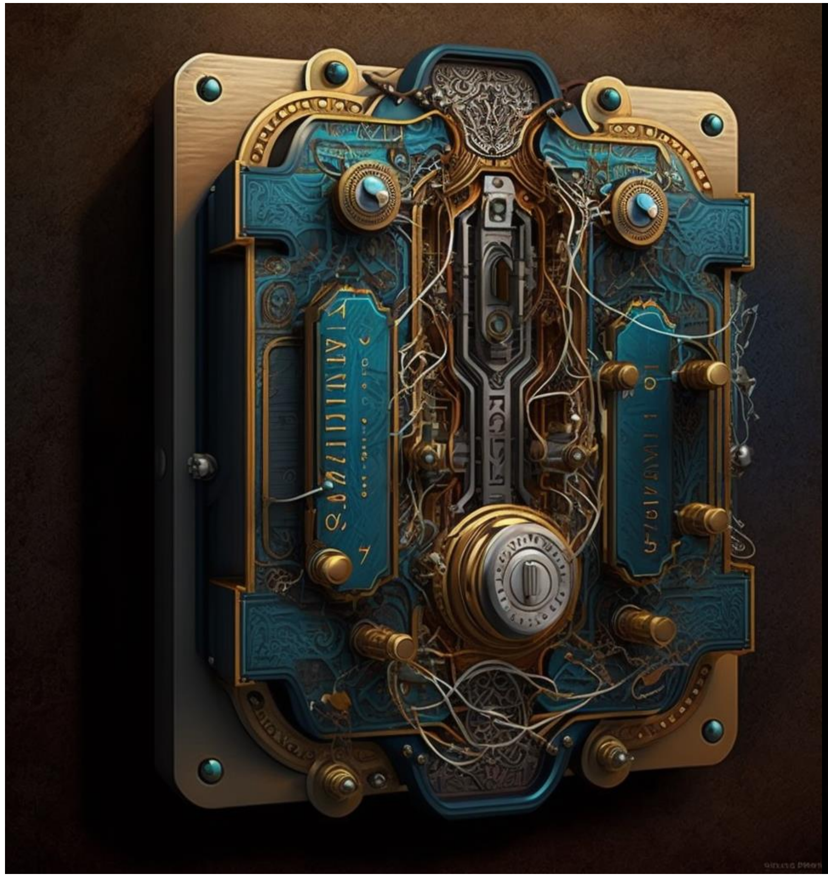
All You Need to Know about the Quantum Switch

Hippolyte Dourdent, ICFO, QIT group, Barcelona

Credits: Midjourney (and all the artists who did not give their consent for the use of their work)

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30/01/2024

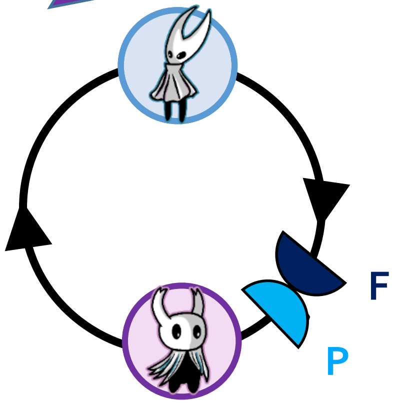
The Quantum Switch, a causal Schrödinger's cat

- Schrödinger's cat: a quantum control of the cat's « alive/dead » state.

« Control » *« Target »*

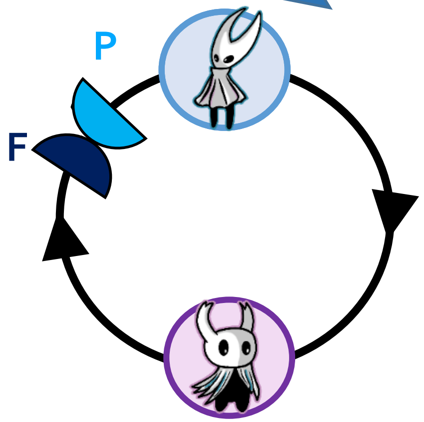
$$\frac{|0\rangle \otimes |alive\rangle + |1\rangle \otimes |dead\rangle}{\sqrt{2}}$$

- The quantum switch: a quantum control of the causal order between Alice and Bob's events
  [Chiribella, D'Ariano & Perinotti,PRA 88, 022318 (2013)]



« Control » *« Target »*

$$\frac{|0\rangle \otimes |"Bob \prec Alice"\rangle + |1\rangle \otimes |"Alice \prec Bob"\rangle}{\sqrt{2}}$$



Bob is in the past of Alice

Alice is in the past of Bob

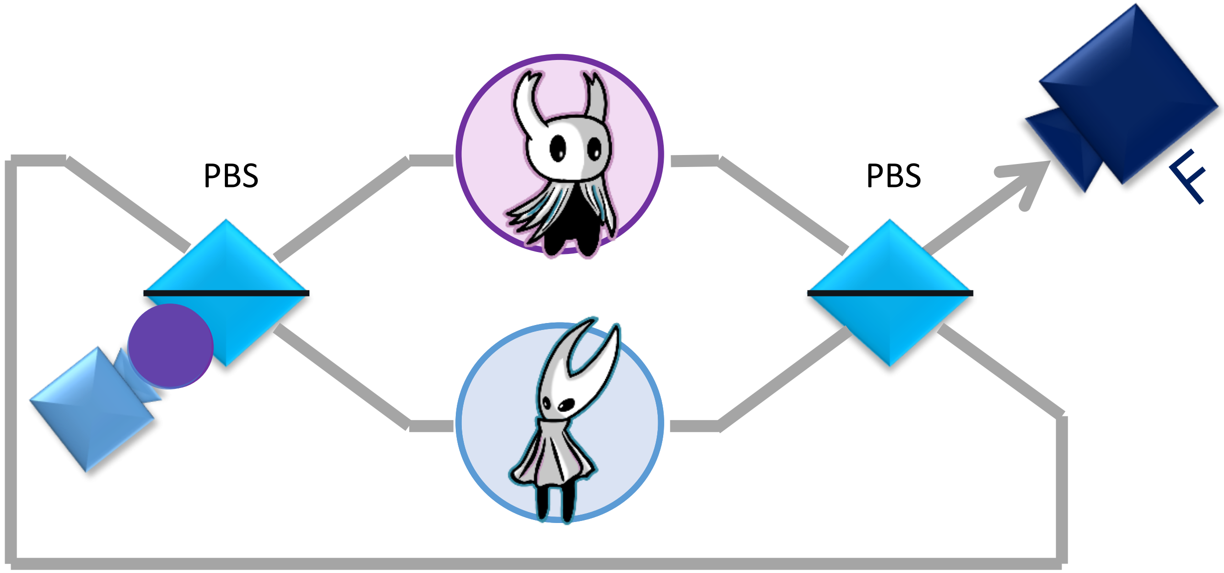
The Quantum Switch: Basic Recipe

- A coumpound system: « control qubit » – « target qubit »
- Three players: Alice and Bob act on the « target » ; Fiona receives both control and target at the end.
- The order of Alice and Bob's operations on the target is coherently controlled by the « control qubit ».

⚠ Crucial feature ⚠

Alice and Bob each acts once and only once, but their causal order is quantumly controlled !

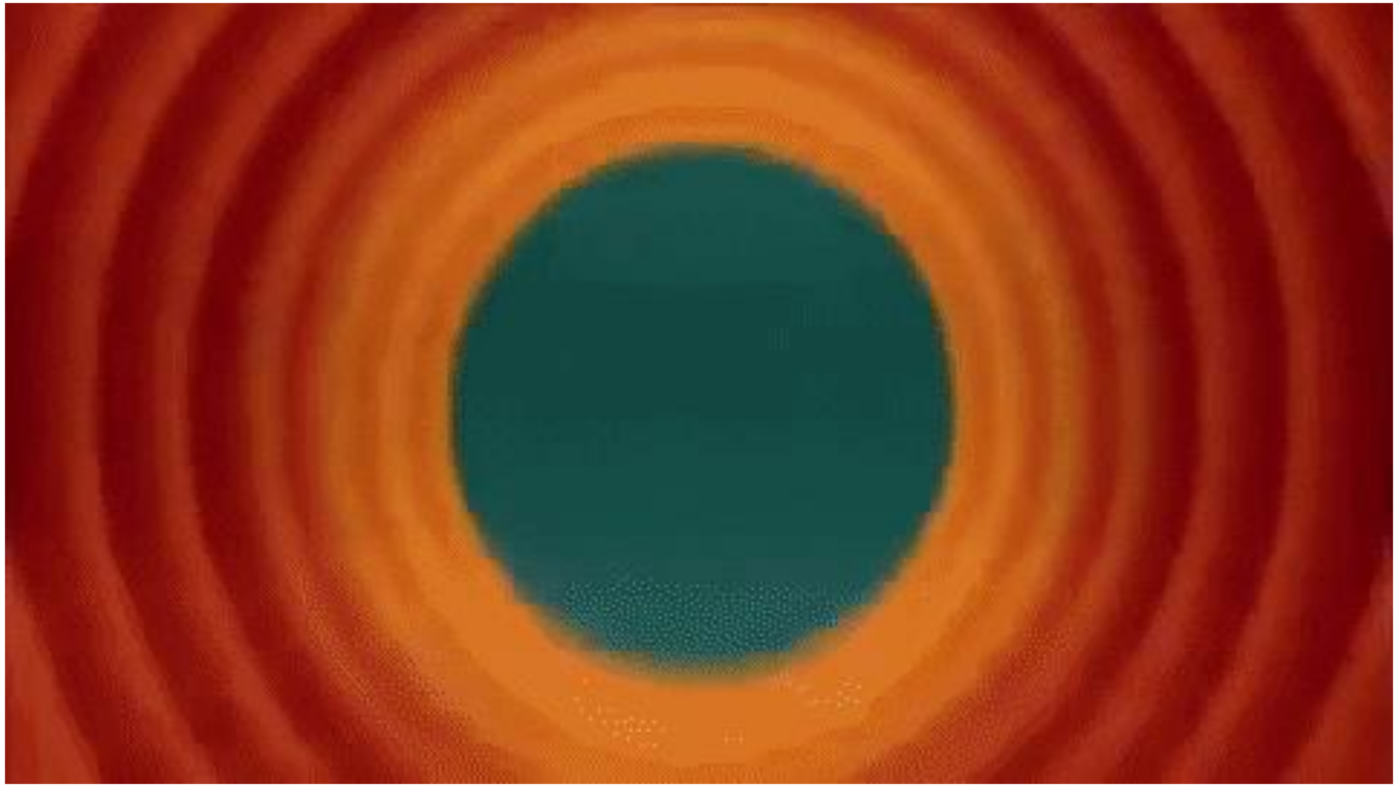
Medium: Photon   Control: Polarisation   Target: Spatial Mode



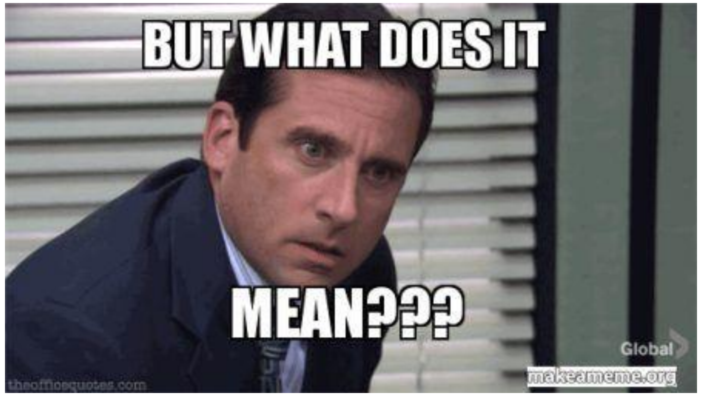
$|0\rangle \otimes |\psi\rangle$

$|1\rangle \otimes |\psi\rangle \rightarrow$

$|+\rangle \otimes |\psi\rangle \rightarrow$



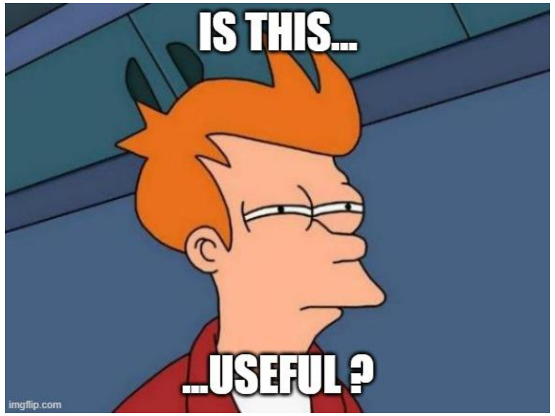
Outline



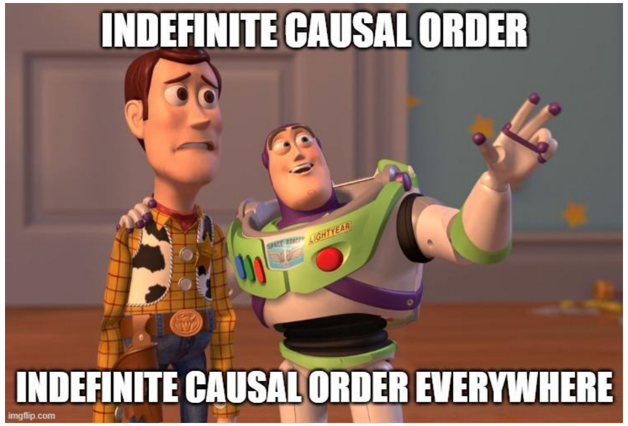
I – Indefinite Causal Orders : Definition.

II – Device-Independent Certification for the Quantum Switch

[Dourdent, Abbott, Supic & Branciard, arXiv:230812760]



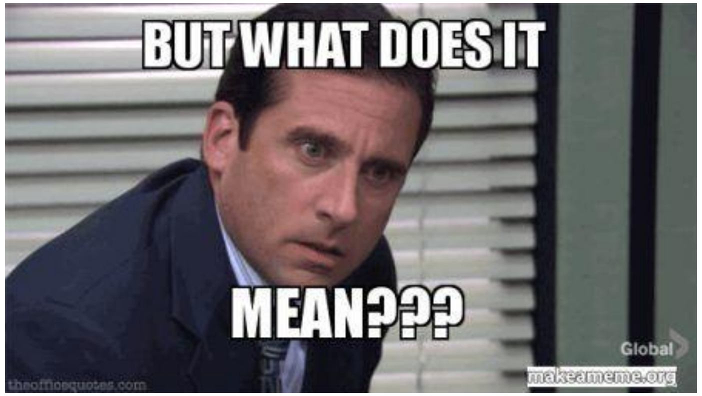
III – Advantages in Quantum Information from the Quantum Switch



IV- Beyond the Quantum Switch

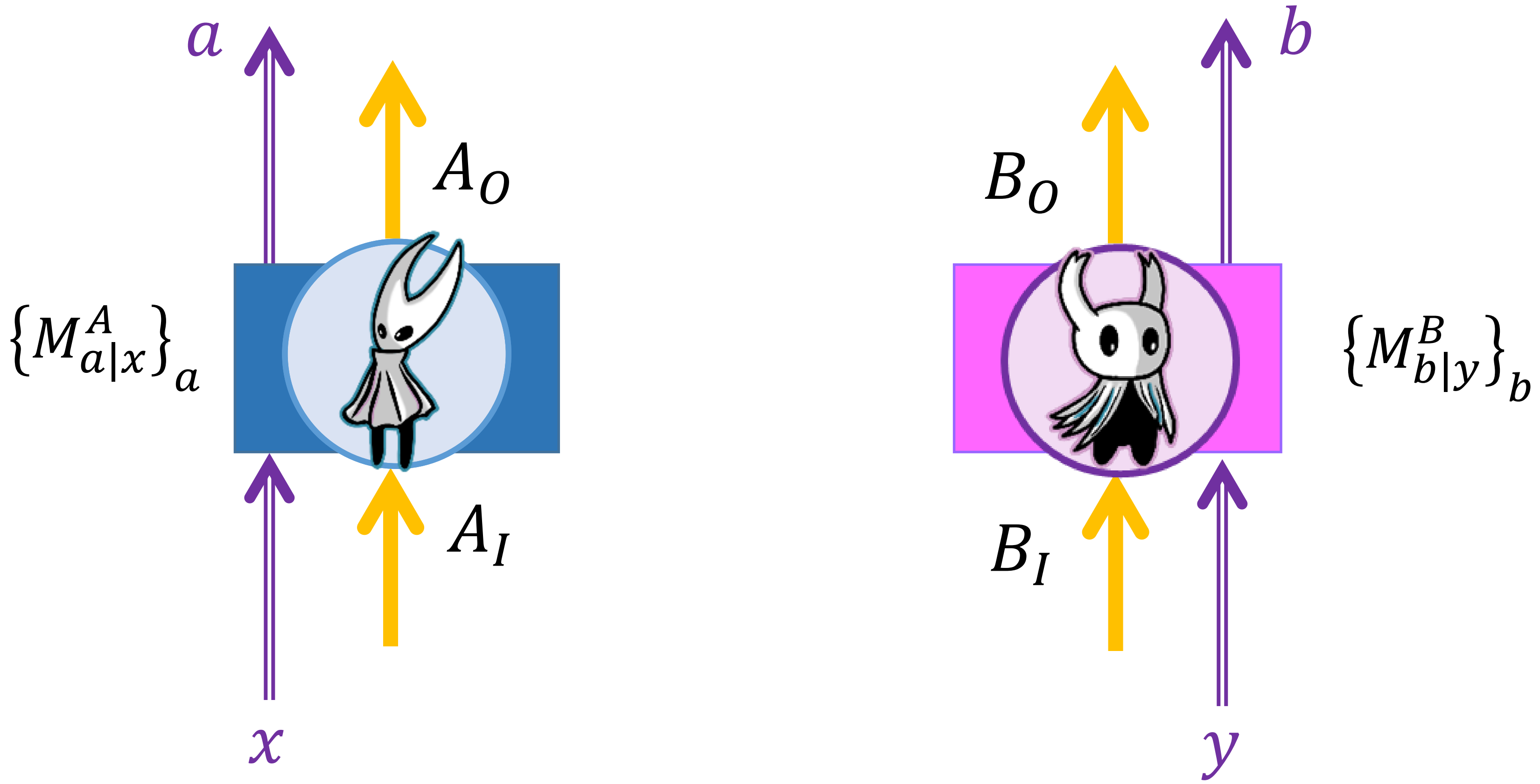
[Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]

Outline

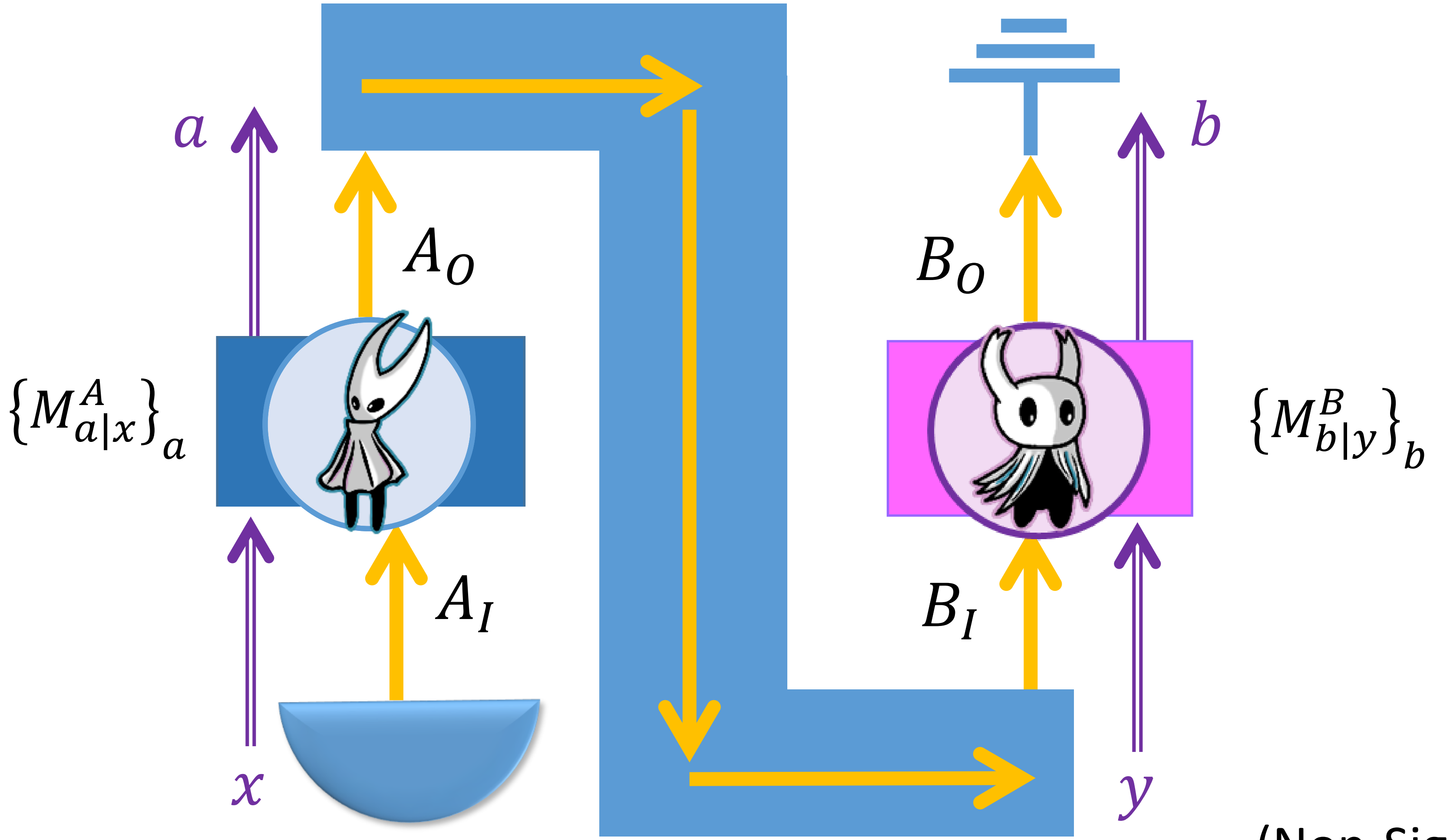


I – Indefinite Causal Orders : Definition.

