Hello Everyone!

Jacopo Surace - December 2024

University of Strathclyde Glasgow

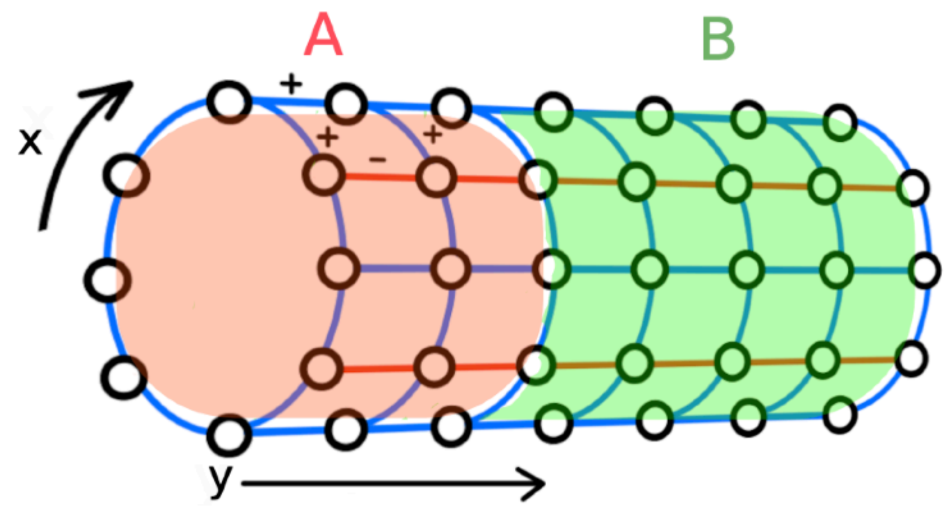
PhD in Condensed Matter,
Glasgow, Scotland, Strathclyde University

Supervisor Luca Tagliacozzo

Entanglement in many-body systems:

Tools that I learned:

- Ising Chains, Entanglement growth, Tensor networks, DMRG, Conformal field theories



Tools that I created:

- Package for simulation of Fermionic Systems in Julia

F_utilities.pkg

GitHub

ICFO

(?)

Postdoc in Quantum Information group
Barcelona, Spain, ICFO

Supervisor Antonio Acín

Many body system certification:

- Certification of bound on energies and other observables of many body systems with SDP and DMRG.

Undecidability theory:

- Undecidability of the membership problem in resource theories. Techniques for proving undecidability. Algorithmic information theory.

Information Geometry:

- Fisher Information, contractivity of channels, non-Markovianity, Reverse Csizar theorem, error correction.

Foundation of probability theory:

- Bayes Theorem, Maximum entropy methods, derivation of probability, De Finetti subjectivism.

PERIMETER INSTITUTE

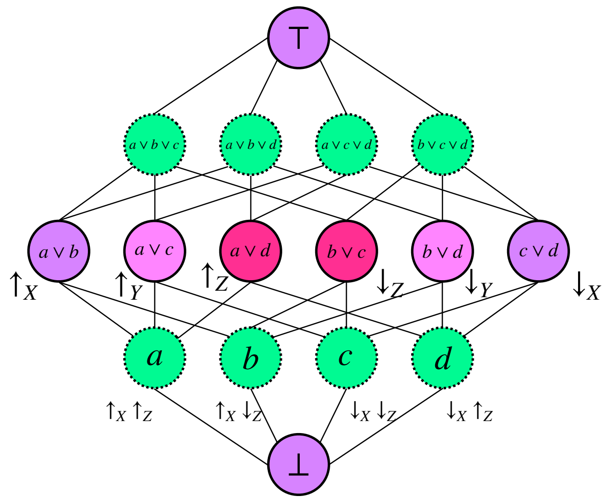
Postdoc in Quantum Foundation group
Waterloo, ON, Canada, Perimeter Institute
Supervisor Robert Spekkens

Symmetries and Local tomography:

- Unitary representation of groups, failure of local tomography in classical, quantum and post-quantum theories and GPT.

Logic, lattices, quasi-probability:

- Reconstruction of probability and quasi-probability from logic, Inaccessibility Hypothesis, epistemic restrictions. Reconstruction of quantum mechanics from logic.



Correlations in time and space:

- State over times, quantum inference, Wigner Friends, time reversal in quantum and classical mechanics.

