]

[1] J. Wechs, O. Oreshkov, in preparation

Subsystem decompositions of quantum circuits

Quantum circuits

- **quantum circuit**: Abstract description of time evolution in quantum theory

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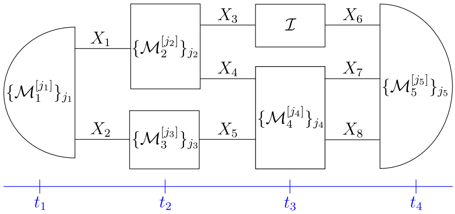
  $\hookrightarrow$ **quantum operations**, represented by **boxes**, which are composed over **quantum systems**, in successive time steps



Quantum circuits

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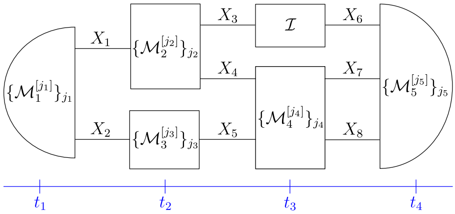
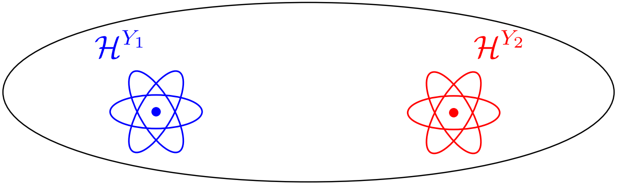


- closed circuit: Composition of all operations corresponds to the joint probability $P(j_1, j_2, j_3, j_4, j_5, j_6)$ of the measurement outcomes

Quantum subsystems

- composite quantum system: described by the **tensor product** of the Hilbert spaces of the individual systems



$\mathcal{H}^{Y_1} \otimes \mathcal{H}^{Y_2}$

Quantum subsystems

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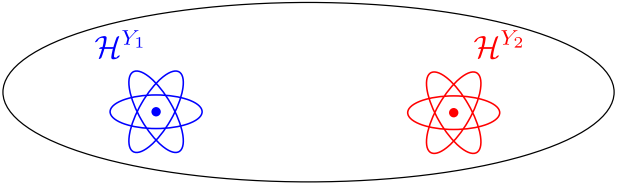

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- formally described through the choice of a **tensor product structure**, i.e., an isomorphism

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(with $\Pi_{i=1}^{n} \dim \mathcal{H}^{Y_n} = \dim \mathcal{H}^Y$).

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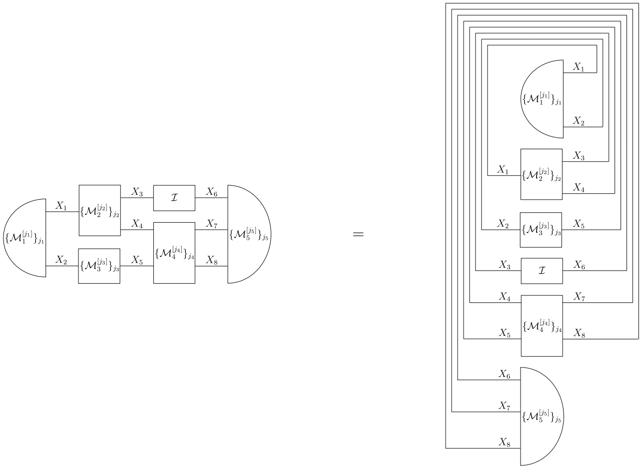
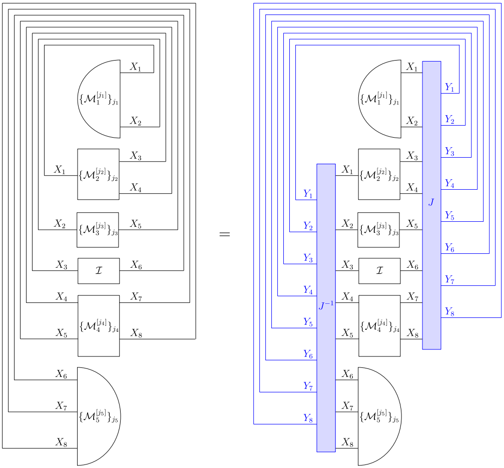
(with $\Pi_{i=1}^{n} \dim \mathcal{H}^{Y_n} = \dim \mathcal{H}^Y$).