A Standard Causal Scenario



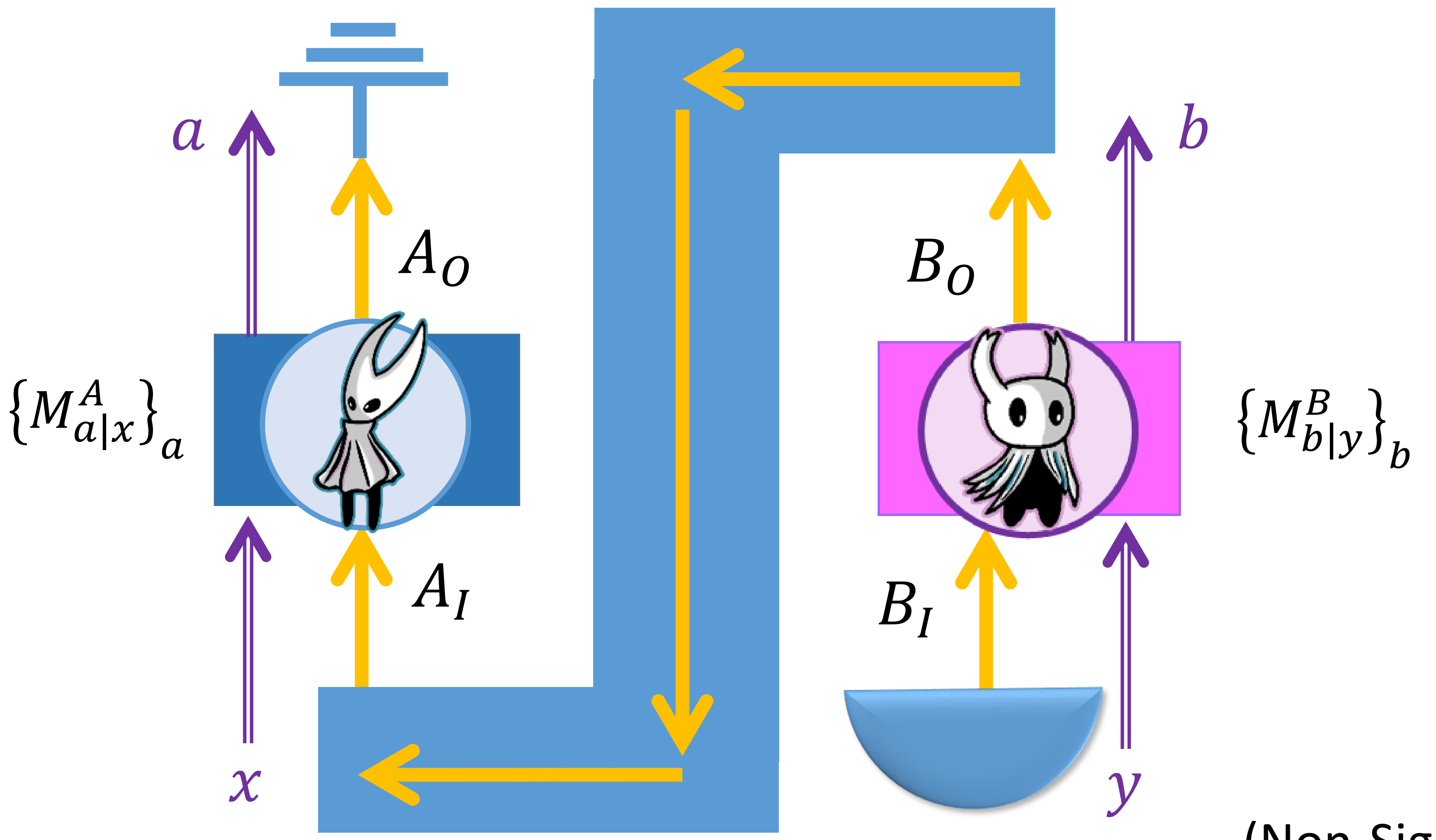
A Causal Scenario



(Non-Signalling $B \nprec A$)

Alice is in the causal past of Bob $A \prec B$ $\forall x, y, y', a \quad \sum_b P^{A\prec B}(a,b|x,y) = \sum_b P^{A\prec B}(a,b|x,y')$

A Causal Scenario



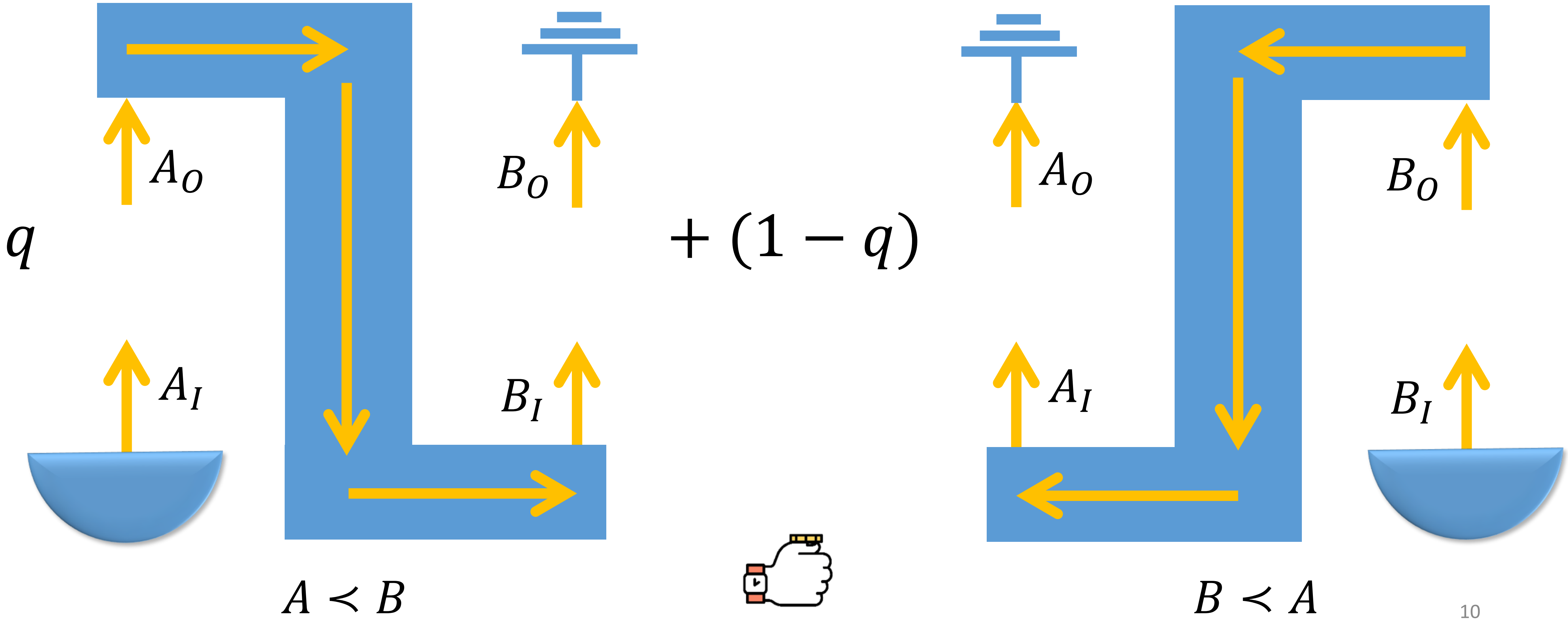
(Non-Signalling $A \nprec B$)

Bob is in the causal past of Alice $B \prec A$ $\forall x, x', y, b \quad \sum_a P^{B\prec A}(a,b|x,y) = \sum_a P^{B\prec A}(a,b|x',y)$

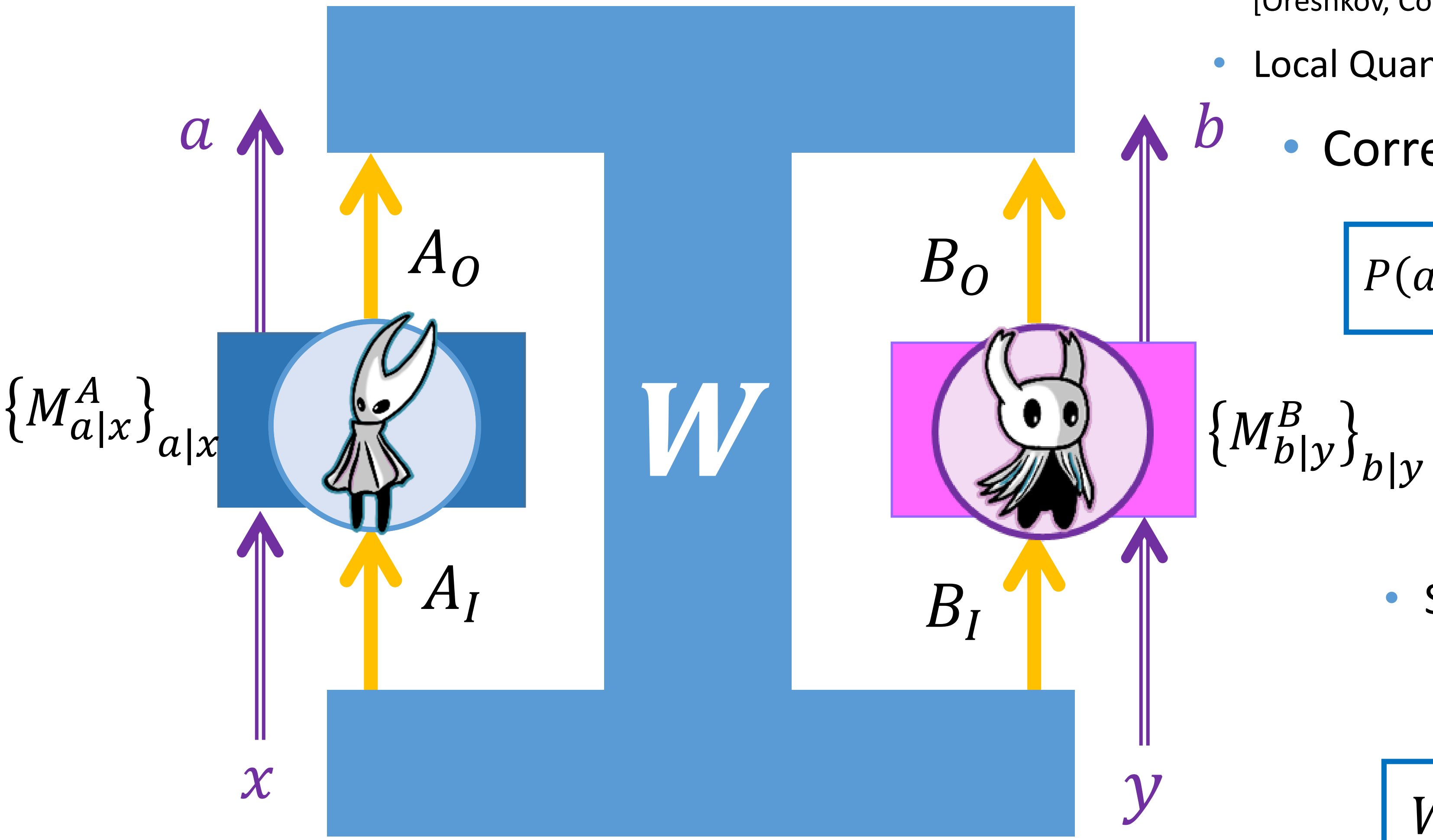
Causal Separability

- A causal resource is *causally separable* if it is compatible with a well defined causal order.

with $q \in [0,1]$



The Process Matrix Formalism



- Beyond the standard quantum formalism:

  [Oreshkov, Costa & Brukner, Nat. Com. 3, 1092 (2012)]

  - Local Quantum Theory
  - No Predefined Global Order

- Correlations : Generalized Born Rule

$$\boxed{P(a,b|x,y) = \mathrm{Tr}\left(\left(M^A_{a|x} \otimes M^B_{b|y}\right)^T . W^{AB}\right)}$$

A generalized channel resource: the process matrix

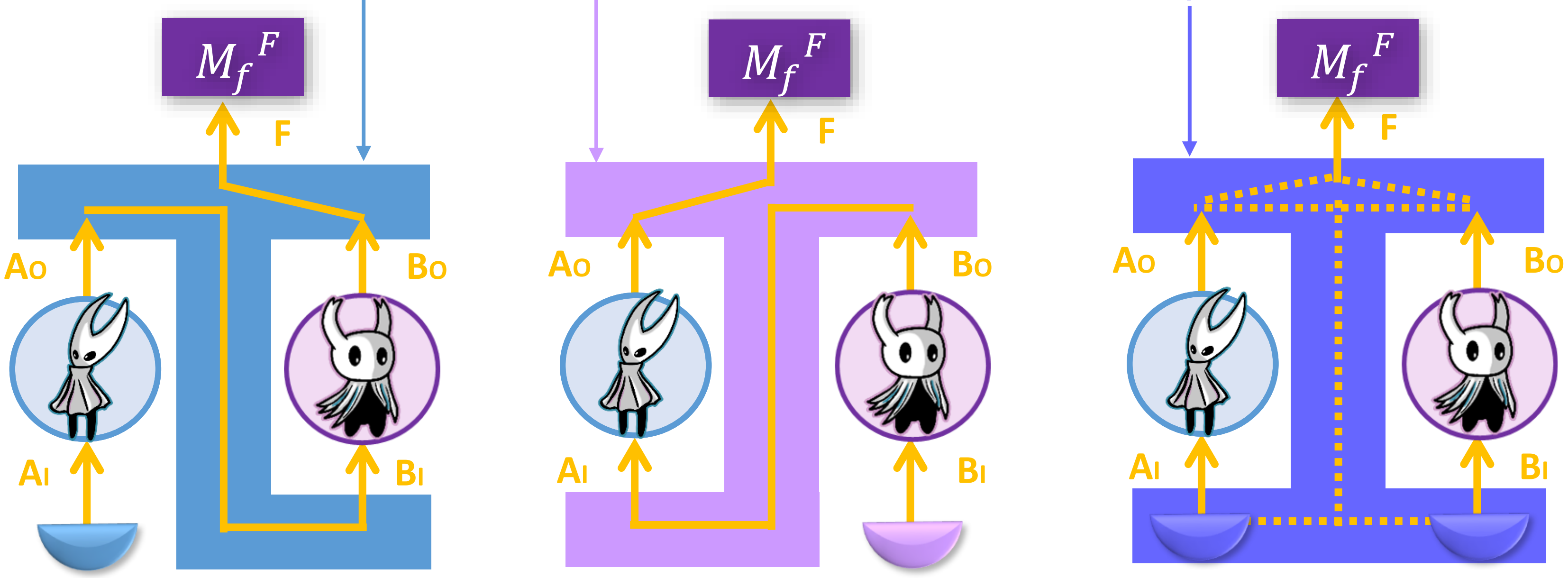
- Some processes W are incompatible with a definite causal order: they are *causally non-separable*.

$$\boxed{W \neq qW^{A \prec B} + (1-\mathrm{q})W^{B \prec A}}$$

with $\mathrm{q} \in [0,1]$

Process Matrix of the Quantum Switch

$$W_{switch} = \frac{1}{2}\left(W^{A\prec B\prec F} + W^{B\prec A\prec F} + |w\rangle^{A\prec B\prec F}\langle w|^{B\prec A\prec F} + |w\rangle^{B\prec A\prec F}\langle w|^{A\prec B\prec F}\right)$$



Alice $\prec$ Bob $\prec$ Fiona Bob $\prec$ Alice $\prec$ Fiona « Causal interferences »

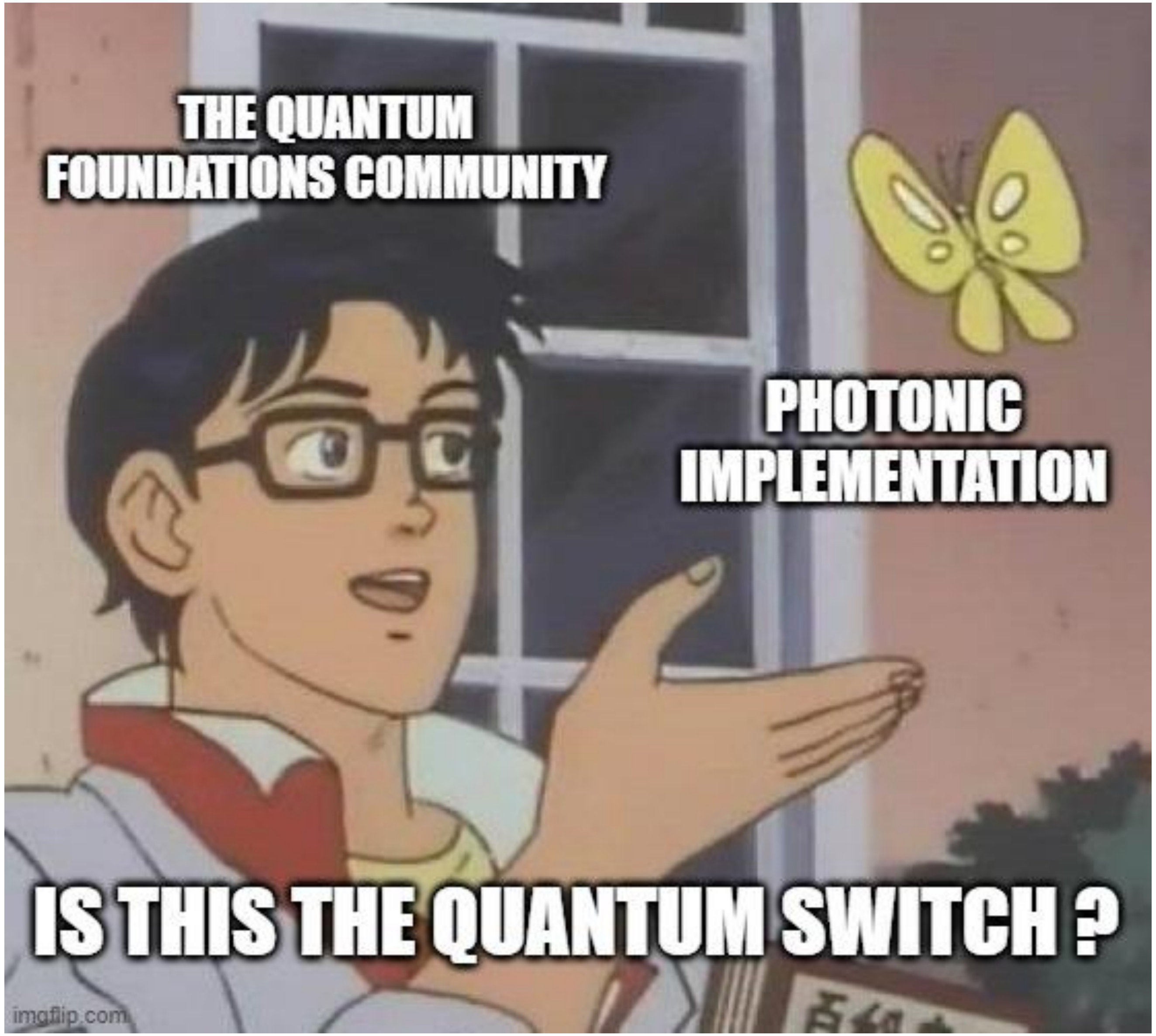
- Because the process matrix of the causal interference terms, it is causally nonseparable.

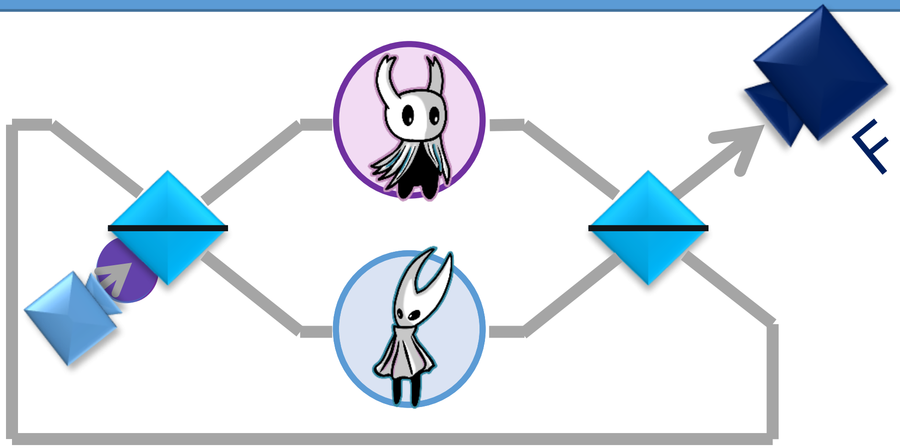
[Araùjo et al., NJP 17, 2015]

$$\boxed{W \neq qW^{A\prec B\prec F} + (1-q)W^{B\prec A\prec F}}$$ with $q \in [0,1]$

$$\Rightarrow \quad W = |w\rangle\langle w| \quad \text{with} \quad |w\rangle = \frac{1}{\sqrt{2}}\left(|\psi\rangle^{A^I}|\mathbb{1}\rangle\rangle^{A^O B^I}|\mathbb{1}\rangle\rangle^{B^O F^t}|0\rangle^{F^c} + |\psi\rangle^{B^I}|\mathbb{1}\rangle\rangle^{B^O A^I}|\mathbb{1}\rangle\rangle^{A^O F^t}|1\rangle^{F^c}\right)$$

Debate on Experimental Implementations of the Quantum Switch





Indefinite Causal Order in a Quantum Switch

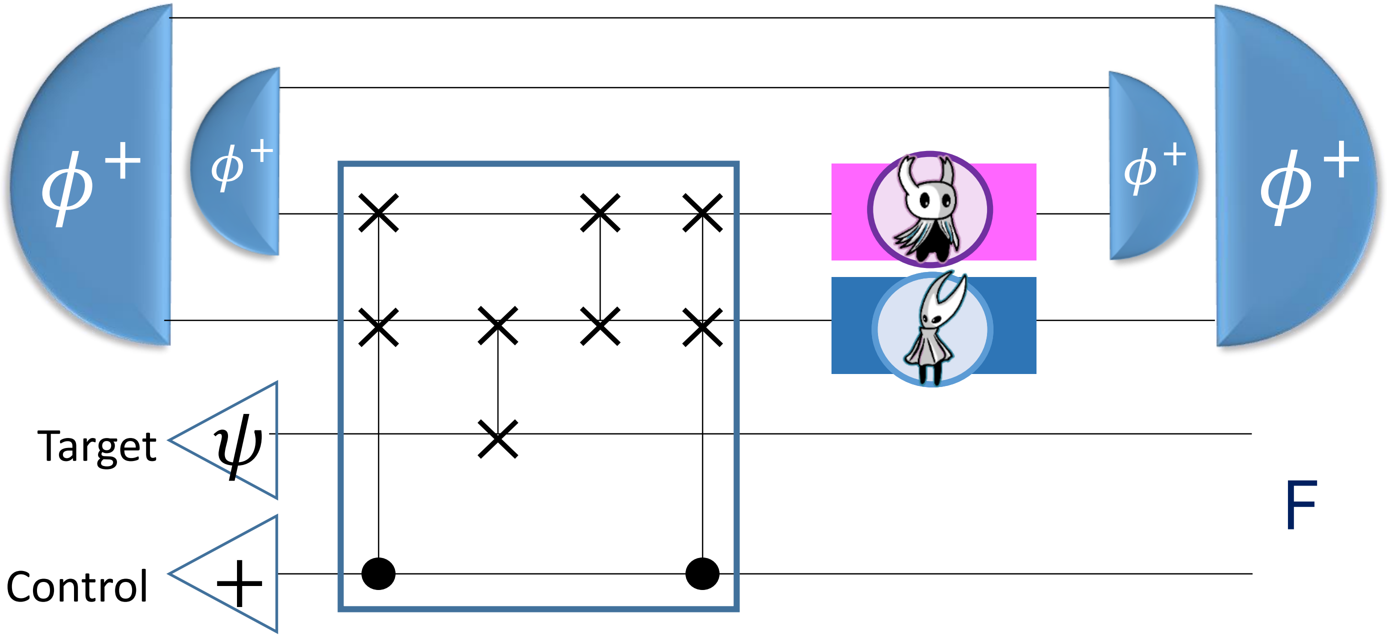
K. Goswami, C. Giarmatzi, M. Kewming, F. Costa, C. Branciard, J. Romero, and A. G. White
Phys. Rev. Lett. **121**, 090503 – Published 31 August 2018

[Goswami et al., PRL 121, 2018]

! Crucial feature !
Alice and Bob each acts once and only once, but their causal order is quantumly controlled !

Is this assumption really satisfied in photonic implementations ?