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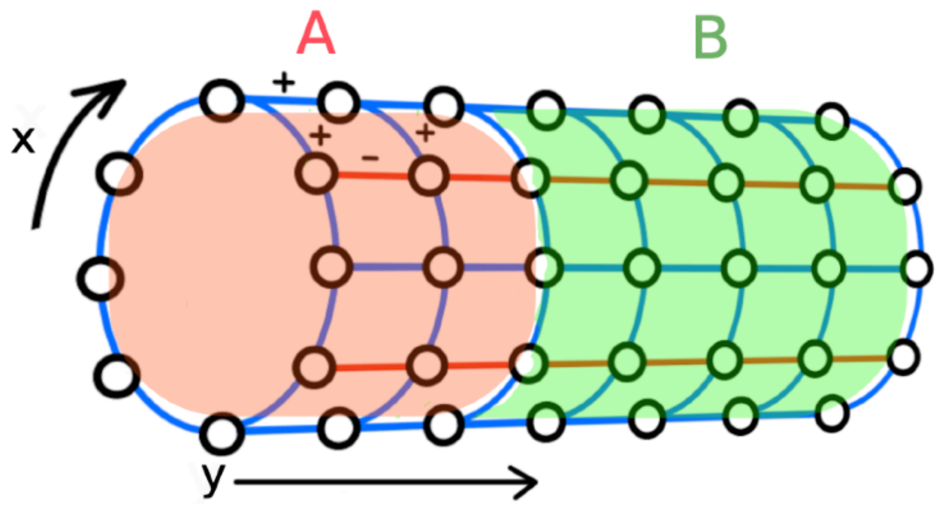

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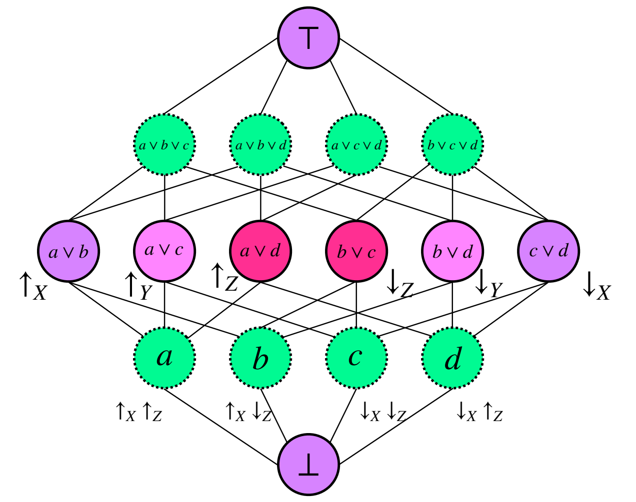

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- State over times, quantum