$\hookrightarrow$ establishes a notion of locality on $\mathcal{H}^Y$, and defines a decomposition of the system Y into subsystems $Y_1, \ldots, Y_n$

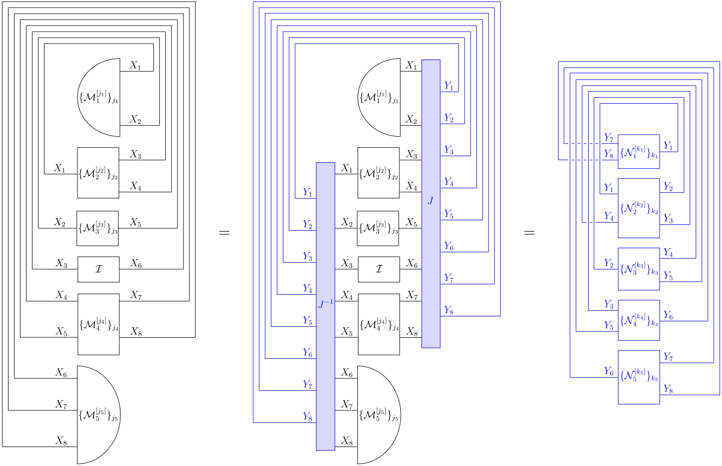


"circuit operation" consisting of the tensor product of all operations
$\rightarrow$ acts on the joint Hilbert space of all systems in the circuit



alternative subsystem decomposition $\rightarrow$ isomorphism J defining another tensor factor decomposition of that joint Hilbert space

Subsystem decompositions of quantum circuits



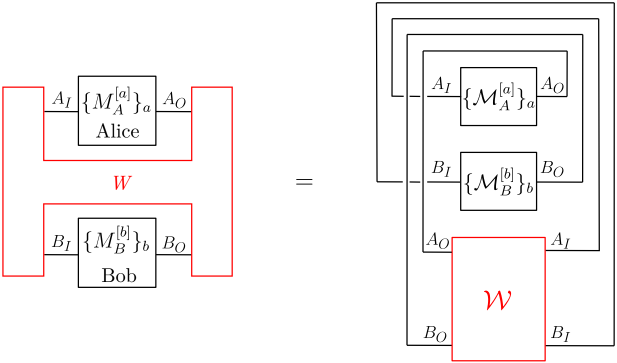
new (possibly cyclic) circuit description with operations acting on new (possibly time-delocalised[1]) systems

[1] O. Oreshkov, Quantum 3, 206 (2019)

Application to processes with indefinite causal order

Quantum processes as circuits with cycles

- quantum processes can be interpreted as circuits with cycles



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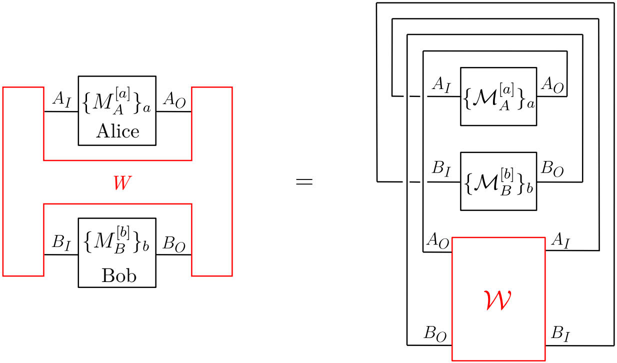
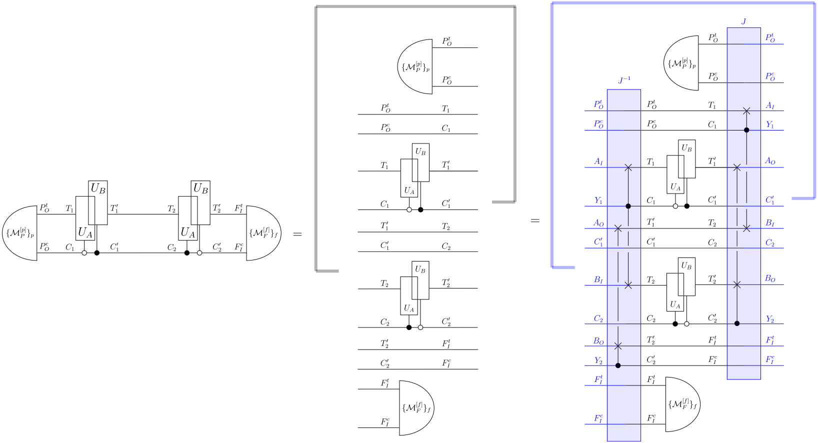