Debate on Experimental Implementations of the Quantum Switch



The quantum switch has a realisation in terms of a post-selection-closed-time-like-curve (P-CTC)

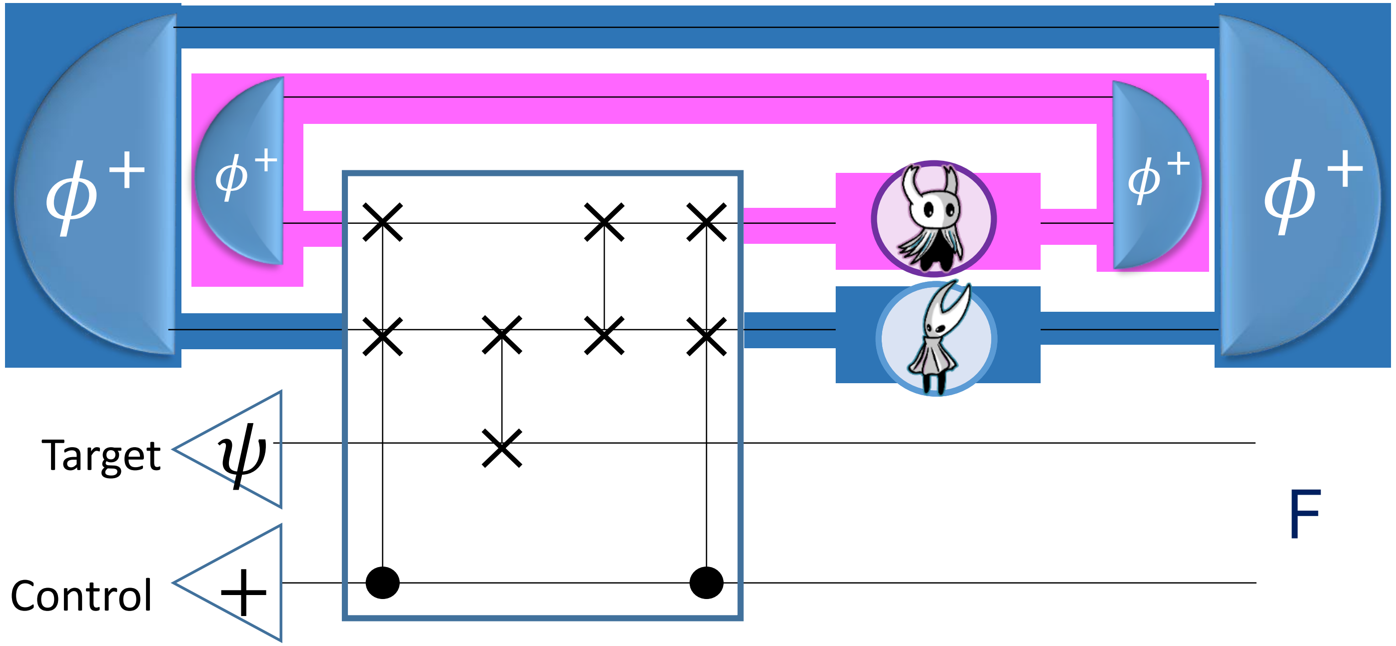
Araùjo, Guérin & Baumeler, Quantum computation with indefinite causal structures, PRA 96, (2017)

Definition of a complexity class for the problems that can be efficiently solved by process matrices:

$$\mathrm{BQP}_{\ell\mathrm{CTC}} \subseteq \mathrm{PP}$$

Are photonic implementations of the quantum switch really implementing a P-CTC ?

Debate on Experimental Implementations of the Quantum Switch

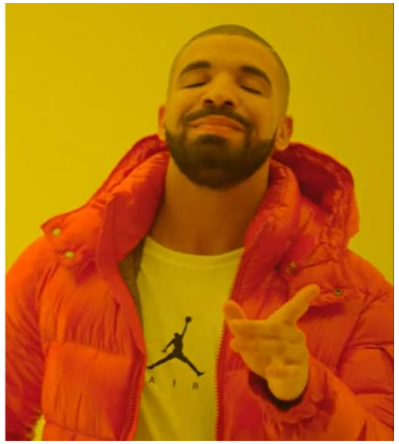


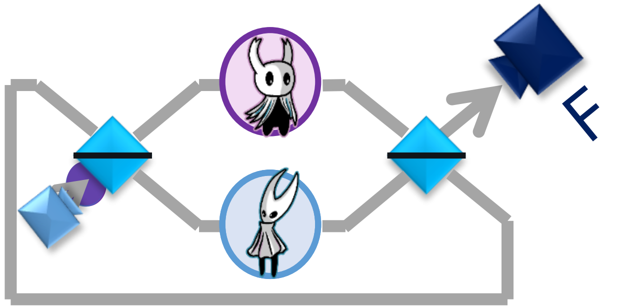
- If

an Event = Space-localized-time-delocalized quantum operation

Time-delocalized quantum subsystems and operations: on the existence of processes with indefinite causal structure in quantum mechanics

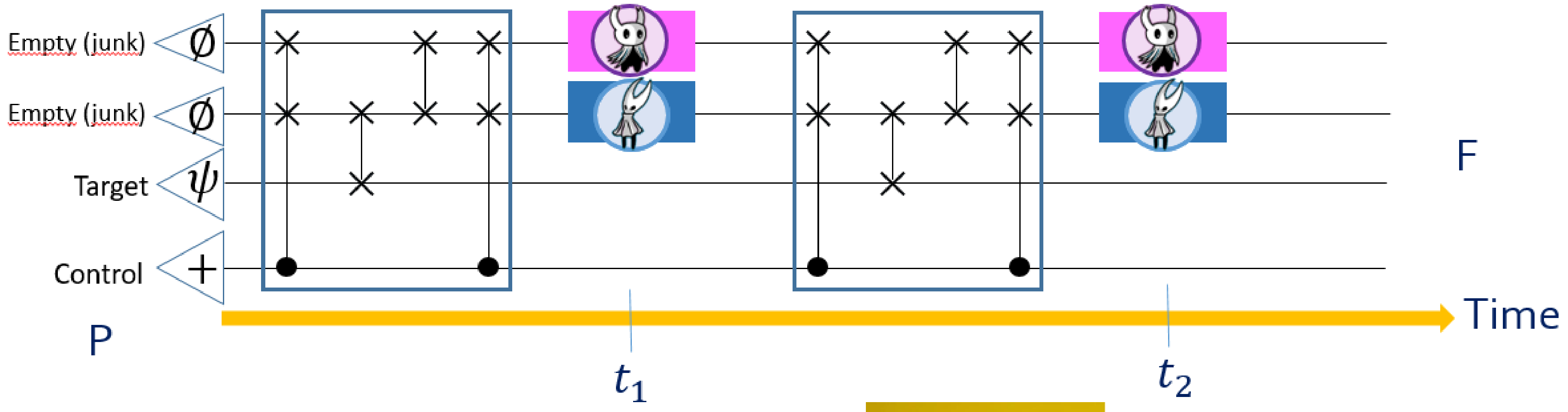
Ognyan Oreshkov



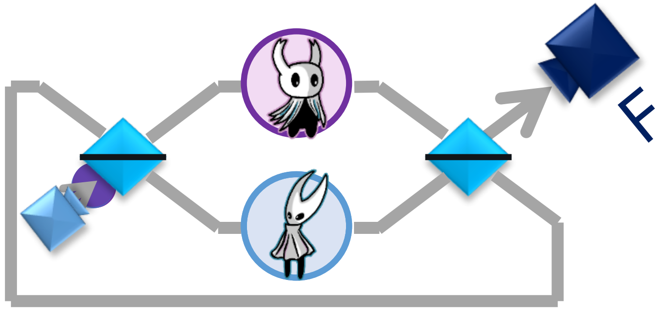


[Oreshkov, Quantum 3, 206 (2019)]

Debate on Experimental Implementations of the Quantum Switch



- If an Event = Space-time point



Causal orders, quantum circuits and spacetime: distinguishing between definite and superposed causal orders

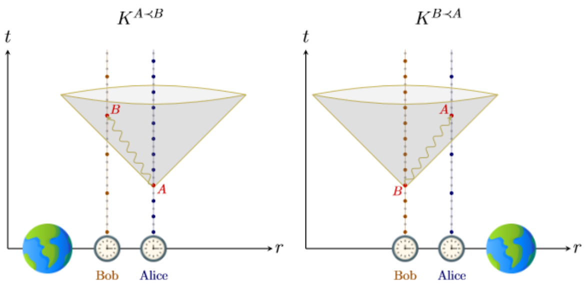
Nikola Paunković[1] and Marko Vojinović[2]

[Paunkovic & Vojinovic, Quantum 4, 275 (2020)]

- Gravitational Quantum Switch

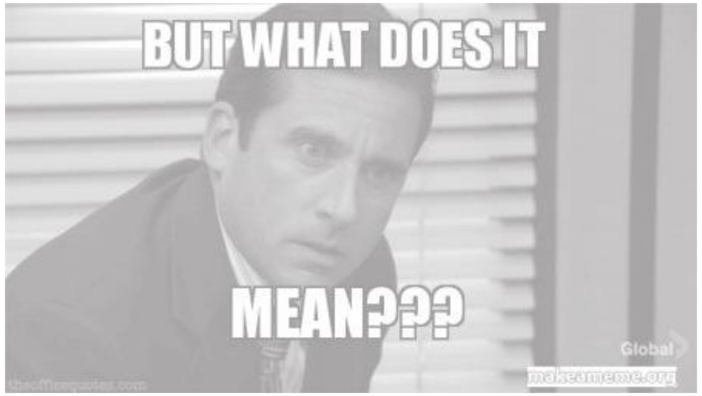
[Zych, Costa, Pikovski & Brukner, Nat. Com. 10, 2019]

[Paunkovic & Vojinovic, Quantum 4, 2020]   [Moller et al. PRA 104, 2021]



(b) Gravitational quantum switch

Outline



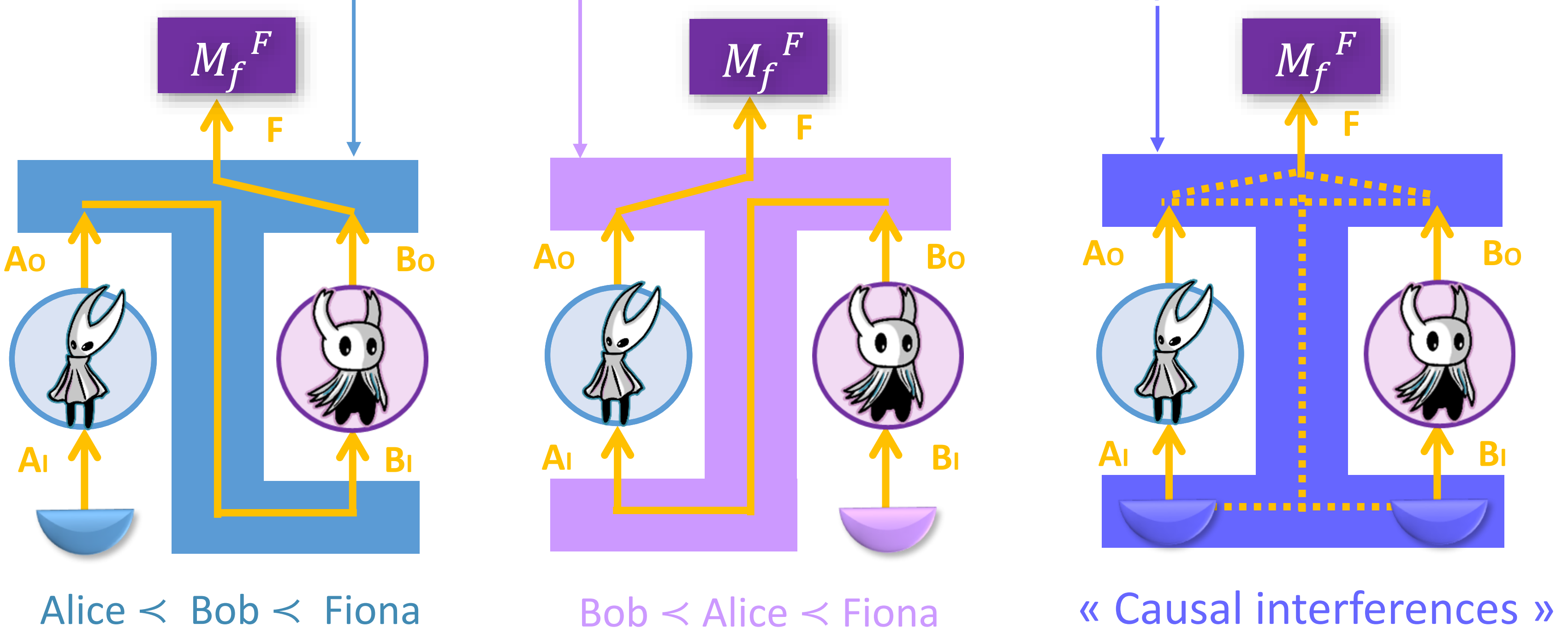
I – Indefinite Causal Orders : Definition.

II – Device-Independent Certification for the Quantum Switch

[Dourdent, Abbott, Supic & Branciard, arXiv:230812760]

Process Matrix of the Quantum Switch

$$W_{switch} = \frac{1}{2}\left(\underbrace{W^{A\prec B\prec F}} + \underbrace{W^{B\prec A\prec F}} + \underbrace{|w\rangle^{A\prec B\prec F}\langle w|^{B\prec A\prec F} + |w\rangle^{B\prec A\prec F}\langle w|^{A\prec B\prec F}}\right)$$



- Because the process matrix of the causal interference terms, it is causally nonseparable.

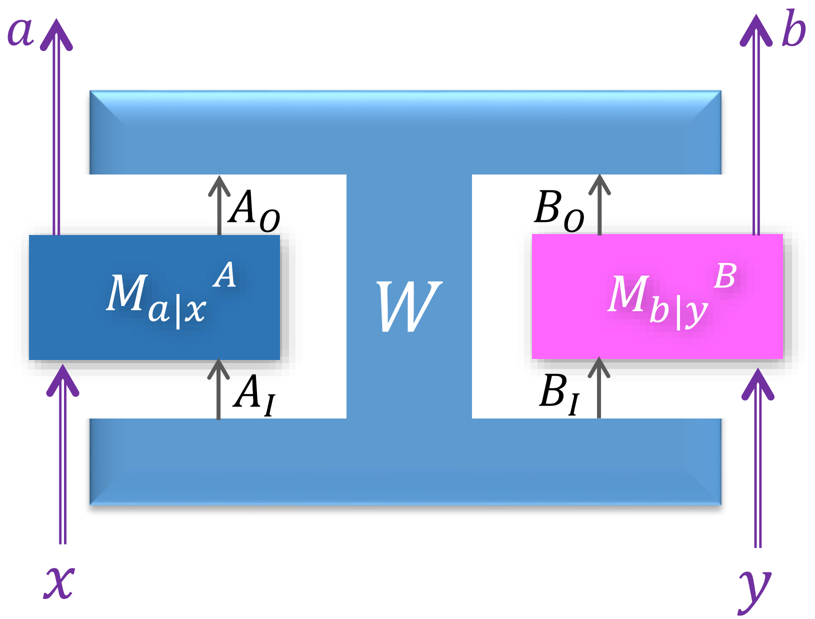
[Araùjo et al., NJP 17, 2015]

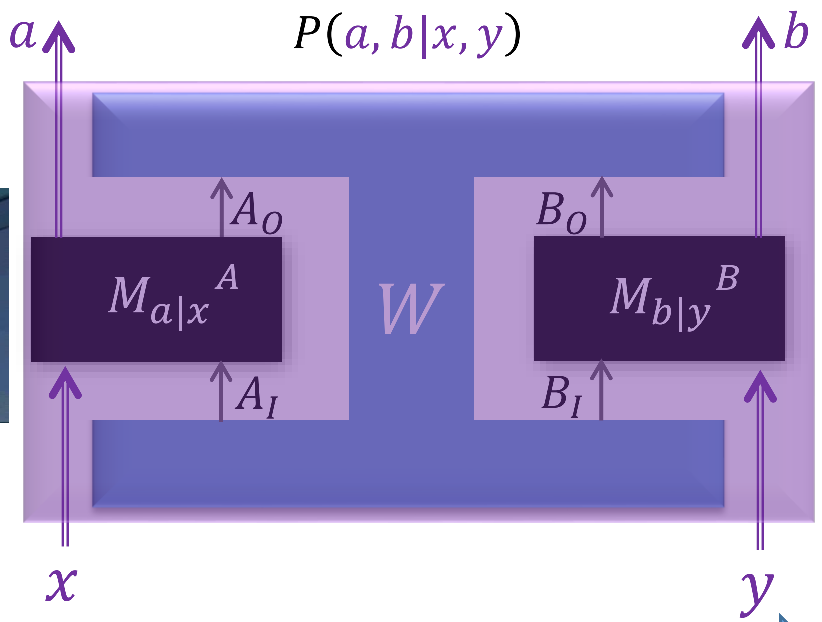
$$\boxed{W \neq qW^{A\prec B\prec F} + (1-\mathrm{q})W^{B\prec A\prec F}}$$ with $\mathrm{q} \in [0,1]$

How can we certify its causal nonseparability ?

Certifying Causal Nonseparability

- Is there a general method to certify experimentally the causal nonseparability of a process matrix ?
    - Which underlying assumptions are necessary to do so ?
- A certification based on « Trust » (analogy with entanglement)





Max. assumptions

Min. assumptions

« Device-Dependent »
(requires complete trust in Alice and Bob)

« **Device-Independent (DI)** »
(Alice and Bob are black boxes)
Require knowledge of observed statistics only.

Interests of certifying Causal Nonseparability without trusting your devices ?

Phenomenological

Causal nonseparability can manifest itself in different ways. Some processes may not be causally nonseparable in a DI way.

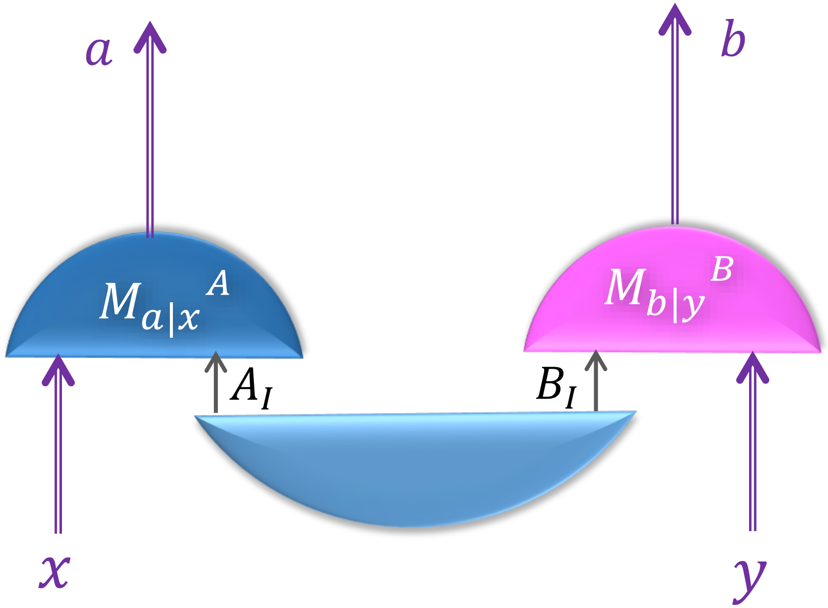
Pragmatical

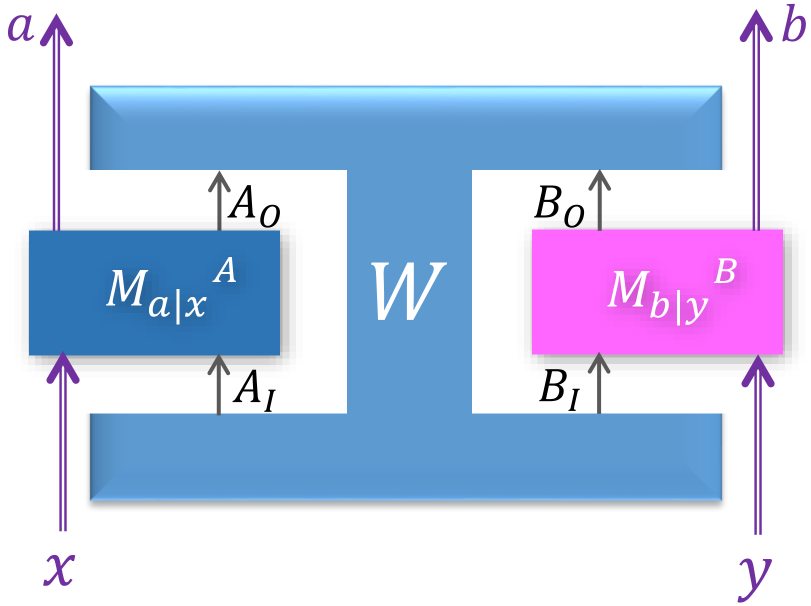
Untrusted devices (black boxes) are associated with eavesdropper in communication, cryptography protocols.

Epistemological

A. Grinbaum, « *How device-independent approaches change the meaning of physical theories* », Studies in History and Philosophy of Modern Physics 58 (2017) 22-30 .

Analogy Entanglement / Causal Nonseparability



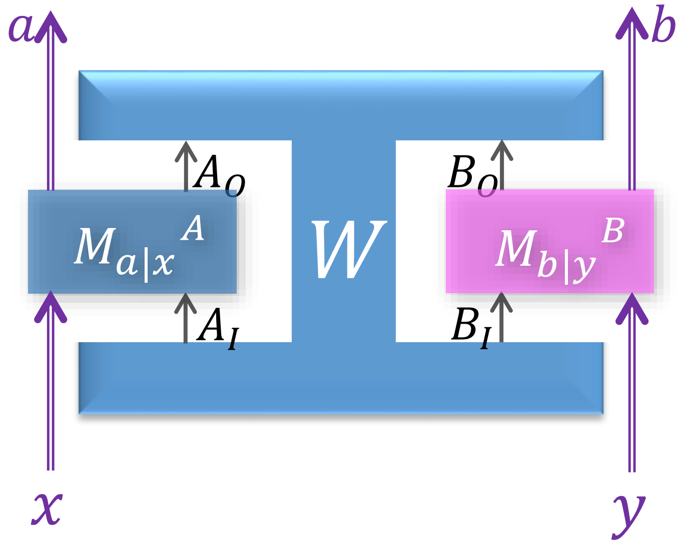


Operations:	• Local Quantum Measurements	• Local Quantum Instruments
Resource:	• Entangled (nonseparable) state	• Causally nonseparable process matrix
Device-Dependent certification:	• Entangled witnesses	• [Non]Causal witnesses [Araùjo et al., NJP 17, 2015]

A « universal » certification of Causal Nonseparability

Device-Dependent	Entangled vs separable states	⇝	Causally (non)separable process matrices
	Entanglement witnesses	⇝	**Causal witnesses**

[Araújo et al., New J. Phys. 17 (2015) 102001] [Branciard,Sci. Rep. 6, 26018 (2016)]



Alice ■, Bob ■ are trusted.

- Dual cone of causally separable process matrices:

$$(\mathcal{W}^{csep})^* = \{S | \forall\, W \in \mathcal{W}^{csep}, Tr(S^T W) \geq 0\}$$

- For any causally nonseparable process matrix, there exists a ***causal witness***

$$S = \sum_{abxy} \gamma_{abxy} M_{a|x}{}^{A} \otimes M_{b|y}{}^{B}$$

- From Alice and Bob's measurements, we can calculate a specific quantity.

$$\langle S \rangle = Tr(S^T W) = \sum_{abxy} \gamma_{abxy}\, P\left(a, b \middle| M_{a|x}{}^{A}, M_{b|y}{}^{B}\right) < 0$$

- If this quantity is negative, then the process is causally nonseparable.