inference, Wigner Friends, time reversal in quantum and classical mechanics.

State retrieval beyond Bayes' retrodiction and reverse processes

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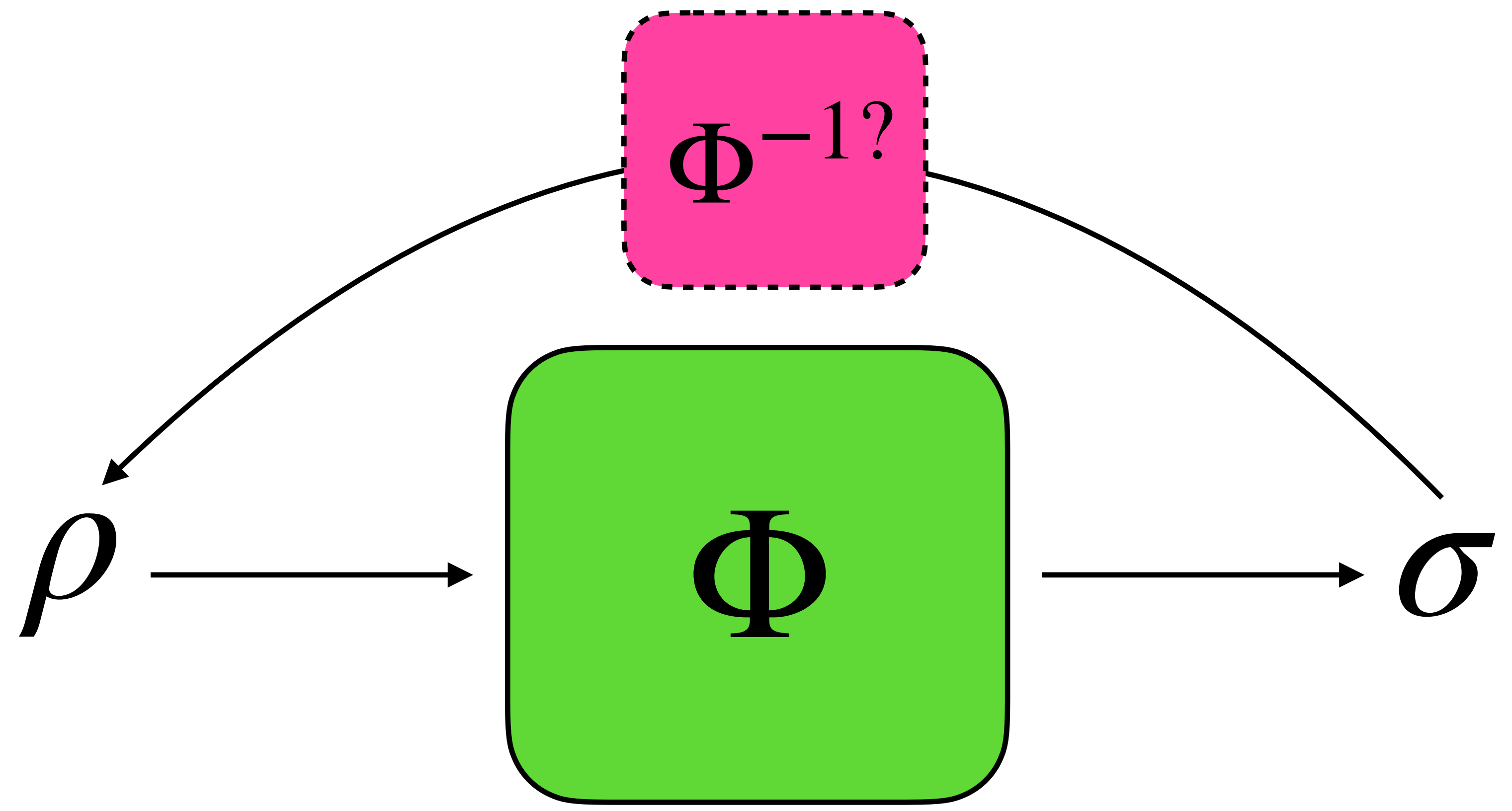
How to reverse a not reversible channel?