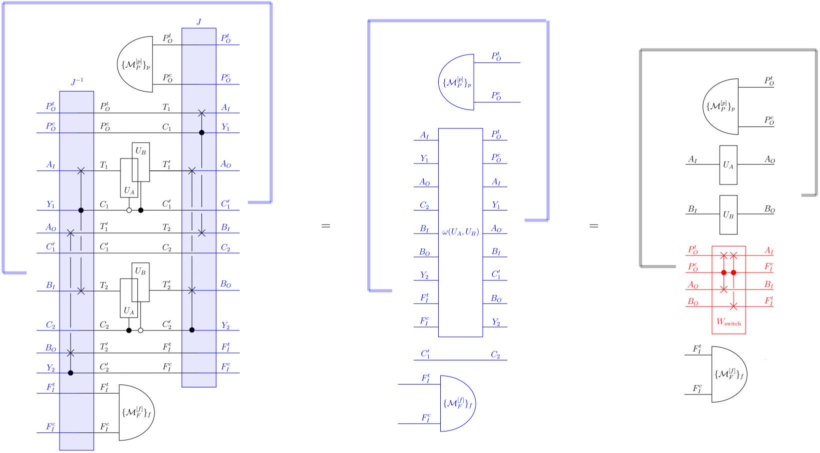
- certain indefinite causal order processes can be related to a temporal circuit via a subsystem transformation

Example: The quantum switch



$\hookrightarrow$ input and output systems A_I, A_O, B_I, B_O in the process matrix description: **Time-delocalised subsystems** of the time-local systems in the temporal circuit

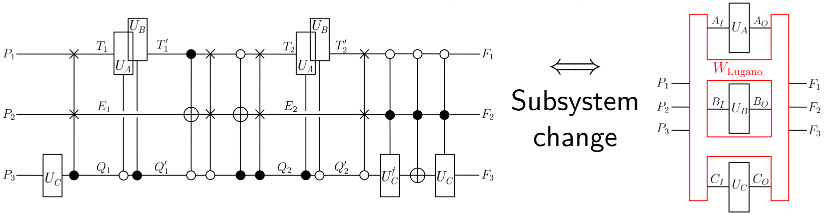
Example: The quantum switch



$\hookrightarrow$ new subsystem description $\equiv$ “fine-grained” process matrix perspective (need to compose over the systems Y_1, Y_2, C_1', C_2)

Certain processes that violate causal inequalities can be mapped to a temporal circuit through a subsystem change.[1]

↪ example: the "**Lugano process**"(see e.g.[2,3])



↪ requires new types of time-delocalised systems

↪ causal inequality violation with classical "**time-delocalised variables**"[1]

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[1] J.Wechs, C.Branciard, O.Oreshkov, Nat. Commun. 14, 1471 (2023)

[2] Ä.Baumeler, S.Wolf, New J. Phys. 18, 013036 (2016)

[3] M.Araújo, A.Feix, M.Navascués, Č.Brukner, Quantum 1, 10 (2017).

Conclusion and open questions

Certain processes with indefinite causal order can be mapped to a standard, temporal quantum circuit through a subsystem change. In that sense, they have a realisation within standard physics.

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- generalisations to other types of processes?
- transformations between "causal perspectives" and link to quantum reference frames/quantum equivalence principle?[1,2]

[1] E.Castro-Ruiz, F.Giacomini, A.Belenchia, Č. Brukner, Nat. Commun. 11, 2672 (2020)

[2] L.Hardy, arXiv:1903.01289 [quant-ph]

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- generalisations to other types of processes?
- transformations between "causal perspectives" and link to quantum reference frames/quantum equivalence principle?[1,2]
- implications of this perspective on quantum information processing with indefinite causal structures?

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Thank you for your attention!