Witnessing causal nonseparability

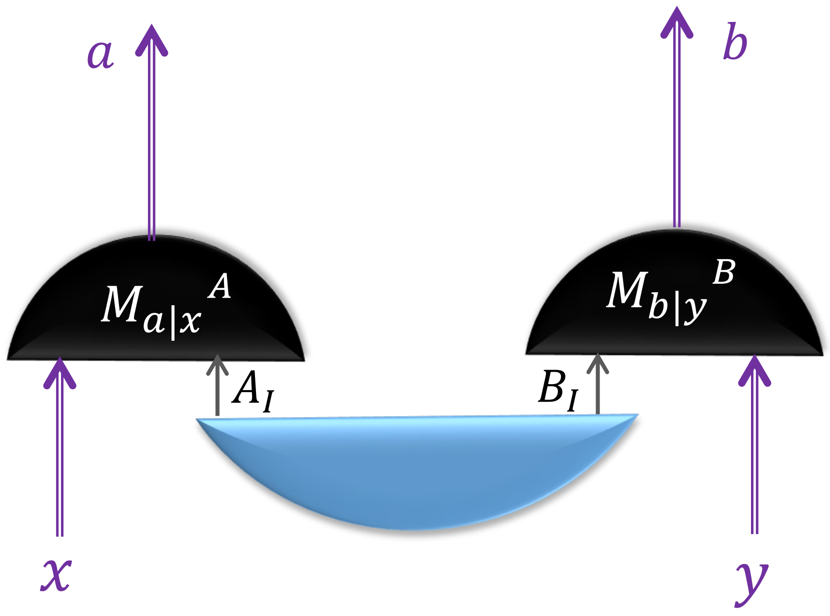
Mateus Araújo[1,2], Cyril Branciard[3], Fabio Costa[1,2,4], Adrien Feix[1,2], Christina Giarmatzi[4,5] and Časlav Brukner[1,2]

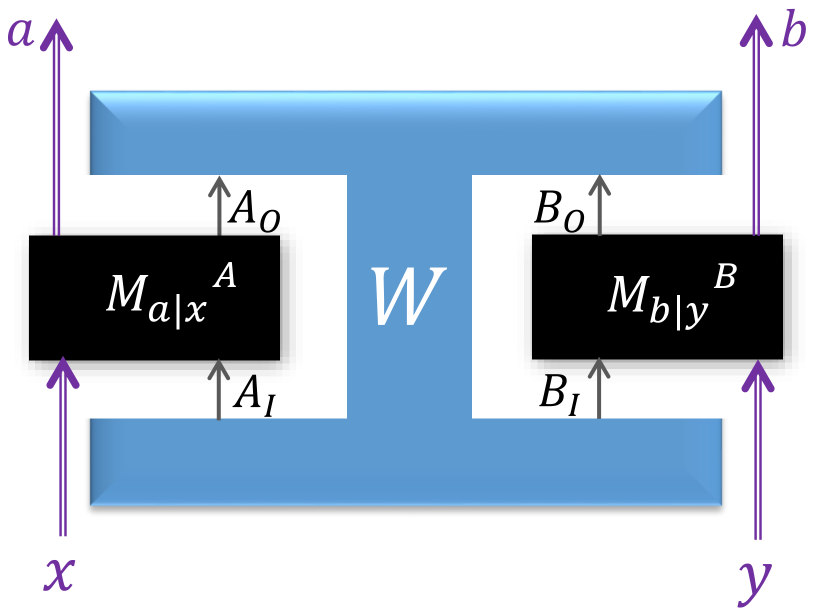
Published 19 October 2015 • © 2015 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

New Journal of Physics, Volume 17, October 2015

[Araùjo et al., NJP 17, 2015]

Analogy Entanglement / Causal Nonseparability



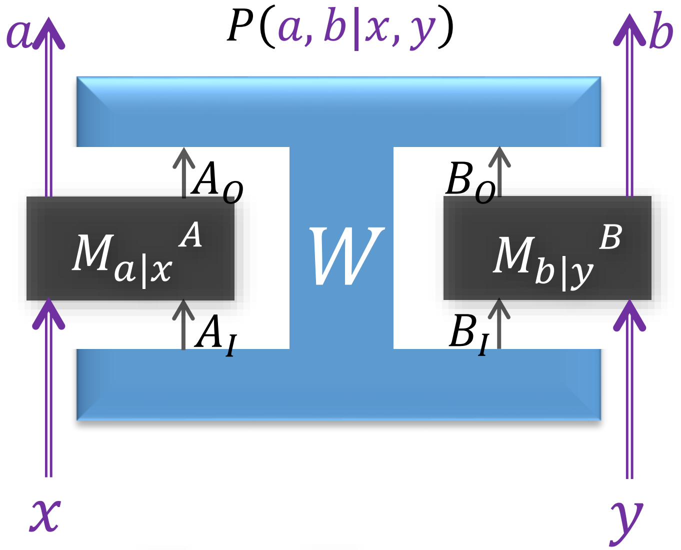


Operations:	• Local Quantum Measurements	• Local Quantum Instruments
Resource:	• Entangled (nonseparable) state	• Causally nonseparable process matrix
Device-Independent certification:	• Nonlocal correlations • Violation of Bell inequalities	• Noncausal correlations • Violation of causal inequalities

The « strongest » certification of Causal Nonseparability

Device-Independent	(Non)local correlations	↭	**(Non)causal correlations**
	Local polytope	↭	Causal polytope
	Bell inequalities	↭	Causal inequalities

[Oreshkov, Costa & Brukner, Nat. Com. 3, 1092 (2012)] [Branciard et al., New J. Phys. 18, 013008 (2016)]



Alice , Bob are untrusted.

- Correlations: $\boxed{P(a,b|x,y) = \mathrm{Tr}\left(\left(M_{a|x}^{A} \otimes M_{b|y}^{B}\right)^{T} . W^{AB}\right)}$

Correlations compatible with the causal order $A \prec B$

$$\forall x, y, y', a \quad \sum_b P^{A\prec B}(a,b|x,y) = \sum_b P^{A\prec B}(a,b|x,y')$$

Correlations compatible with the causal order $B \prec A$

$$\forall x, x', y, b \quad \sum_a P^{B\prec A}(a,b|x,y) = \sum_a P^{B\prec A}(a,b|x',y)$$

ARTICLE

Received 29 May 2012 | Accepted 17 Aug 2012 | Published 2 Oct 2012 DOI: 10.1038/ncomms2076

Quantum correlations with no causal order

Ognyan Oreshkov[1,2], Fabio Costa[1] & Časlav Brukner[1,3]

[Oreshkov, Costa & Brukner, Nat. Com. 3, 2012]

The simplest causal inequalities and their violation

Cyril Branciard[5,1], Mateus Araújo[5,2,3], Adrien Feix[2,3], Fabio Costa[2,3,4] and Časlav Brukner[2,3]

Published 23 December 2015 • © 2016 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

New Journal of Physics, Volume 18, January 2016

[Branciard et al., NJP 18, 2016]

- Correlations incompatible with a definite causal order are ***noncausal***.

$$\boxed{P(a,b|x,y) \neq \lambda P^{A\prec B}(a,b|x,y) + (1-\lambda) P^{B\prec A}(a,b|x,y)} \quad \text{with } \lambda \in [0,1]$$

Noncausal correlations can violate causal inequalities.

Certification of Causal Nonseparability of the Quantum Switch

- **Device-Dependent Certification**

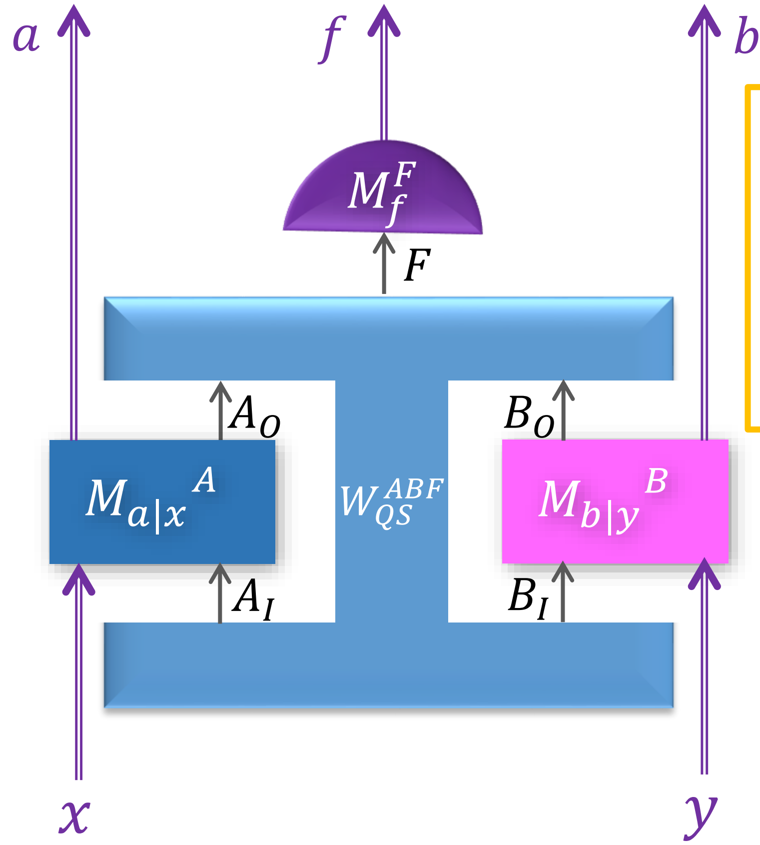
- Measuring a *causal witness* $P\left(a,b,f \middle| M_{a|x}{}^{A}, M_{b|y}{}^{B}, M_f{}^{F}\right)$

Witnesses of causal nonseparability: an introduction and a few case studies

Cyril Branciard

Scientific Reports 6, Article number: 26018 (2016) | Cite this article

[Branciard, Scientific Reports 6, 2016]

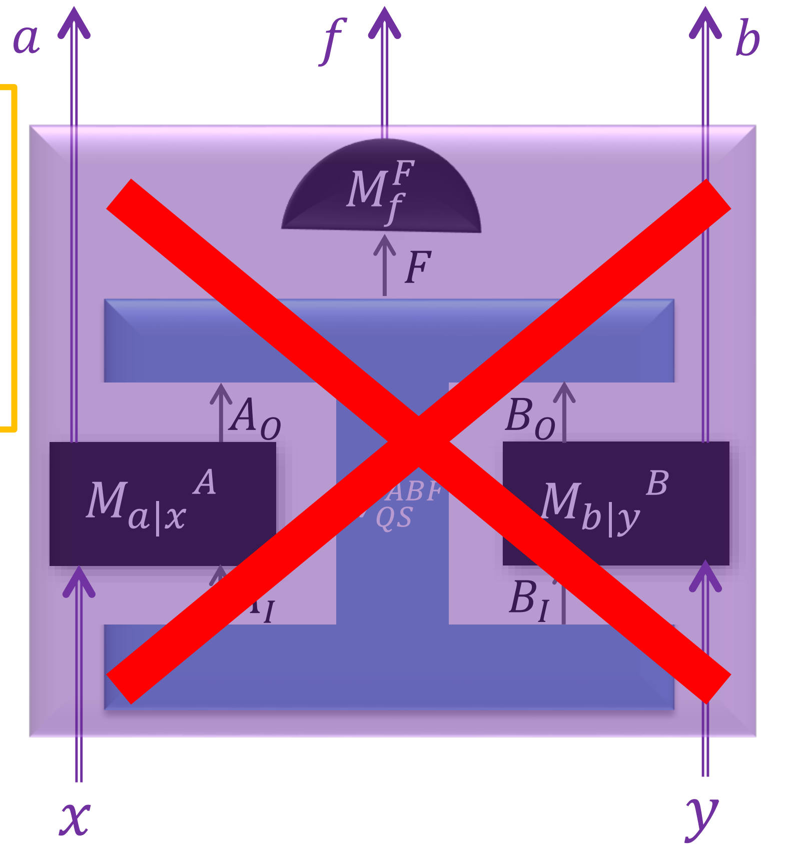


The quantum switch is
causally nonseparable,
but it is «***causal*** »
(it cannot generate noncausal
correlations in this DI scenario).

- **Device-Independent Certification**

- Violation of *Causal Inequality* $P(a,b,f|x,y)$

→ Impossible for the Quantum Switch

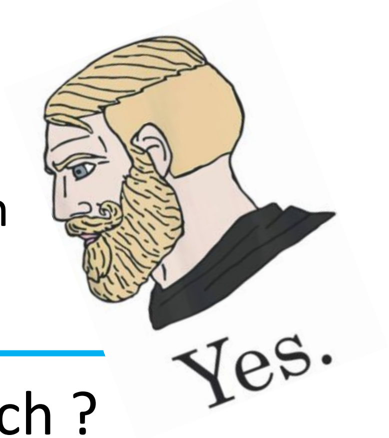


Phenomenological Interest from Device-Independent Certification

Not all causally nonseparable processes can violate a causal inequality.

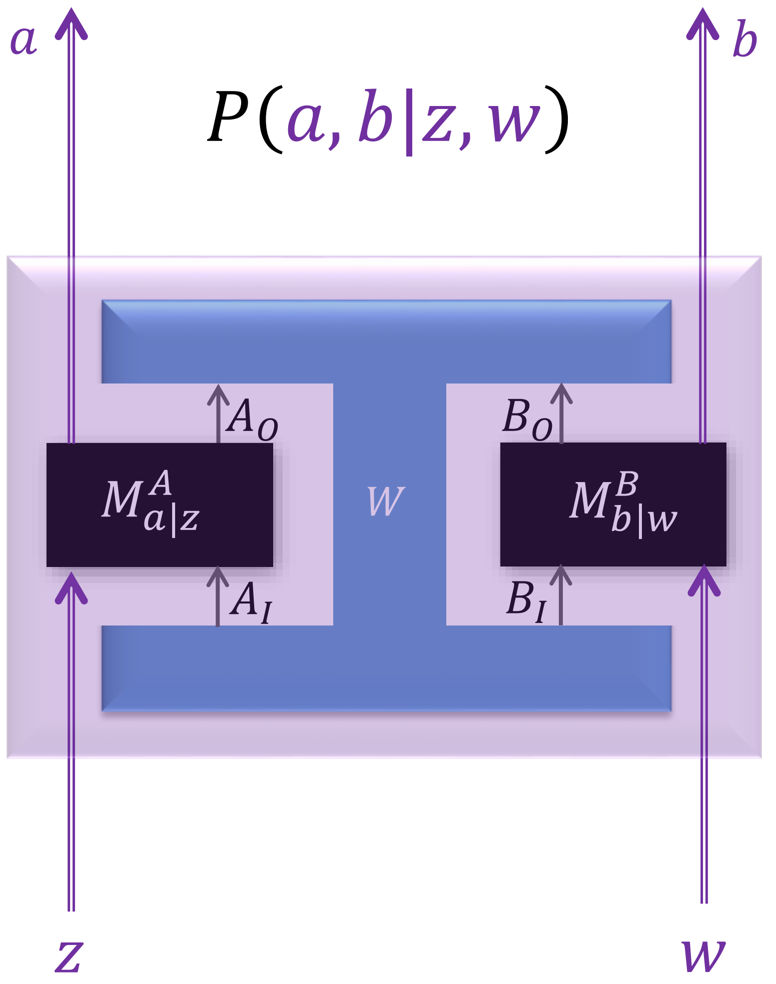
[Araùjo et al., NJP 17, 2015]
[Oreshkov & Giarmatzi, NJP 18, 2016]
[Wechs et al., PRX Quantum 2, 2021]

It is unclear whether the causal nonseparability of any process with a physical realization can manifest itself in a DI way.



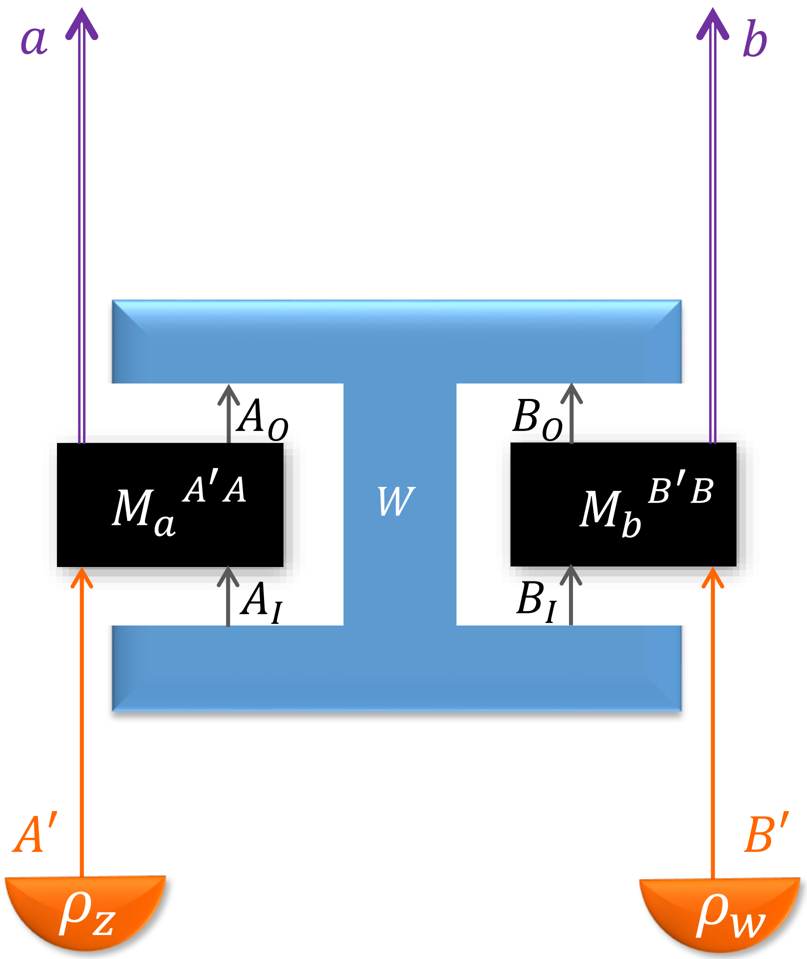
Are there « alternative DI » certifications that include the quantum switch ?

Towards the Network-Device-Independent Certification of Causal Nonseparability



Analoguous method for certification of entanglement:
[J. Bowles, I. Šupić, D. Cavalcanti & A. Acin, PRL 121, 2018]

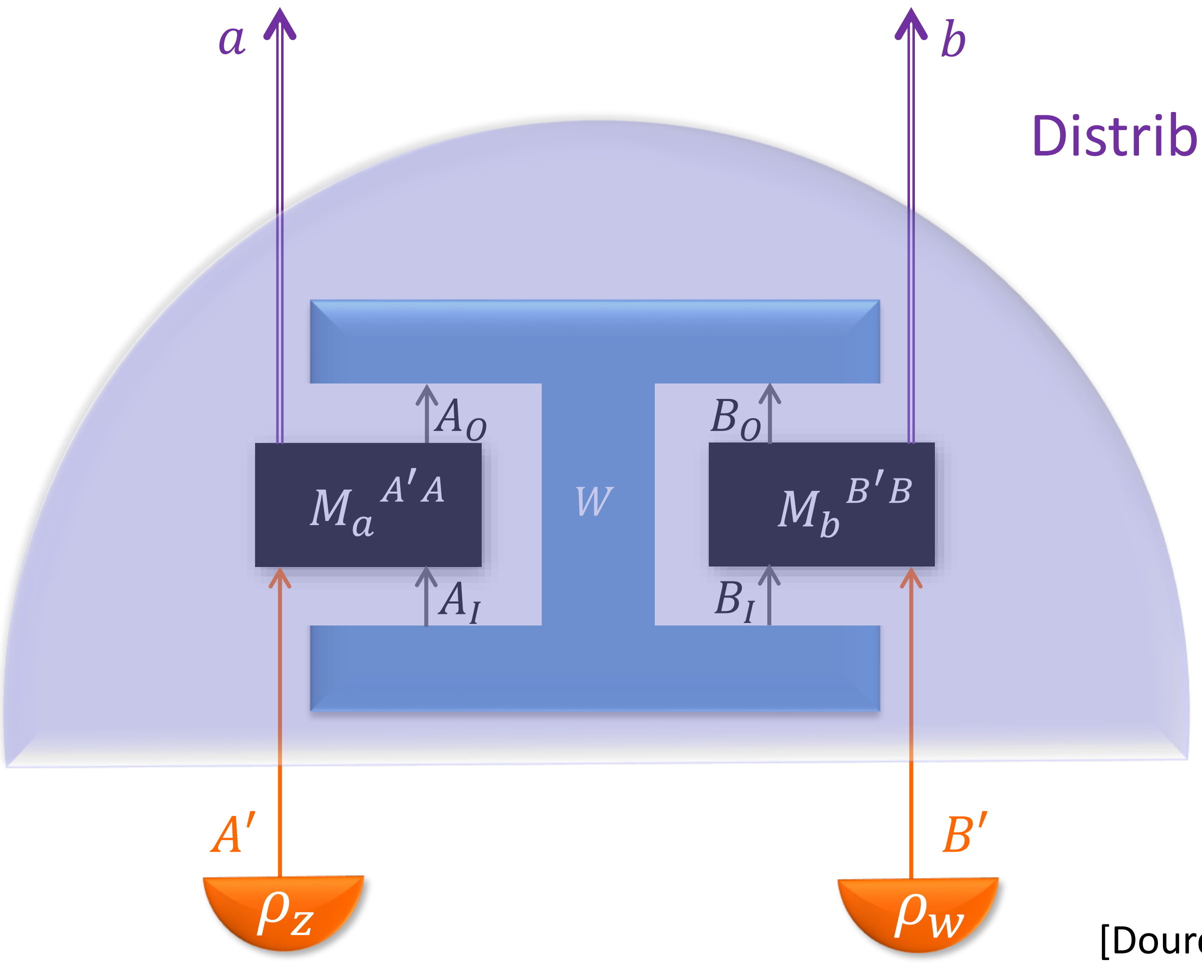
Towards the Network-Device-Independent Certification of Causal Nonseparability



Semi-Device-Independent Certification of Causal Nonseparability with Trusted Quantum Inputs

Hippolyte Dourdent, Alastair A. Abbott, Nicolas Brunner, Ivan Šupić, and Cyril Branciard
Phys. Rev. Lett. **129**, 090402 – Published 24 August 2022

Step 1: Semi-Device-Independent Certification with Trusted Quantum Inputs (SDIQI)



Distributed POVM (D-POVM):

$$\left(E_{a,b}^{A'B'}\right)_{a,b}$$

[Dourdent et al.,PRL **129**, 090402 (2022)]

$$P(a,b|\rho_z^{A'},\rho_w^{B'}) = \mathrm{Tr}\left(\left(M_a^{A'A} \otimes M_b^{B'B}\right)^T . (W^{AB} \otimes \rho_z^{A'} \otimes \rho_w^{B'})\right) = \mathrm{Tr}\left(\left(E_{a,b}^{A'B'}\right)^T . \rho_z^{A'} \otimes \rho_w^{B'}\right)$$

Causally Separable Distributed POVM

- <u>Definition</u>

A bipartite D-POVM $\mathbb{E}^{A'B'}$ that can be decomposed as a convex mixture of D-POVMs compatible with the causal orders $A' \prec B'$ and $B' \prec A'$

$$\mathbb{E}^{A'B'} = q\mathbb{E}^{A'\prec B'} + (1-q)\mathbb{E}^{B'\prec A'}$$

$$\mathbb{E}^{A'\prec B'} := \left(E_{a,b}^{A'\prec B'}\right)_{a,b} \qquad \mathbb{E}^{B'\prec A'} := \left(E_{a,b}^{B'\prec A'}\right)_{a,b}$$

$$\forall a \sum_b E_{a,b}^{A'\prec B'} = E_a^{A'} \otimes \mathbf{1}^{B'} \qquad \forall b \sum_a E_{a,b}^{B'\prec A'} = E_b^{B'} \otimes \mathbf{1}^{A'}$$

with $q \in [0,1]$ is said to be causally separable.