Intuitive analogy

Mathematical definition

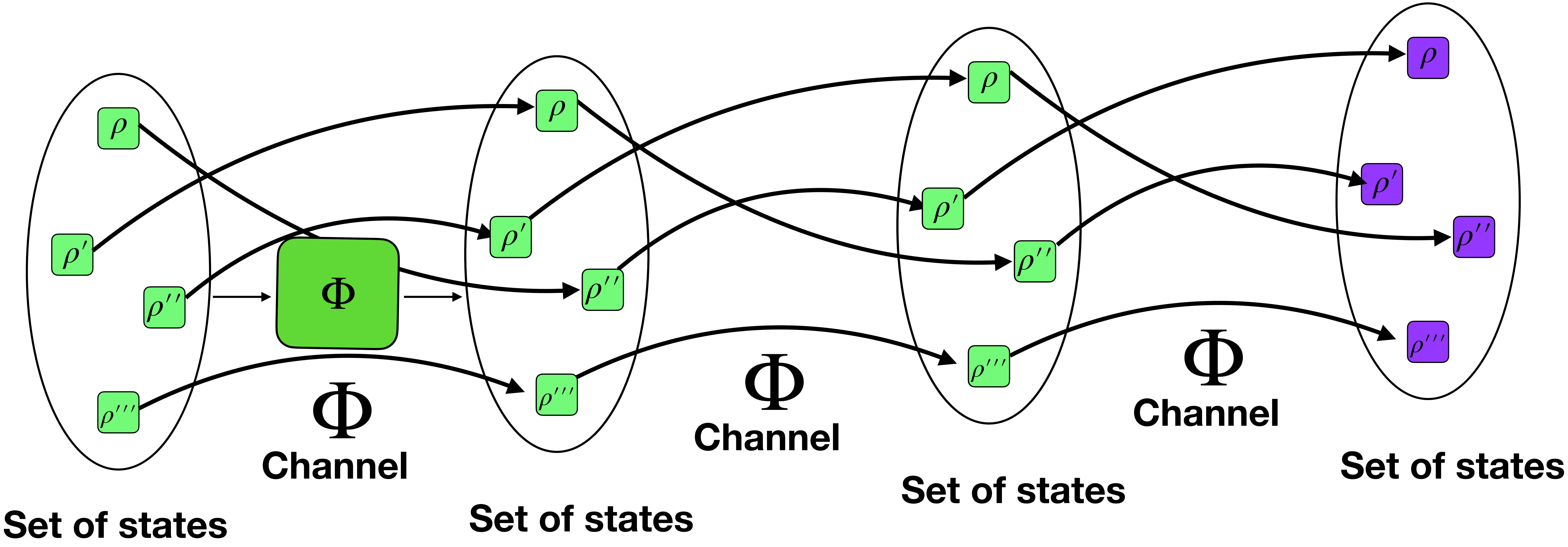


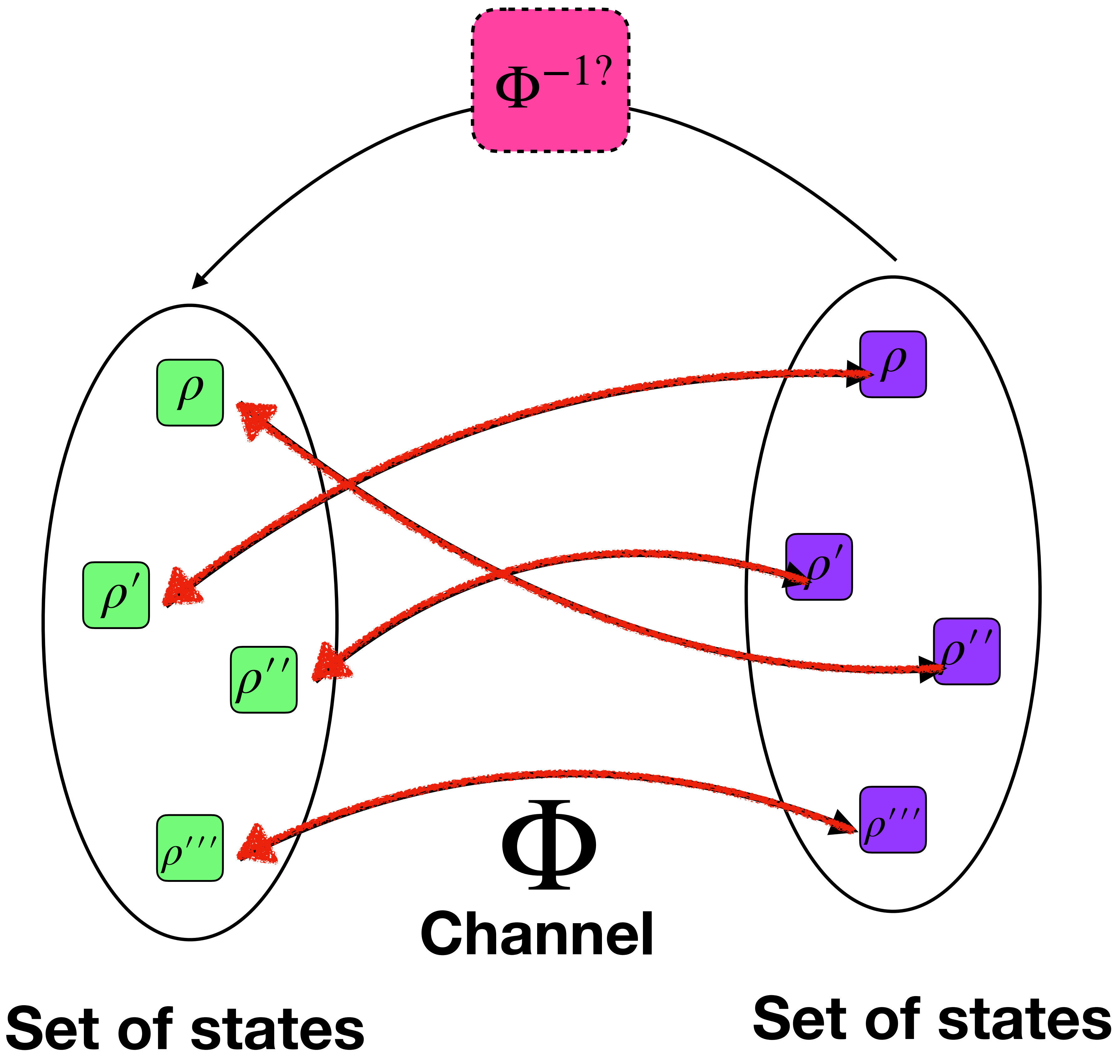
How to formalise the intuitive idea of the reverse of a not reversible channel?