- A causally separable process matrix can only generate causally separable D-POVMs.

$$\boxed{A \prec B \ \rightarrow A' \prec B'}$$

- Any causally separable D-POVMs can be realised by local operations on a causally separable process matrix.

$$\boxed{A' \prec B' \rightarrow A \prec B}$$

SDIQI Certification of Causal Nonseparability

- If a causally nonseparable process matrix can generate a causally nonseparable D-POVM, we say that its causal nonseparability can be certified in a *Semi-Device-Independent with trusted Quantum Inputs* (SDIQI) manner.

- Protocol: Find quantum systems and operations such that the generated D-POVM is causally nonseparable, whose causal nonseparability can be certified with the violation of a ***witness inequality***.

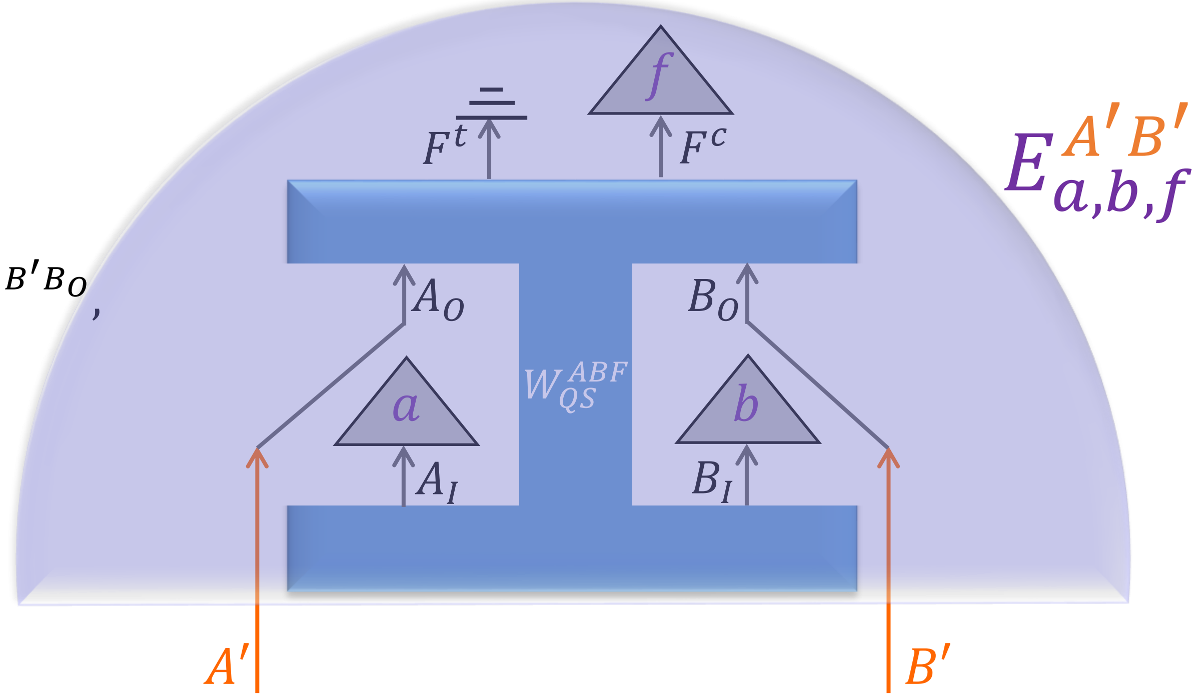
- Causally nonseparable D-POVM from the quantum switch:

    - For the operations:

$$M_a^{A'A} = |a\rangle\langle a|^{A_I} \otimes |\mathbf{1}\rangle\rangle\langle\langle\mathbf{1}|^{A'A_O},\ M_b^{B'B} = |b\rangle\langle b|^{B_I} \otimes |\mathbf{1}\rangle\rangle\langle\langle\mathbf{1}|^{B'B_O},$$

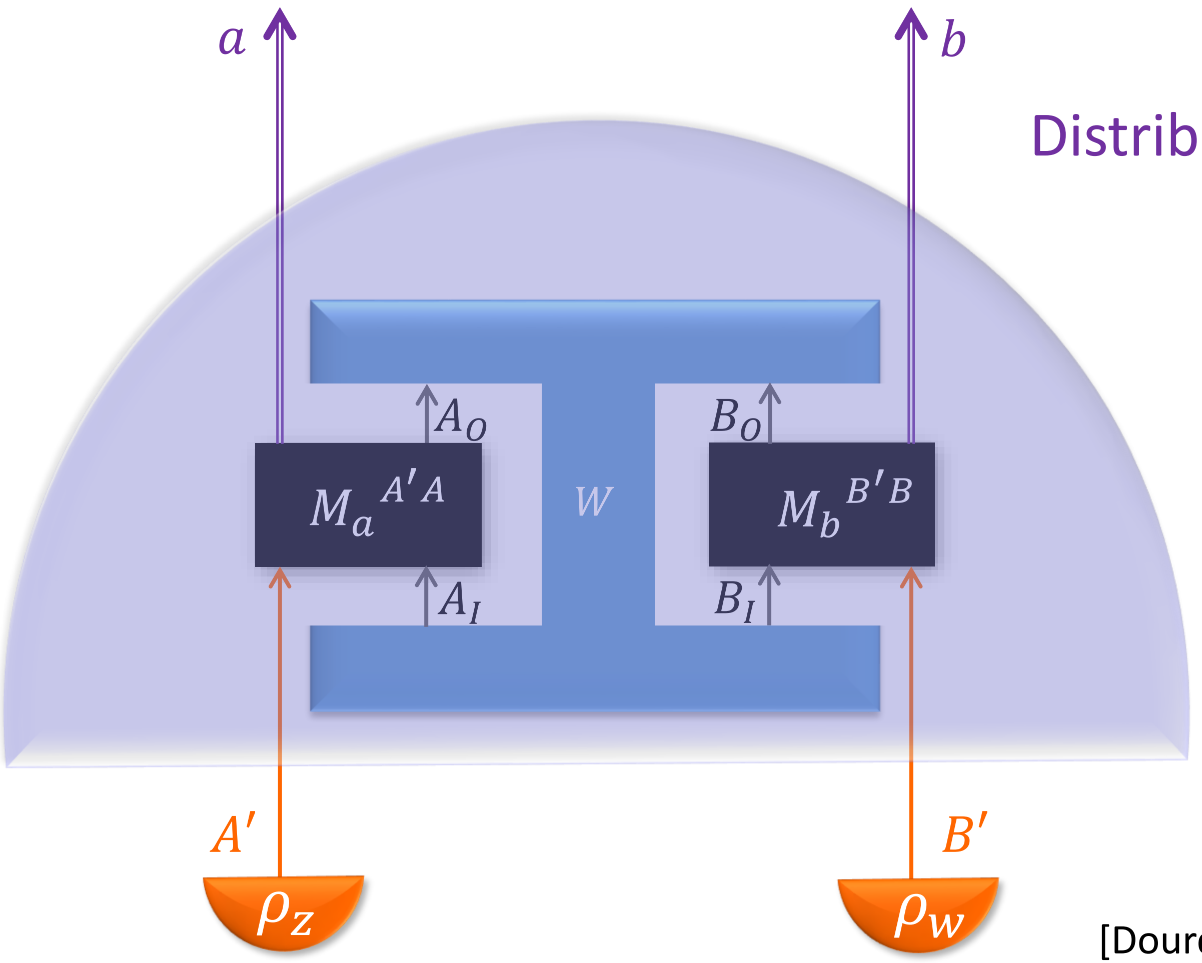
$$M_{f=\pm}^{F} = \mathbf{1}^{F^t} \otimes |f\rangle\langle f|^{F^c}$$

$E_{a,b,f}^{A'B'}$ is *causally nonseparable*.



- The quantum switch is causally nonseparable in a SDIQI way.

Step 1: Semi-Device-Independent Certification with Trusted Quantum Inputs (SDIQI)



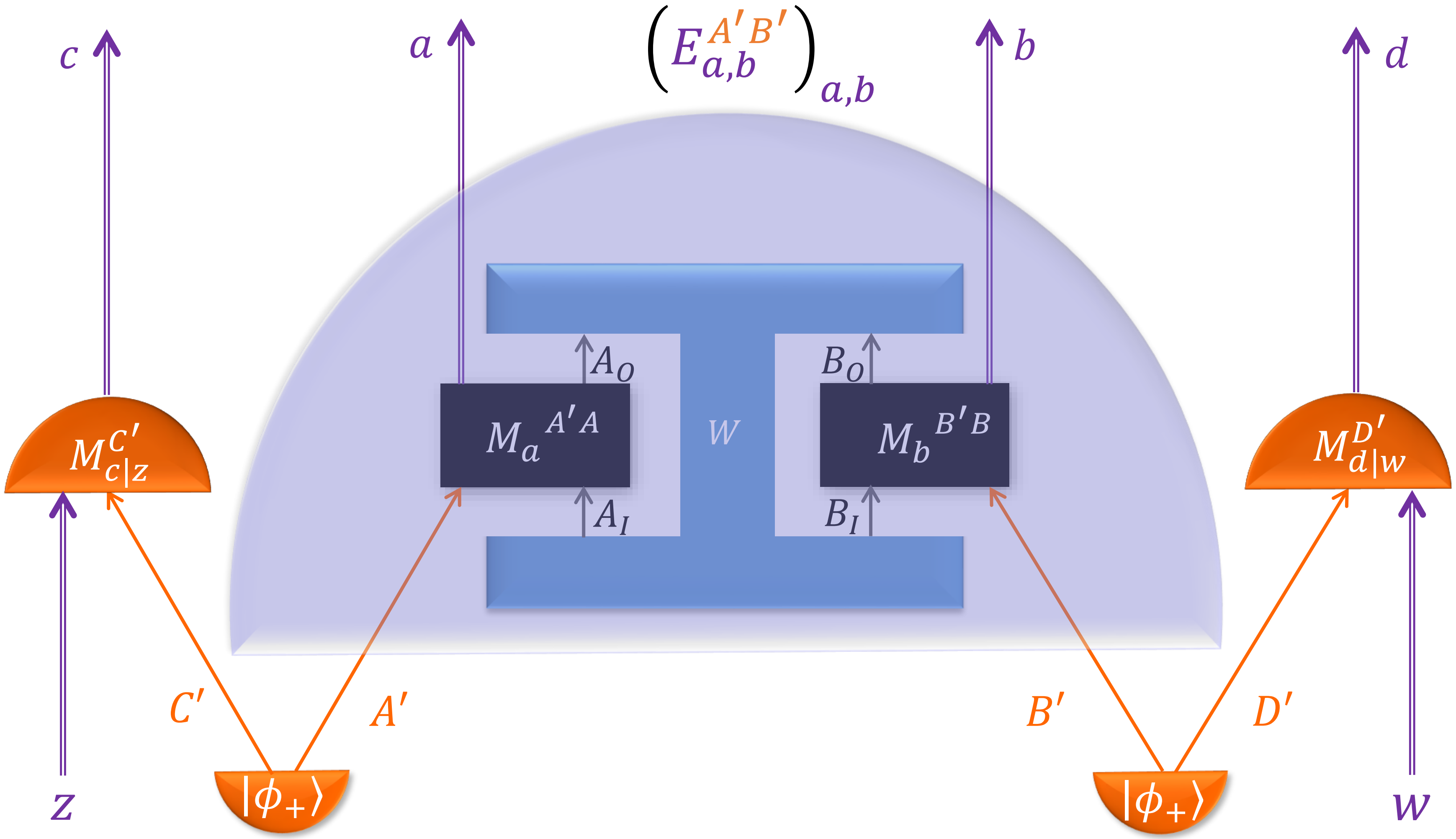
Distributed POVM (D-POVM):

$$\left(E_{a,b}^{A'B'}\right)_{a,b}$$

[Dourdent et al.,PRL **129**, 090402 (2022)]

$$P(a,b|\rho_z^{A'},\rho_w^{B'}) = \mathrm{Tr}\left(\left(M_a^{A'A}\otimes M_b^{B'B}\right)^T.(W^{AB}\otimes\rho_z^{A'}\otimes\rho_w^{B'})\right) = \mathrm{Tr}\left(\left(E_{a,b}^{A'B'}\right)^T.\rho_z^{A'}\otimes\rho_w^{B'}\right)$$

Step 2:The Network-SDIQI Certification of Causal Nonseparability

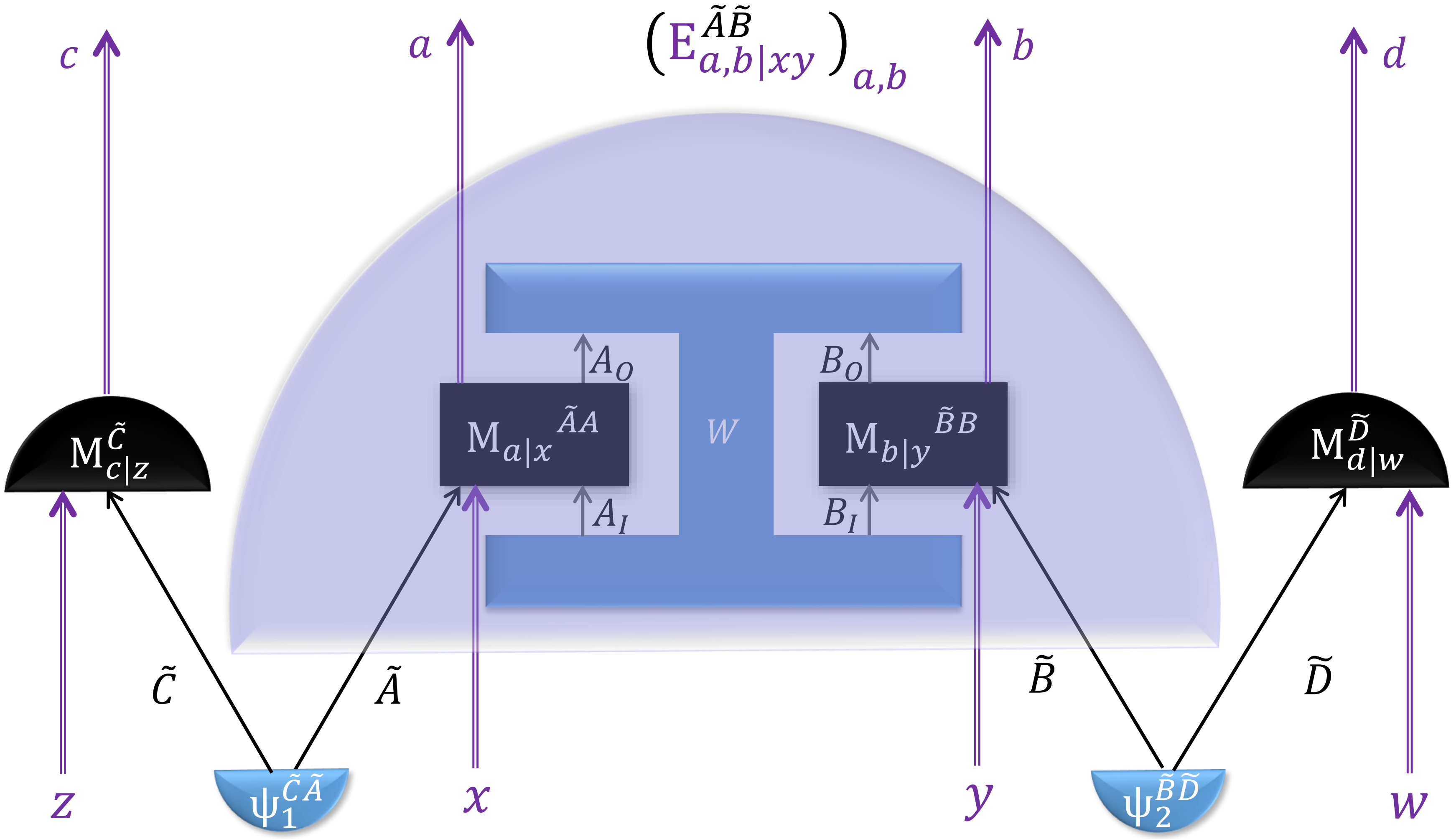


$$P(c,a,b,d|z,w) = \mathrm{Tr}\left(\left(M_{c|z}^{C'} \otimes E_{a,b}^{A'B'} \otimes M_{d|w}^{D'}\right)^T \cdot \left(|\phi_+\rangle\langle\phi_+|^{C'A'} \otimes |\phi_+\rangle\langle\phi_+|^{B'D'}\right)\right)$$

satisfy $\quad P(0,a,b,0|z,w) = P(0|z)P(0|w)P(a,b|\rho_z^{A'}, \rho_w^{B'})$

Relax the assumptions with self-testing techniques.

Step 3: The Network-DI Certification of Causal Nonseparability



Network-Device-Independent Certification of Causal Nonseparability

Hippolyte Dourdent, Alastair A. Abbott, Ivan Šupić, Cyril Branciard

[Dourdent et al., arXiv:230812760]

The Network-DI Certification of Causal Nonseparability

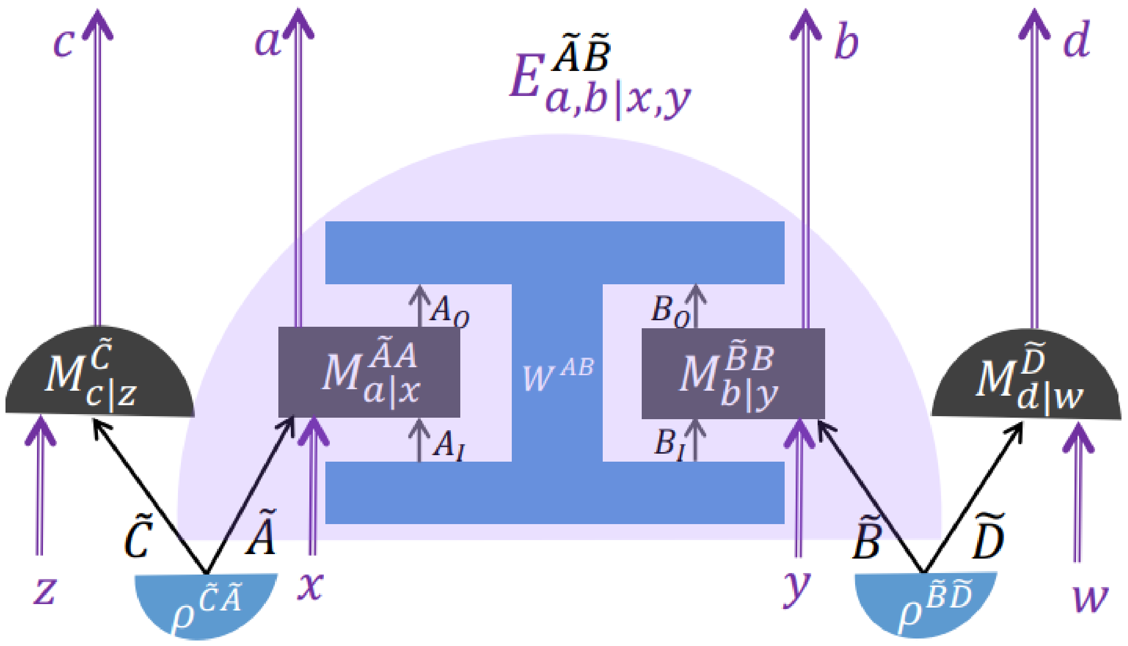
- « Device-Independent »:
  Require knowledge of observed statistics only.

Network-Device-Independent Certification of Causal Nonseparability

Hippolyte Dourdent, Alastair A. Abbott, Ivan Šupić, Cyril Branciard

[Dourdent et al., arXiv:230812760]

- Network: Causal Scenario + 2 Additional Separated Parties



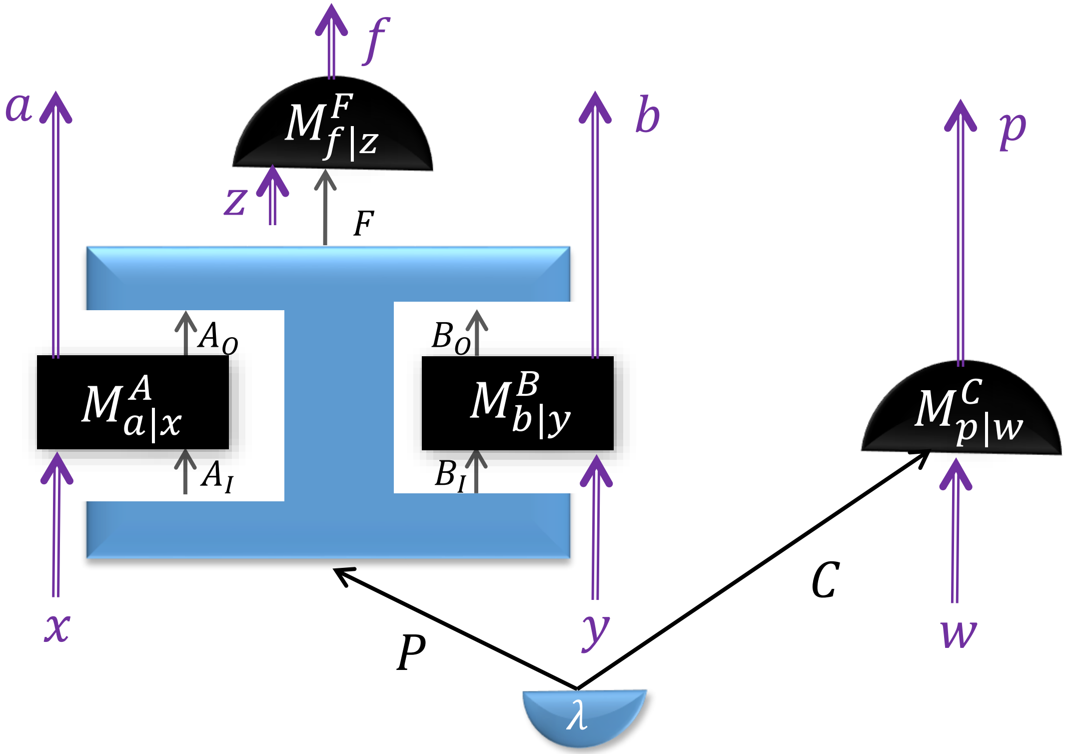
- Scope: All Causally Nonseparable Process Matrices that can generate a causally nonseperable D-POVM.