We start from some examples

Discrete state space, discrete time

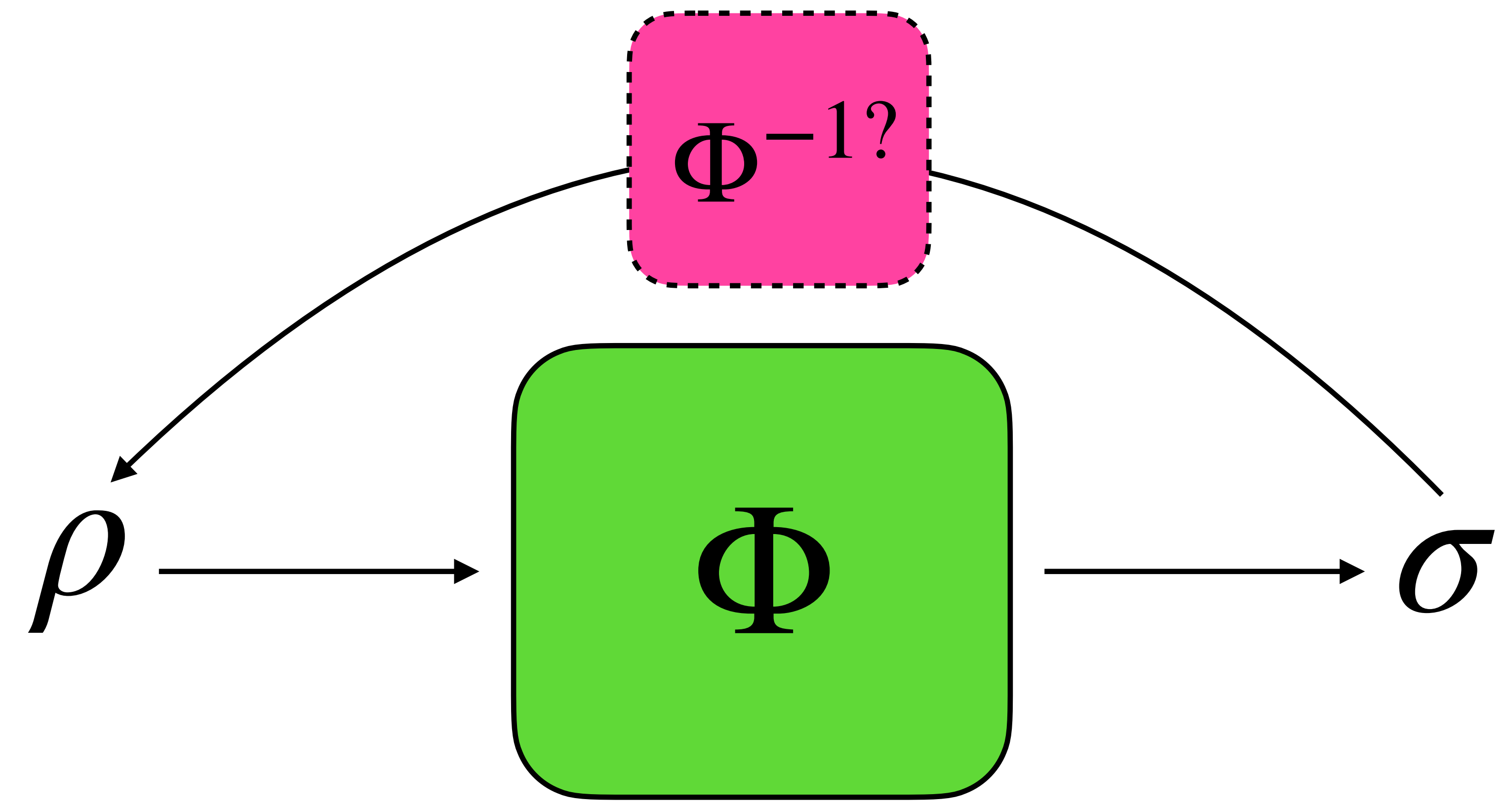
Channels ➡ Maps between states

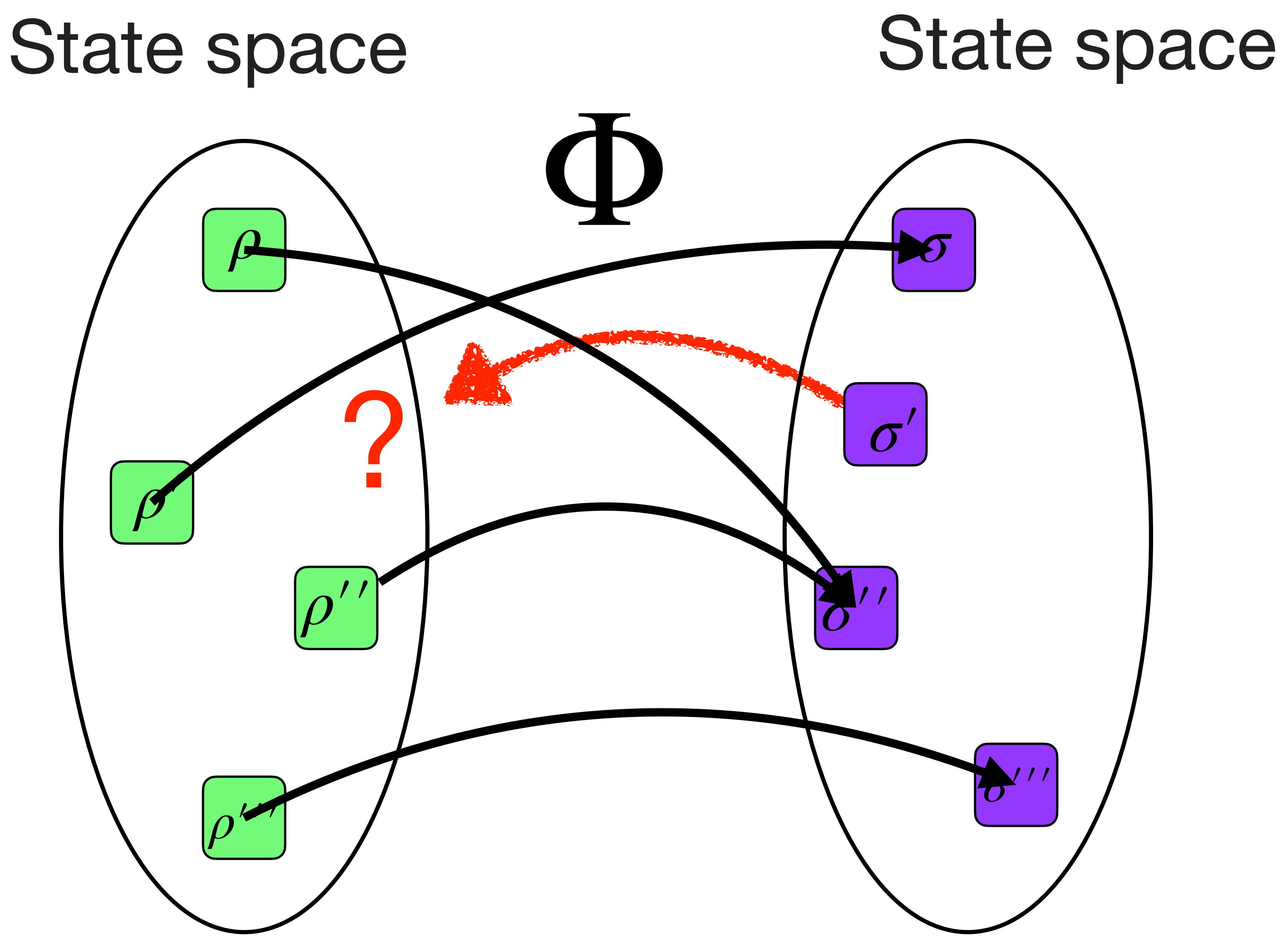


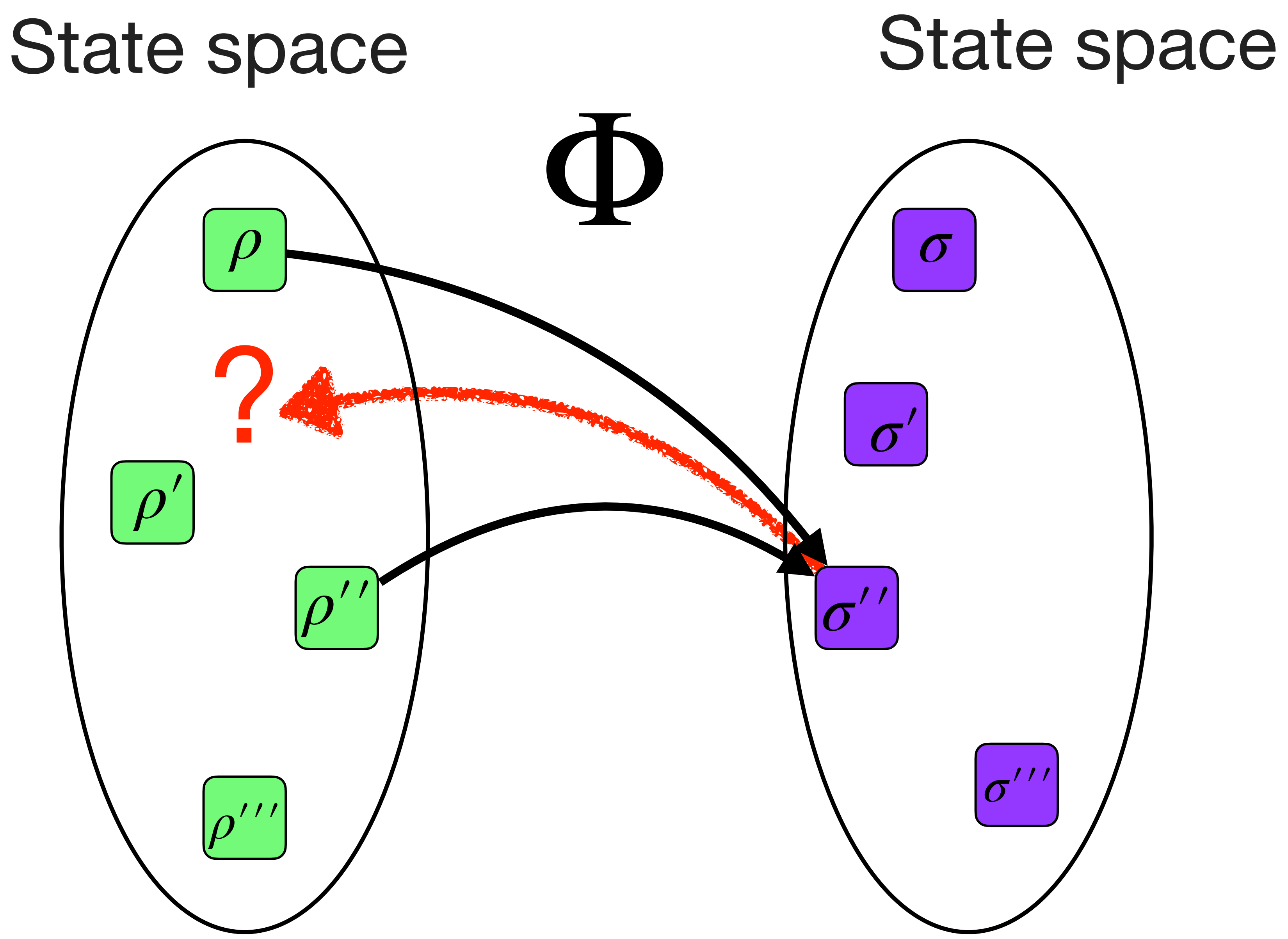


This is reversible

Let’s make it not reversible