DI and Theory Independent Certification of Relativistic Indefinite Causal Orders

[Van der Lugt et al, Nat. Com. 14, 2023]

Device-independent certification of indefinite causal order in the quantum switch

Tein van der Lugt[*1,2], Jonathan Barrett[†1,2,3], and Giulio Chiribella[†1,2,3,4]

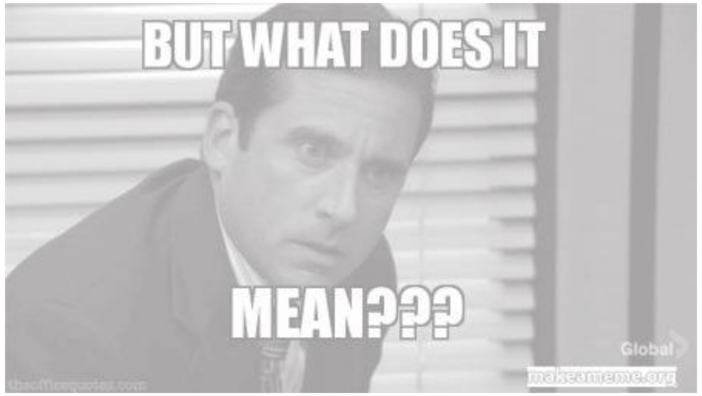


Max. correlations from λ between Charlie and the causal order Alice/Bob

- DRF Inequality: $$P(p=0, b=x|w=0) + \overbrace{P(p=1, a=y|w=0)}^{\leq 1} + \overbrace{P(f \oplus p = zw|x=y=0)}^{\leq \frac{3}{4}} \leq \frac{7}{4}$$

Test nonclassical correlations between P and F (CHSH ineq.)

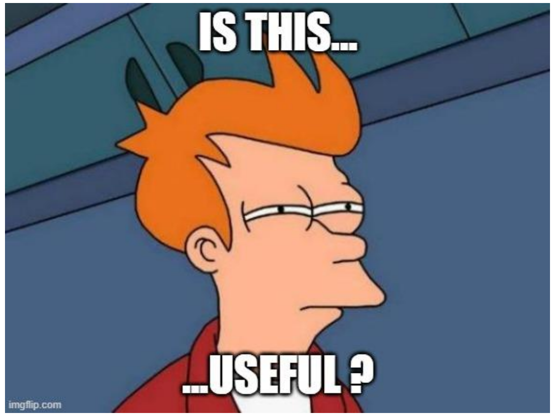
Outline



I – Indefinite Causal Orders : Definition.

II – Device-Independent Certification for the Quantum Switch

[Dourdent, Abbott, Supic & Branciard, arXiv:230812760]



III – Advantages in Quantum Information from the Quantum Switch

The Quantum Switch : Origins

Beyond Quantum Computers

G. Chiribella, G. M. D'Ariano, P.Perinotti, B. Valiron

https://doi.org/10.48550/arXiv.0912.0195

Quantum computations without definite causal structure

Giulio Chiribella, Giacomo Mauro D'Ariano, Paolo Perinotti, and Benoit Valiron
Phys. Rev. A **88**, 022318 – Published 14 August 2013

"The processing of quantum states, however, is not the ultimate physical model of computation that can be conceived within the quantum framework."

« *Higher-order quantum computation* »

Quantum Circuit Architecture

G. Chiribella, G. M. D'Ariano, and P. Perinotti
Phys. Rev. Lett. **101**, 060401 – Published 4 August 2008

Transforming quantum operations: Quantum supermaps

G. Chiribella[1], G. M. D'Ariano[1] and P. Perinotti[1]
Published 11 July 2008 • Europhysics Letters Association
Europhysics Letters, Volume 83, Number 3

Theoretical framework for quantum networks

Giulio Chiribella, Giacomo Mauro D'Ariano, and Paolo Perinotti
Phys. Rev. A **80**, 022339 – Published 31 August 2009

"there exist higher-order computations that are admissible in principle—i.e. their existence does not lead to any paradoxical or unphysical effect—and yet cannot be realized by inserting a single use of the input black box in a quantum circuit with fixed causal ordering of the gates."

La Reproduction Interdite (Not to be Reproduced), Magritte, Brussels, 1937

The Quantum Switch, Advantage in Information Processing

- A Simple Quantum Advantage in Discrimination Task

Perfect discrimination of no-signalling channels via quantum superposition of causal structures

Giulio Chiribella
Phys. Rev. A **86**, 040301(R) – Published 10 October 2012

Quantum computation with programmable connections between gates

Timoteo Colnaghi [a], Giacomo Mauro D'Ariano [a b], Stefano Facchini [a], Paolo Perinotti [a b]

Physics Letters A
Volume 376, Issue 45, 1 October 2012, Pages 2940-2943

Computational Advantage from Quantum-Controlled Ordering of Gates

Mateus Araújo, Fabio Costa, and Časlav Brukner
Phys. Rev. Lett. **113**, 250402 – Published 18 December 2014

Exponential Communication Complexity Advantage from Quantum Superposition of the Direction of Communication

Philippe Allard Guérin, Adrien Feix, Mateus Araújo, and Časlav Brukner
Phys. Rev. Lett. **117**, 100502 – Published 1 September 2016

Computational Advantage from the Quantum Superposition of Multiple Temporal Orders of Photonic Gates

Márcio M. Taddei, Jaime Cariñe, Daniel Martínez, Tania García, Nayda Guerrero, Alastair A. Abbott, Mateus Araújo, Cyril Branciard, Esteban S. Gómez, Stephen P. Walborn, Leandro Aolita, and Gustavo Lima
PRX Quantum **2**, 010320 – Published 8 February 2021

Advantages from the Quantum Switch

Enhanced Communication with the Assistance of Indefinite Causal Order

Daniel Ebler, Sina Salek, and Giulio Chiribella
Phys. Rev. Lett. **120**, 120502 – Published 22 March 2018

Indefinite causal order enables perfect quantum communication with zero capacity channels

Giulio Chiribella[1,2,3], Manik Banik[4], Some Sankar Bhattacharya[9,1], Tamal Guha[5], Mir Alimuddin[5], Arup Roy[6], Sutapa Saha[5], Sristy Agrawal[7,8] and Guruprasad Kar[5]

Published 19 March 2021 • © 2021 The Author(s). Published by IOP Publishing Ltd on behalf of the Institute of Physics and Deutsche Physikalische Gesellschaft

New Journal of Physics, Volume 23, March 2021

Quantum Refrigeration with Indefinite Causal Order

David Felce and Vlatko Vedral
Phys. Rev. Lett. **125**, 070603 – Published 11 August 2020

Noisy quantum metrology with the assistance of indefinite causal order

François Chapeau-Blondeau
Phys. Rev. A **103**, 032615 – Published 29 March 2021

Work extraction from coherently activated maps via quantum switch

Kyrylo Simonov, Gianluca Francica, Giacomo Guarnieri, and Mauro Paternostro
Phys. Rev. A **105**, 032217 – Published 31 March 2022

Increasing communication capacity via superposition of order

K. Goswami, Y. Cao, G. A. Paz-Silva, J. Romero, and A. G. White
Phys. Rev. Research **2**, 033292 – Published 24 August 2020

Quantum communication in a superposition of causal orders

Sina Salek, Daniel Ebler, Giulio Chiribella

Superposition of causal order as a metrological resource for quantum thermometry

Chiranjib Mukhopadhyay, Manish K. Gupta, Arun Kumar Pati

Information Bleaching, No-Hiding Theorem and Indefinite Causal Order

Abhay Srivastav, Arun Kumar Pati

Advantages from the Quantum Switch

Enhanced Communication with the Assistance of Indefinite Causal Order

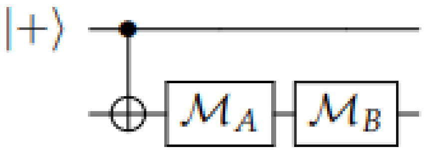
Daniel Ebler, Sina Salek, and Giulio Chiribella
Phys. Rev. Lett. **120**, 120502 – Published 22 March 2018

Communication Through Coherent Control of Quantum Channels

Alastair A. Abbott[1,2], Julian Wechs[2], Dominic Horsman[3], Mehdi Mhalla[3], and Cyril Branciard[2]

Communication through quantum-controlled noise

Philippe Allard Guérin, Giulia Rubino, and Časlav Brukner
Phys. Rev. A **99**, 062317 – Published 17 June 2019



Channel capacity enhancement with indefinite causal order

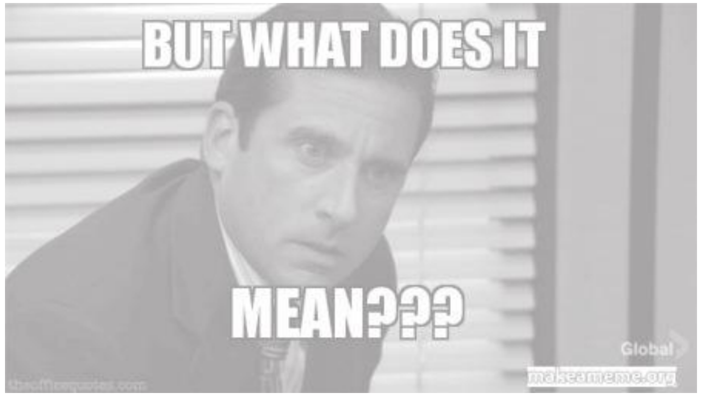
Nicolas Loizeau and Alexei Grinbaum
Phys. Rev. A **101**, 012340 – Published 24 January 2020

Reassessing the advantage of indefinite causal orders for quantum metrology

Raphaël Mothe, Cyril Branciard, Alastair A. Abbott

Tool to Compare Quantum Circuits
with and without indefinite causal orders ?

Outline



I – Indefinite Causal Orders : Definition.

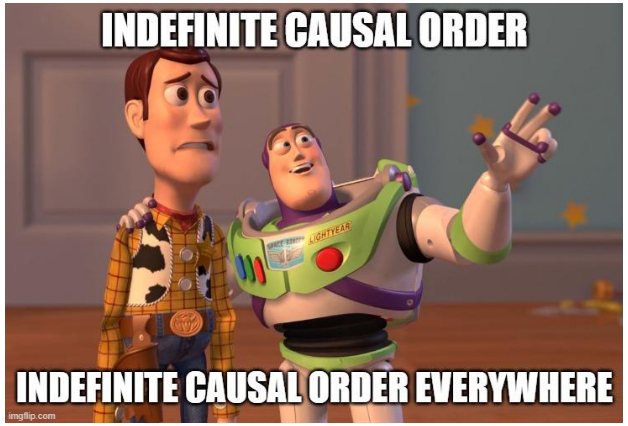
II – Device-Independent Certification for the Quantum Switch

[Dourdent, Abbott, Supic & Branciard, arXiv:230812760]

III – Advantages in Quantum Information from the Quantum Switch

IV- Beyond the Quantum Switch

[Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]



Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]

Choi matrix of a map:

$$M := (\mathcal{I}^X \otimes \mathcal{M})\big(|\mathbb{1}\rangle\!\rangle\langle\!\langle\mathbb{1}|^{XX}\big) = \sum_{i,i'} |i\rangle\langle i'|^X \otimes \mathcal{M}\big(|i\rangle\langle i'|^X\big) \quad \in \mathcal{L}(\mathcal{H}^{XY})$$

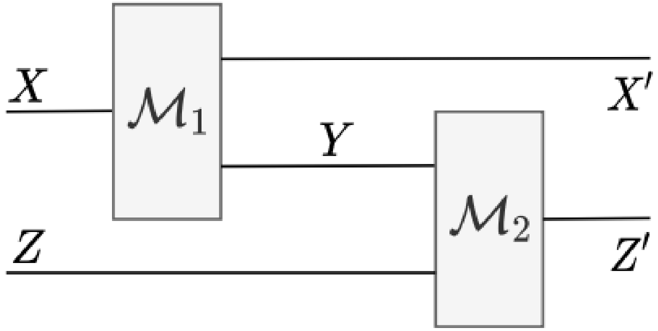


FIG. 2. Composition of two linear maps $\mathcal{M}_1 : \mathcal{L}(\mathcal{H}^X) \to \mathcal{L}(\mathcal{H}^{X'Y})$ and $\mathcal{M}_2 : \mathcal{L}(\mathcal{H}^{YZ}) \to \mathcal{L}(\mathcal{H}^{Z'})$ (as indicated by the labels on the wires, to be read from left to right). The Choi matrix of the composed map $\mathcal{M} := (\mathcal{I}^{X'} \otimes \mathcal{M}_2) \circ (\mathcal{M}_1 \otimes \mathcal{I}^Z)$ is obtained as the *link product* of the Choi matrices of $\mathcal{M}_1$ and $\mathcal{M}_2$, as in Eq. (9)—and similarly for the "pure case" of two linear operators $V_1 : \mathcal{H}^X \to \mathcal{H}^{X'Y}$ and $V_2 : \mathcal{H}^{YZ} \to \mathcal{H}^{Z'}$, as in Eq. (8).

Link product:

[Chiribella, D'Ariano, Perinotti, PRL 2008, PRA 2009]

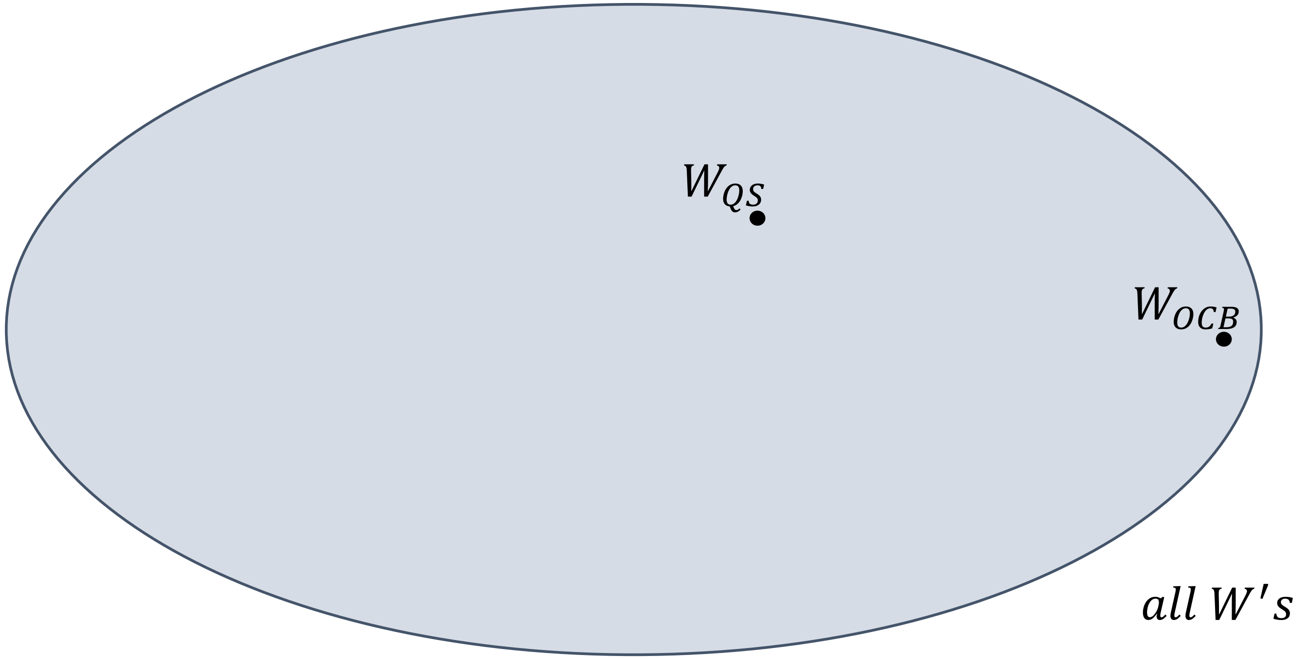
$$M = M_1 * M_2 \quad \in \mathcal{L}(\mathcal{H}^{XX'ZZ'})$$

Similarly, the link product of any two operators $A \in \mathcal{L}(\mathcal{H}^{XY})$ and $B \in \mathcal{L}(\mathcal{H}^{YZ})$ is defined as [1, 2][5]

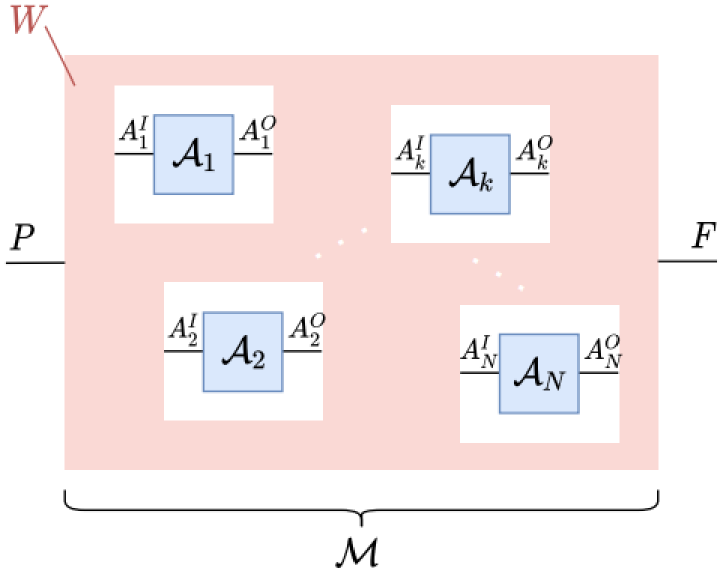
$$\begin{aligned} A * B &:= \big(\mathbb{1}^{XZ} \otimes \langle\!\langle\mathbb{1}|^{YY}\big)(A \otimes B)\big(\mathbb{1}^{XZ} \otimes |\mathbb{1}\rangle\!\rangle^{YY}\big) \\ &= \mathrm{Tr}_Y\big[(A^{T_Y} \otimes \mathbb{1}^Z)(\mathbb{1}^X \otimes B)\big] \\ &= \sum_{ii'} A^X_{ii'} \otimes B^Z_{ii'} \quad \in \mathcal{L}(\mathcal{H}^{XZ}) \qquad (7) \end{aligned}$$

Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]

PROCESS MATRICES



Process Matrices as Quantum Supermaps

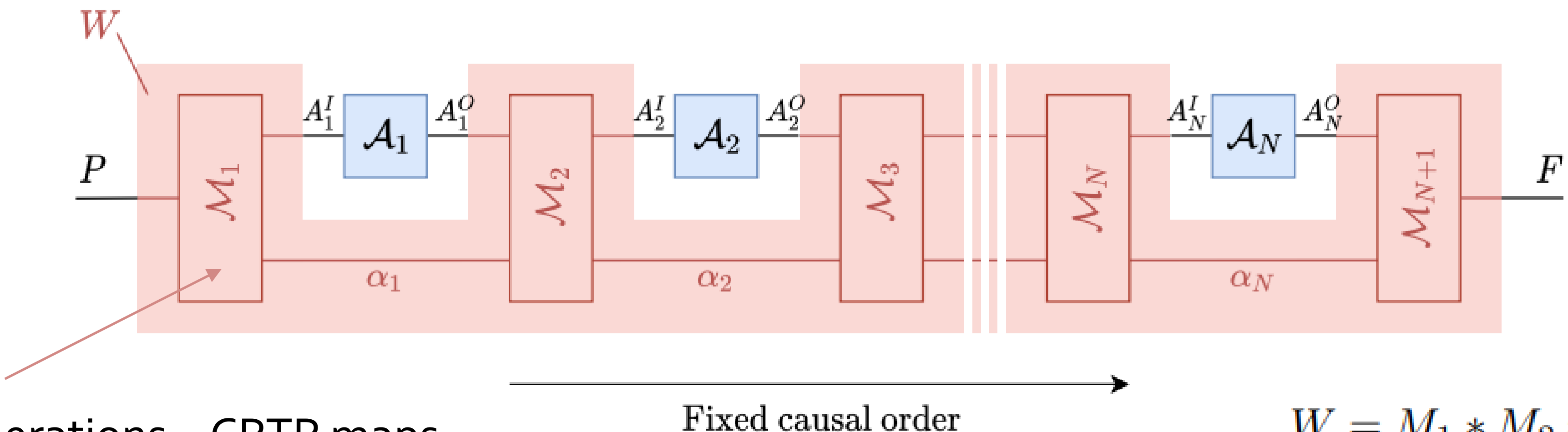


- "Process matrices" W [Oreshkov, Costa & Brukner, Nat Comms 2012]

$$\begin{aligned} M &= \mathrm{Tr}_{A_N^{IO}}\left[(A_1^T \otimes \cdots \otimes A_N^T \otimes \mathbb{1}^{PF})W\right] \\ &= (A_1 \otimes \cdots \otimes A_N) * W \quad \in \mathcal{L}(\mathcal{H}^{PF}), \end{aligned}$$

$\boxed{\text{Which one are physical ? Let us start with what we know !}}$

Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]



- Internal operations = CPTP maps

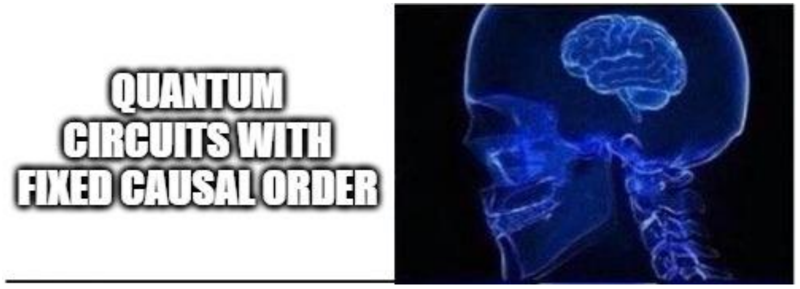
$$W = M_1 * M_2 * \cdots * M_N * M_{N+1}$$

Proposition 2 (Characterisation of QC-FOs). *For a given matrix* $W \in \mathcal{L}(\mathcal{H}^{PA_N^{IO}F})$, *let us define the reduced matrices (for* $1 \le n \le N$, *and relative to the fixed order* $(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_N)$) $W_{(n)} := \frac{1}{d_n^O d_{n+1}^O \cdots d_N^O} \mathrm{Tr}_{A_n^O A_{\{n+1,\ldots,N\}}^{IO} F} W \in \mathcal{L}(\mathcal{H}^{PA_{\{1,\ldots,n-1\}}^{IO} A_n^I})$.