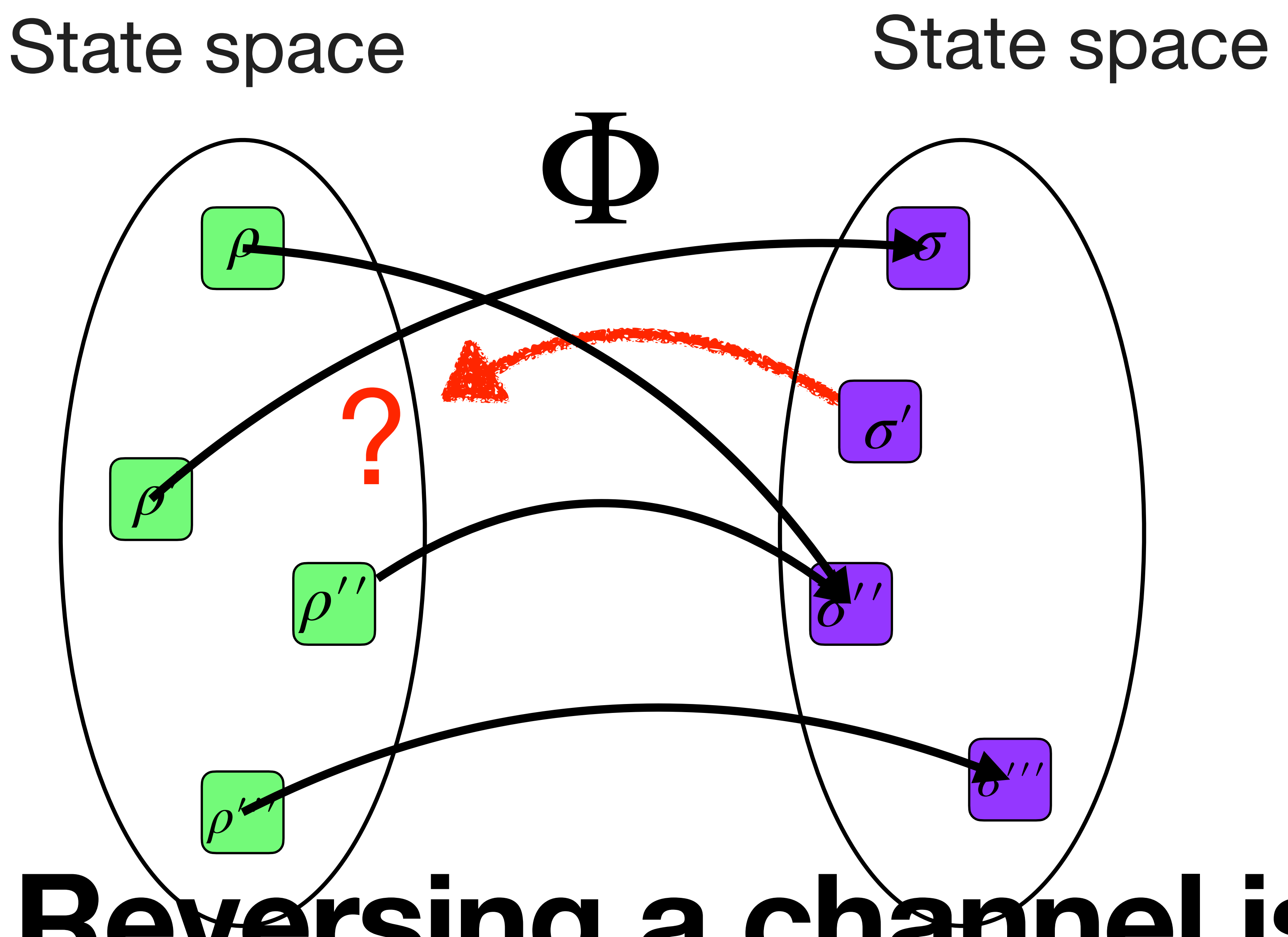
First Problem: not surjective

Well… we never reach that state in the image so it does not make sense to go back from there.