The process matrix $W \in \mathcal{L}(\mathcal{H}^{PA_N^{IO}F})$ *of a quantum circuit with the fixed causal order* $(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_N)$ *is a positive semidefinite matrix such that its reduced matrices* $W_{(n)}$ *just defined satisfy*

$$\begin{aligned} &\mathrm{Tr}_{A_1^I} W_{(1)} = \mathbb{1}^P, \\ &\forall n = 1, \ldots, N-1, \quad \mathrm{Tr}_{A_{n+1}^I} W_{(n+1)} = W_{(n)} \otimes \mathbb{1}^{A_n^O}, \\ &\text{and} \quad \mathrm{Tr}_F W = W_{(N)} \otimes \mathbb{1}^{A_N^O}. \end{aligned} \tag{19}$$

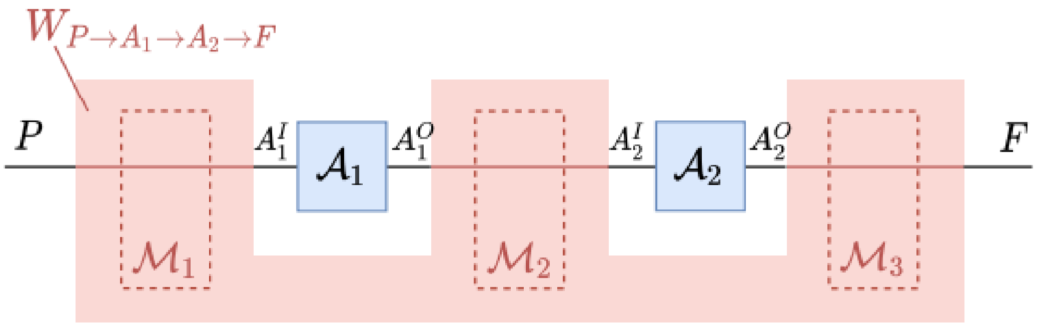
Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]

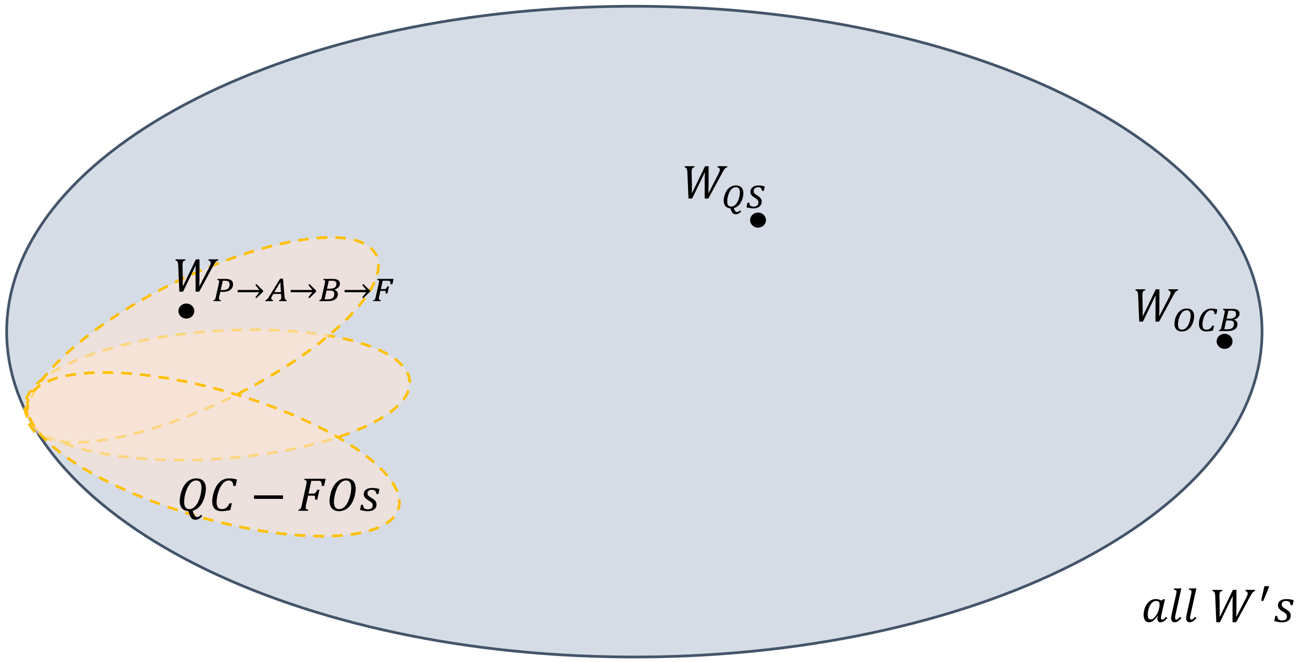


- “Quantum combs”

[Chiribella, D’Ariano & Perinotti, EPL 2008, PRL 2008, PRA 2009]

- A fixed causally ordered process $W_{P\to A\to B\to F}$

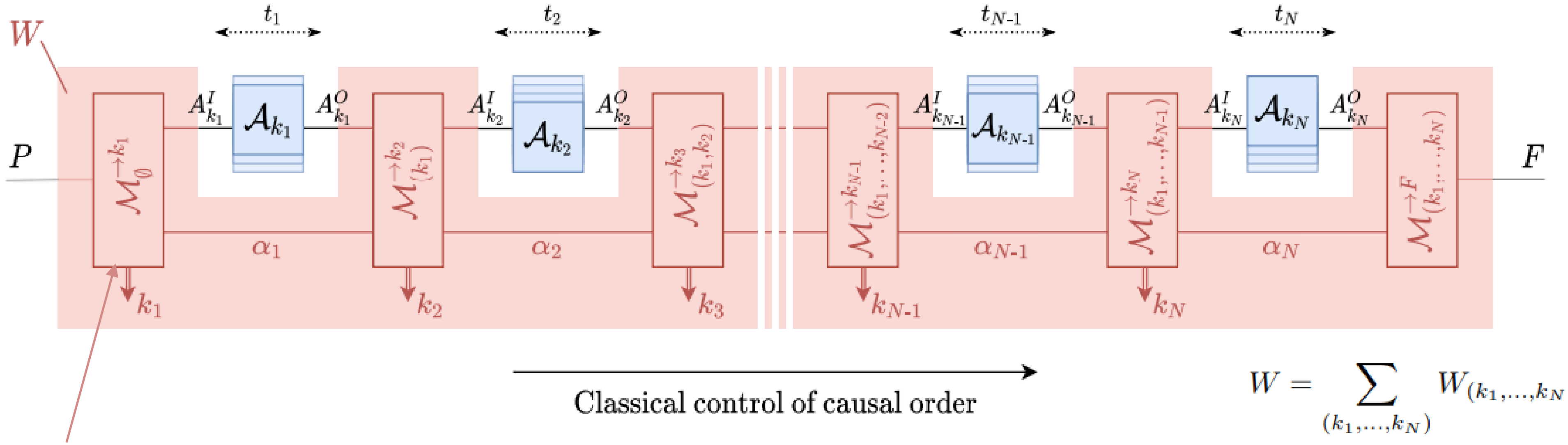




- “Process matrices” W [Oreshkov, Costa & Brukner, Nat Comms 2012]

- What kind of quantum circuits are incompatible with definite causal orders ?

Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]



$$W = \sum_{(k_1,\dots,k_N)} W_{(k_1,\dots,k_N,F)} \tag{28}$$

- Internal operations = Quantum Instruments

where

$$W_{(k_1,\dots,k_N,F)} := M_\emptyset^{\to k_1} * M_{(k_1)}^{\to k_2} * M_{(k_1,k_2)}^{\to k_3} * \cdots \cdots * M_{(k_1,\dots,k_{N-1})}^{\to k_N} * M_{(k_1,\dots,k_N)}^{\to F} \in \mathcal{L}(\mathcal{H}^{PA_\mathcal{N}^{IO}F}). \tag{29}$$

Proposition 5 (Characterisation of QC-CCs). *The process matrix $W \in \mathcal{L}(\mathcal{H}^{PA_\mathcal{N}^{IO}F})$ of a quantum circuit with classical control of causal order can be decomposed in terms of positive semidefinite matrices $W_{(k_1,\dots,k_n)} \in \mathcal{L}(\mathcal{H}^{PA^{IO}_{\{k_1,\dots,k_{n-1}\}}A^I_{k_n}})$ and $W_{(k_1,\dots,k_N,F)} \in \mathcal{L}(\mathcal{H}^{PA_\mathcal{N}^{IO}F})$, for all nonempty ordered subsets $(k_1,\dots,k_n)$ of $\mathcal{N}$ (with $1 \le n \le N$, $k_i \neq k_j$ for $i \neq j$), in such a way that*

$$W = \sum_{(k_1,\dots,k_N)} W_{(k_1,\dots,k_N,F)} \tag{30}$$

$$\sum_{k_1} \mathrm{Tr}_{A^I_{k_1}} W_{(k_1)} = \mathbb{1}^P,$$

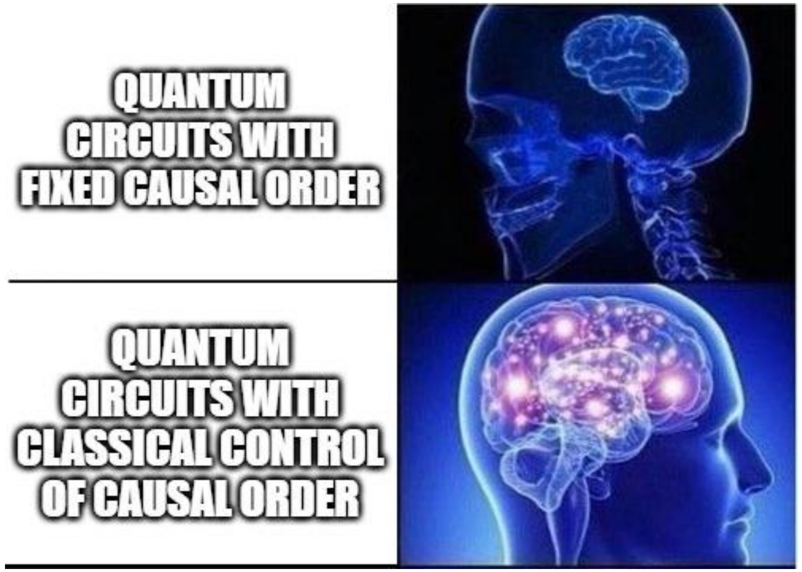
$\forall n = 1,\dots,N-1,\ \forall (k_1,\dots,k_n),$

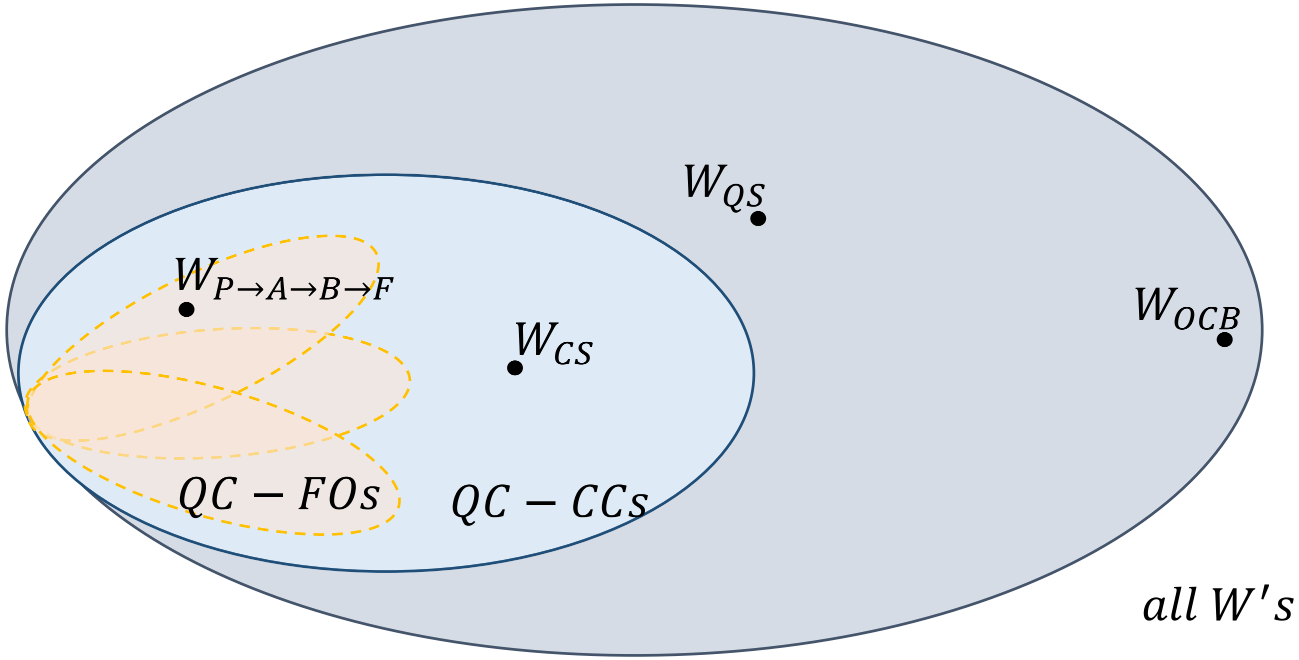
$$\sum_{k_{n+1}} \mathrm{Tr}_{A^I_{k_{n+1}}} W_{(k_1,\dots,k_n,k_{n+1})} = W_{(k_1,\dots,k_n)} \otimes \mathbb{1}^{A^O_{k_n}},$$

and $\forall (k_1,\dots,k_N),$

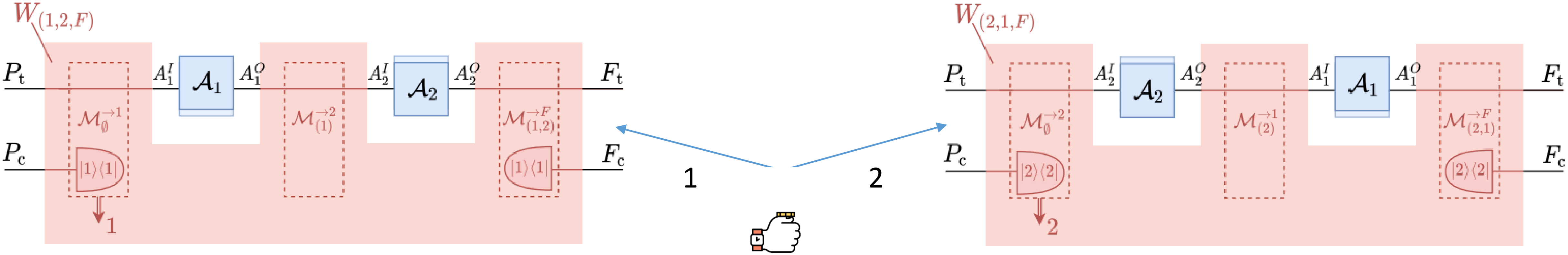
$$\mathrm{Tr}_F\, W_{(k_1,\dots,k_N,F)} = W_{(k_1,\dots,k_N)} \otimes \mathbb{1}^{A^O_{k_N}}. \tag{31}$$

Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]



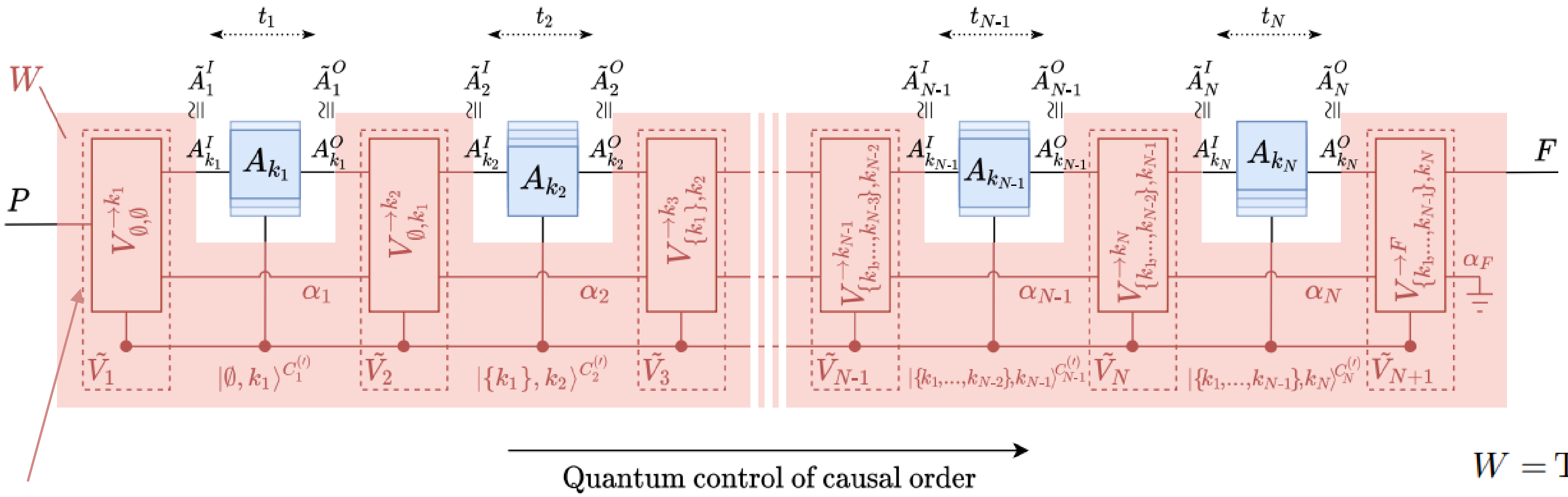


- The classical switch W_{CS}



- **QC-CC = Causally separable process matrices => Beyond ?**

Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]



- Internal coherent operations (Kraus op)

$$W = \mathrm{Tr}_{\alpha_F} |w_{(\mathcal{N},F)}\rangle\!\langle w_{(\mathcal{N},F)}|$$

with $\quad |w_{(\mathcal{N},F)}\rangle := \sum_{(k_1,\dots,k_N)} |w_{(k_1,\dots,k_N,F)}\rangle$

and with

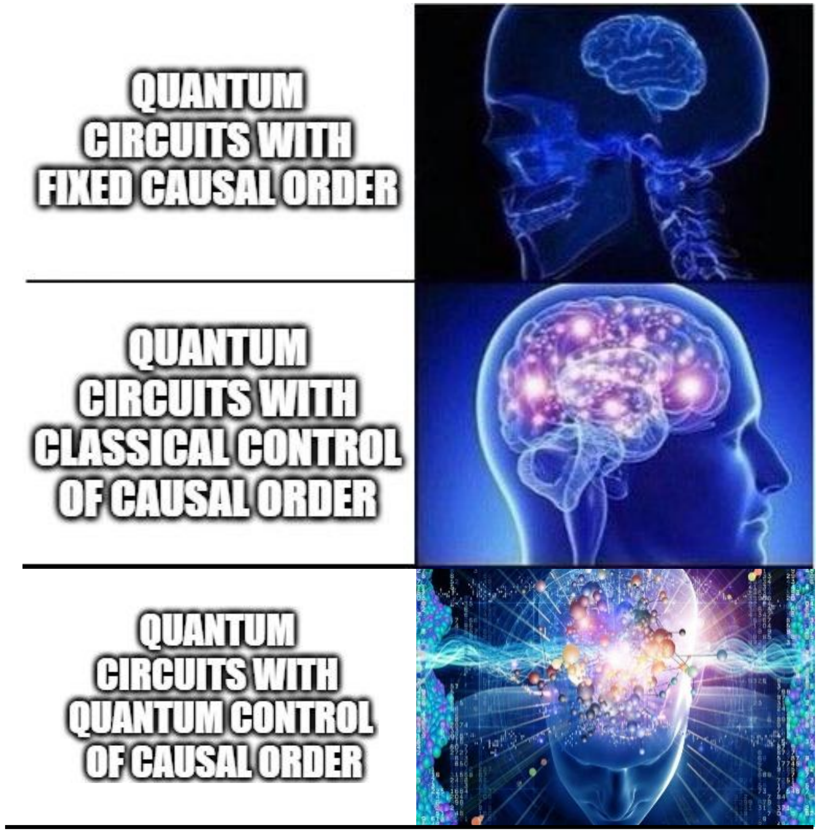
$$\begin{aligned} &|w_{(k_1,\dots,k_N,F)}\rangle \\ &\quad := |V_{\emptyset,\emptyset}^{\to k_1}\rangle\!\rangle * |V_{\emptyset,k_1}^{\to k_2}\rangle\!\rangle * |V_{\{k_1\},k_2}^{\to k_3}\rangle\!\rangle * \cdots \\ &\qquad \cdots * |V_{\{k_1,\dots,k_{N-2}\},k_{N-1}}^{\to k_N}\rangle\!\rangle * |V_{\{k_1,\dots,k_{N-1}\},k_N}^{\to F}\rangle\!\rangle \\ &\qquad \in \mathcal{H}^{PA_N^{IO}F\alpha_F}. \end{aligned} \tag{62}$$

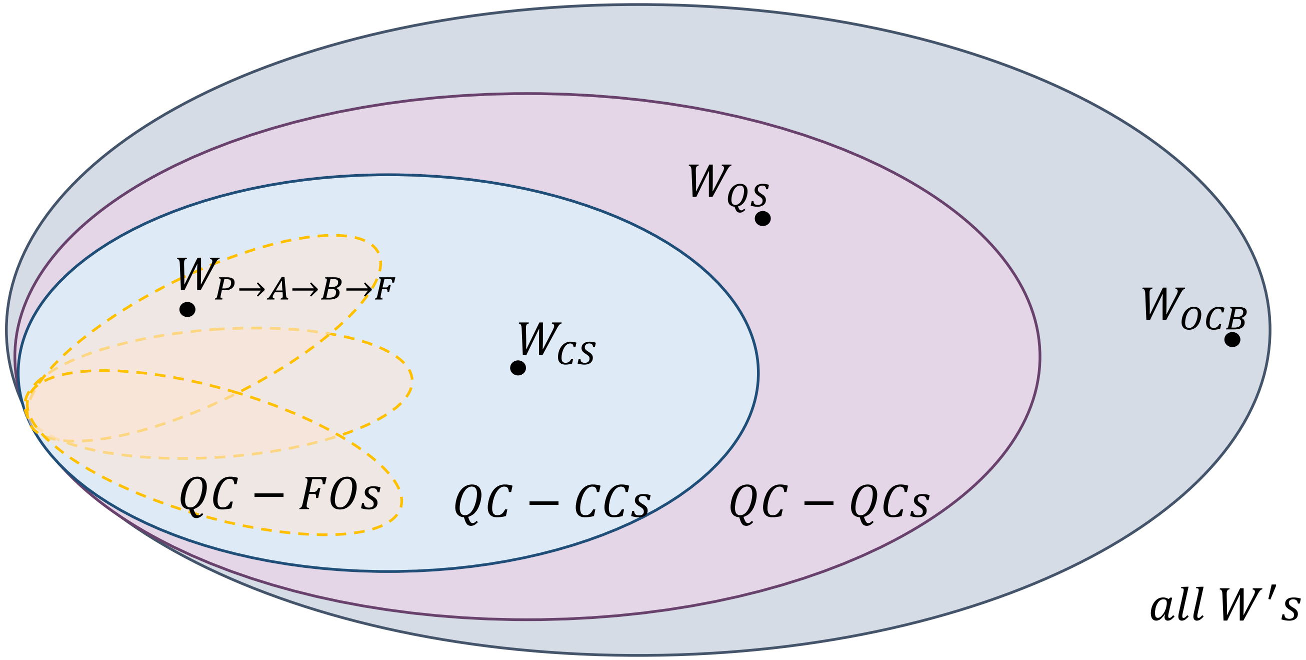
Proposition 7 (Characterisation of QC-QCs). *The process matrix $W \in \mathcal{L}(\mathcal{H}^{PA_N^{IO}F})$ of a quantum circuit with quantum control of causal order is such that there exist positive semidefinite matrices $W_{(\mathcal{K}_{n-1},k_n)} \in \mathcal{L}(\mathcal{H}^{PA_{\mathcal{K}_{n-1}}^{IO}A_{k_n}^I})$, for all strict subsets $\mathcal{K}_{n-1}$ of $\mathcal{N}$ and all $k_n \in \mathcal{N}\backslash\mathcal{K}_{n-1}$, satisfying*

$$\sum_{k_1\in\mathcal{N}} \mathrm{Tr}_{A_{k_1}^I} W_{(\emptyset,k_1)} = \mathbb{1}^P, \qquad \forall \emptyset \subsetneq \mathcal{K}_n \subsetneq \mathcal{N}, \sum_{k_{n+1}\in\mathcal{N}\backslash\mathcal{K}_n} \mathrm{Tr}_{A_{k_{n+1}}^I} W_{(\mathcal{K}_n,k_{n+1})} = \sum_{k_n\in\mathcal{K}_n} W_{(\mathcal{K}_n\backslash k_n,k_n)} \otimes \mathbb{1}^{A_{k_n}^O},$$

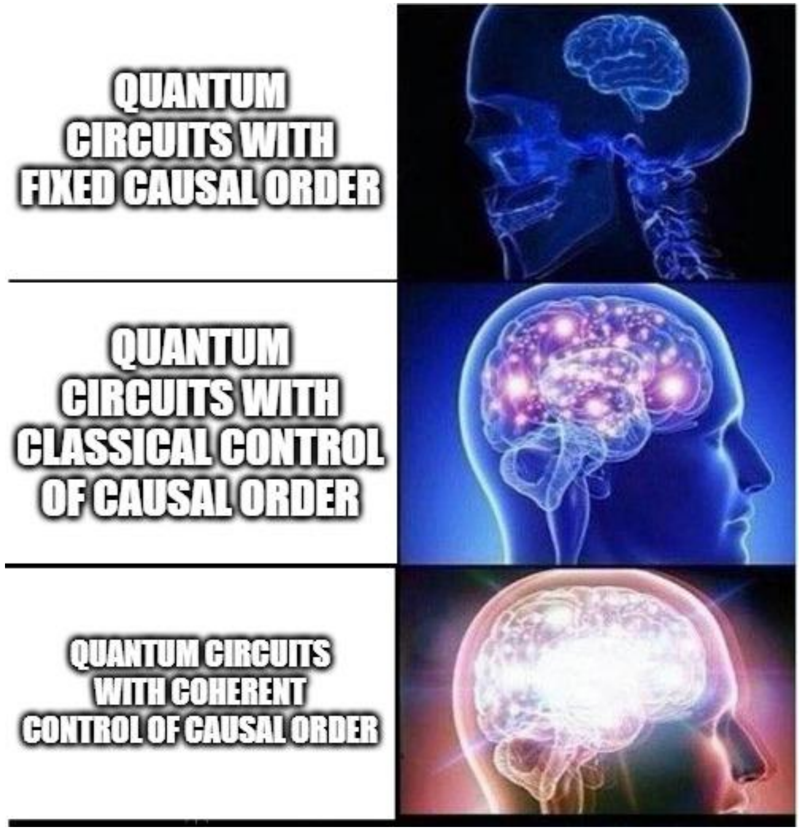
and $\quad \mathrm{Tr}_F W = \sum_{k_N\in\mathcal{N}} W_{(\mathcal{N}\backslash k_N,k_N)} \otimes \mathbb{1}^{A_{k_N}^O}.$

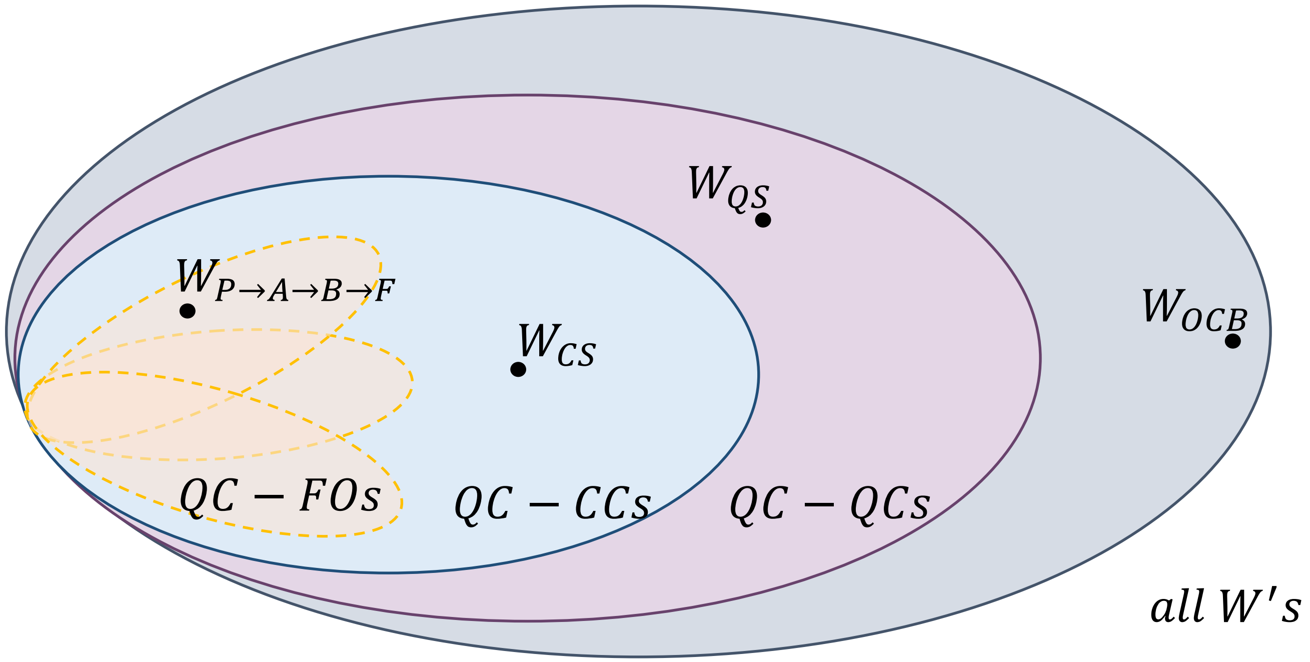
Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]





Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]

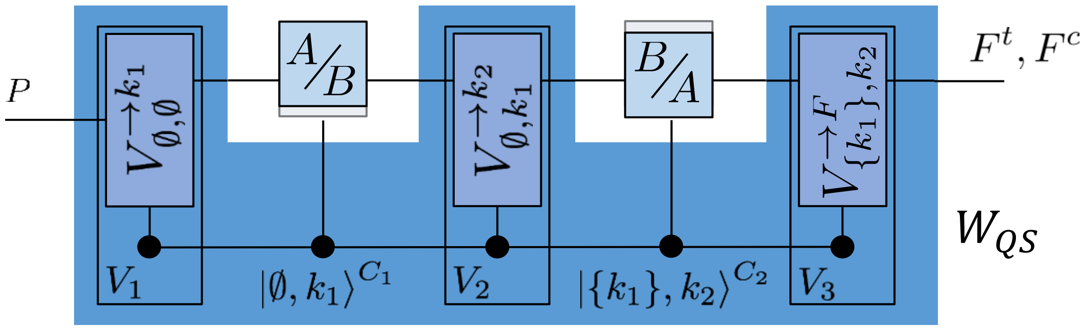




$$|V_{\emptyset,\emptyset}^{\to A}\rangle\rangle = \frac{1}{\sqrt{2}}|\psi\rangle^{A^I}, \quad |V_{\emptyset,\emptyset}^{\to B}\rangle\rangle = \frac{1}{\sqrt{2}}|\psi\rangle^{B^I}$$

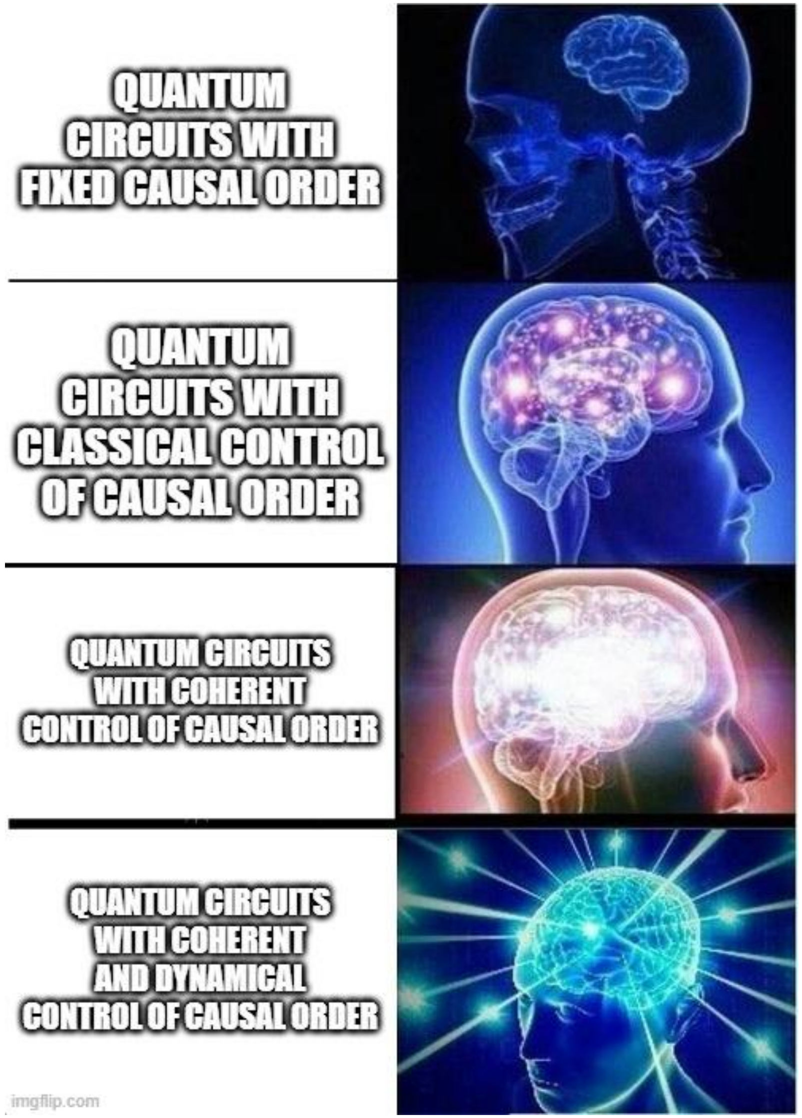
$$|V_{\emptyset,A}^{\to B}\rangle\rangle = |\mathbb{I}\rangle\rangle^{A^O B^I}, \quad |V_{\emptyset,B}^{\to A}\rangle\rangle = |\mathbb{I}\rangle\rangle^{B^O A^I}$$

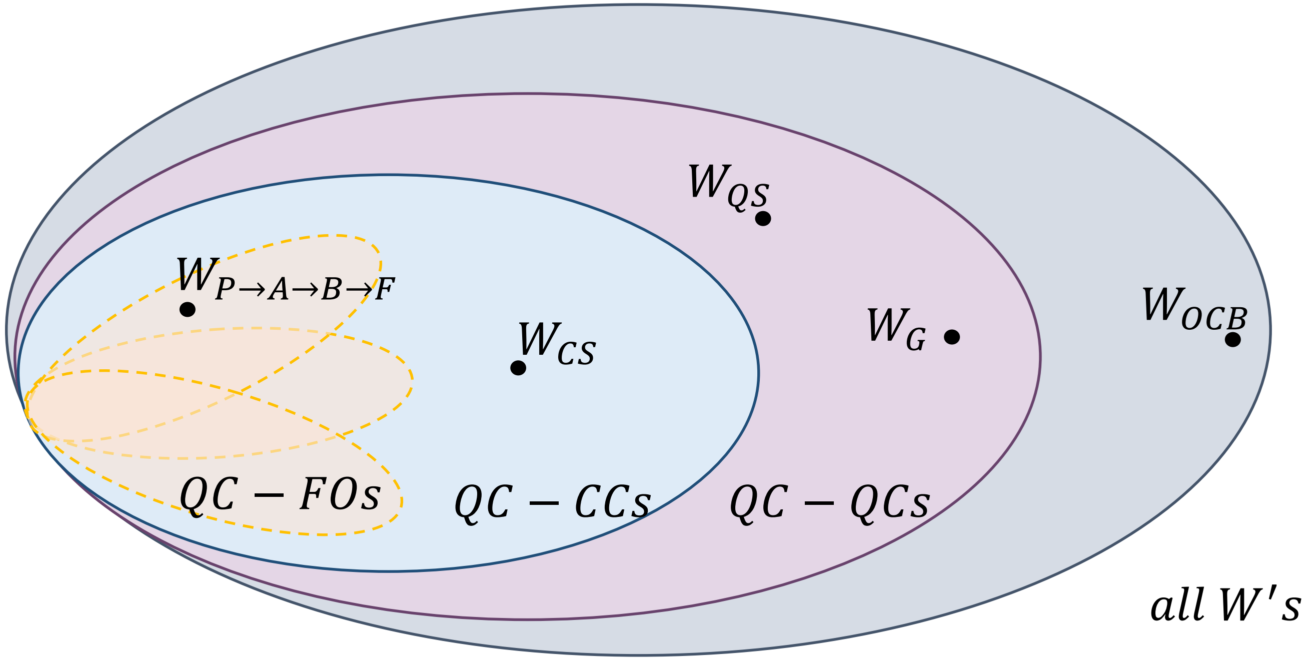
$$|V_{\{A\},B}^{\to F}\rangle\rangle = |\mathbb{I}\rangle\rangle^{B^O F^t}|0\rangle^{F^c}, \quad |V_{\{B\},A}^{\to F}\rangle\rangle = |\mathbb{I}\rangle\rangle^{A^O F^t}|1\rangle^{F^c}$$

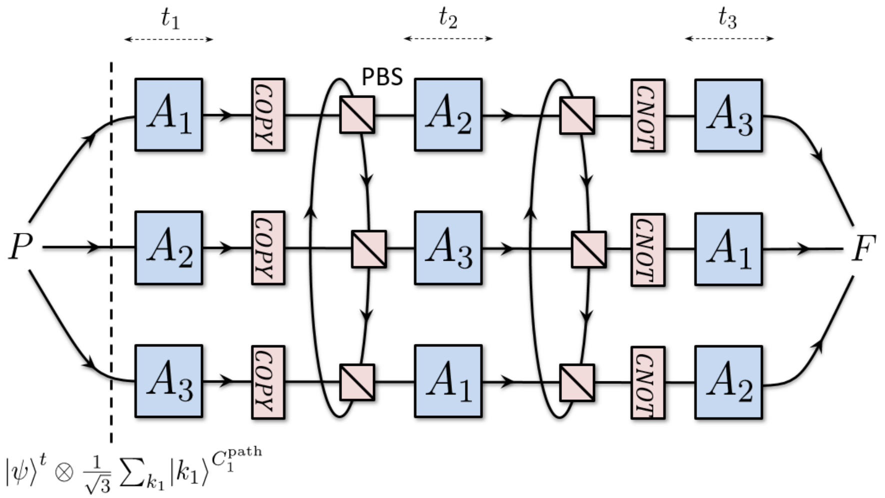


- The quantum switch W_{QS}

Beyond the Quantum Switch, [Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]

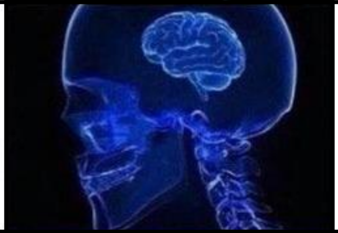






- The Grenoble Process

QI Advantage: K-unitary equivalence determination problem

CLASS OF CIRCUITS		$K=2$	$K=3$
QUANTUM CIRCUITS WITH QUANTUM OPERATIONS IN PARALLEL		0.875^*	0.6919
QUANTUM CIRCUITS WITH FIXED CAUSAL ORDER		0.875^*	0.6998
QUANTUM CIRCUITS WITH CLASSICAL CONTROL OF CAUSAL ORDER		0.875	0.6998
QUANTUM CIRCUITS WITH QUANTUM CONTROL OF CAUSAL ORDER		0.875	0.7080
NONCAUSAL PROCESS MATRICES		0.875	0.7093

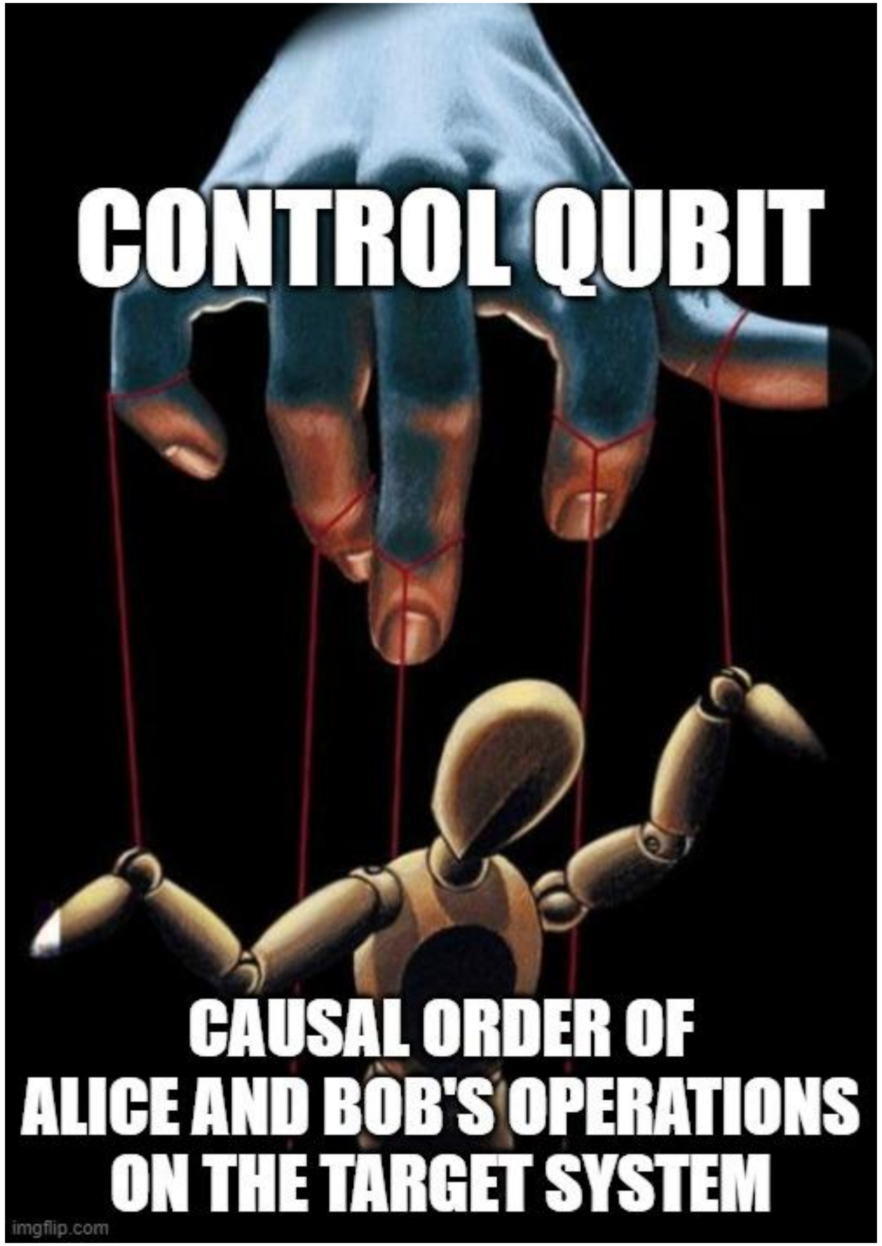
- To quantify how a given class of circuits performs for some task, we optimise over the corresponding higher-order transformations (characterised with SDP constraints) in order to maximise the success probability of the task.

- K *reference boxes* which implement black-box unitary operations $U_1, \dots, U_K$, and a further unknown *target box* that implements one of the U_k with probability $\frac{1}{K}$.
- Aim: to determined which of the reference boxes is implemented by the target box, while using each of the $K+1$ boxes exactly once.

[Wechs, Dourdent, Abbott & Branciard, PRX Quantum 2, 2021]

*[Shimbo, Soeda & Murao, 2021]

Take Away Memes : Conclusion



Noncausality

Causal Nonseparability

Take Away Memes : Conclusion

Network-Device-Independent Certification of Causal Nonseparability

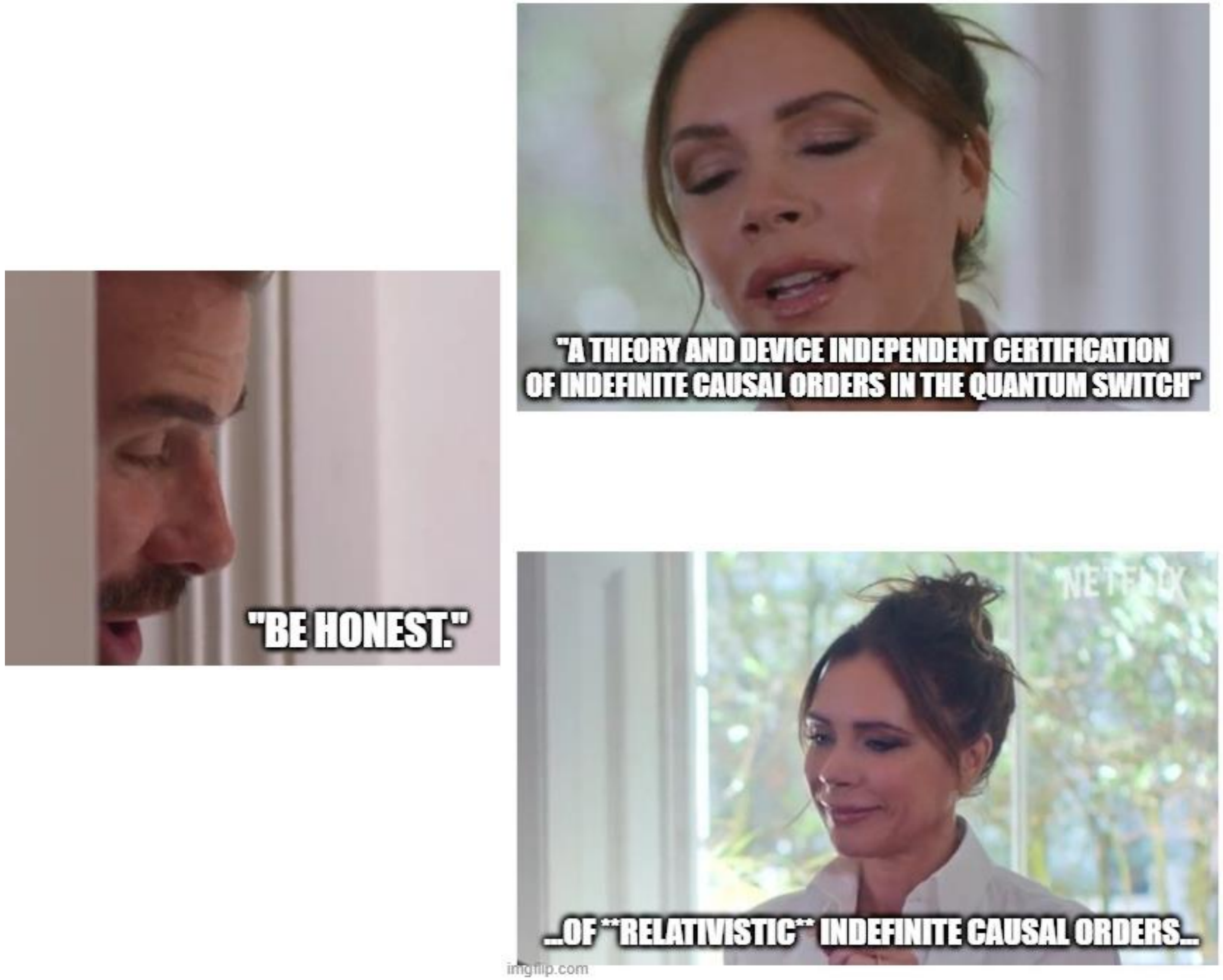
Hippolyte Dourdent, Alastair A. Abbott, Ivan Šupić, Cyril Branciard

[Dourdent et al., arXiv:230812760]

Device-independent certification of indefinite causal order in the quantum switch

Tein van der Lugt*[1,2], Jonathan Barrett†[1,2,3], and Giulio Chiribella†[1,2,3,4]

[Van der Lugt et al, Nat. Com. 14, 2023]



Take Away Memes : Conclusion