Second Problem: not injective

Reversing is not possible because we may forget from where we come from, so we don't know where to go back.

Reversing a channel is quite an ambiguous task.



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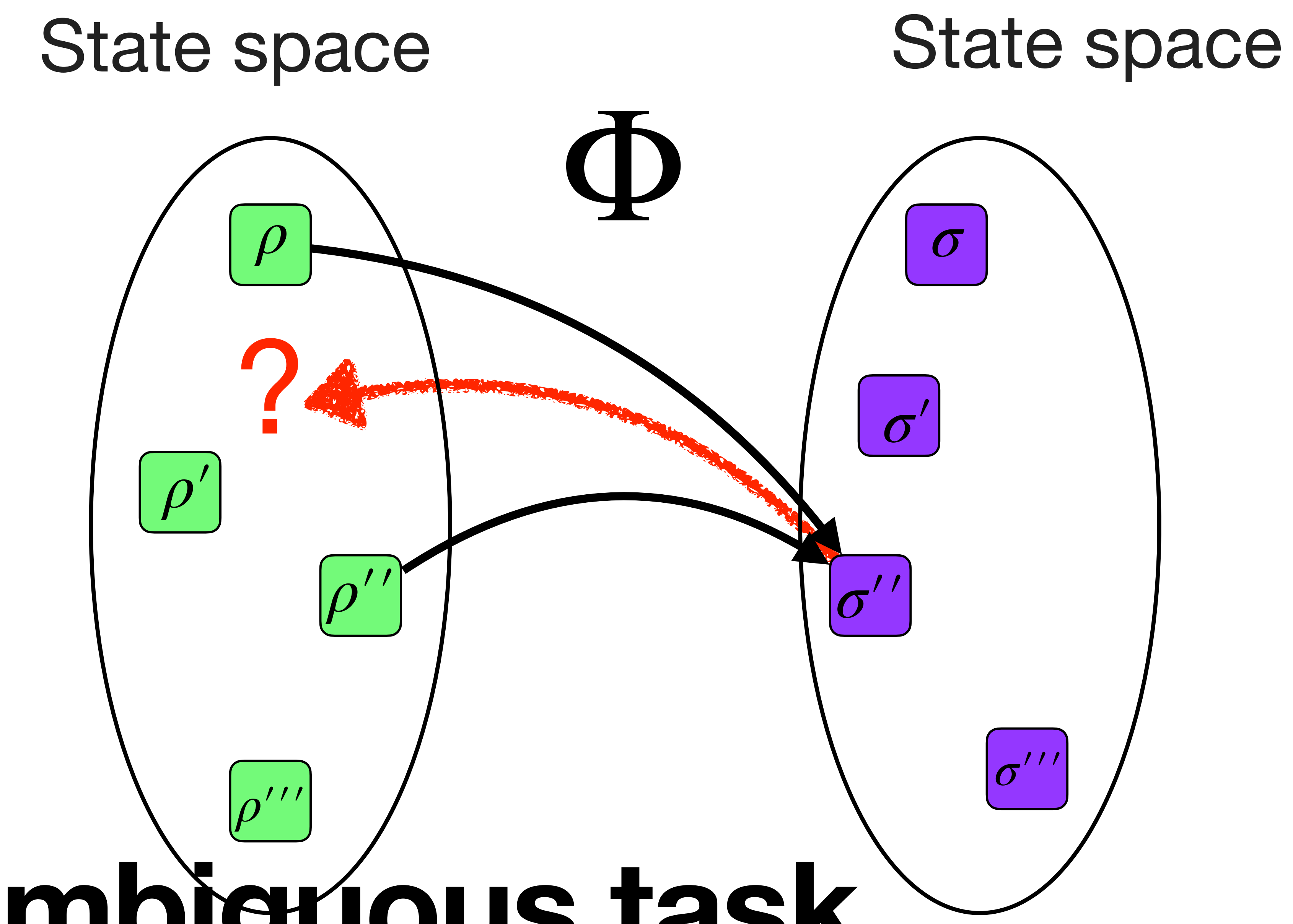
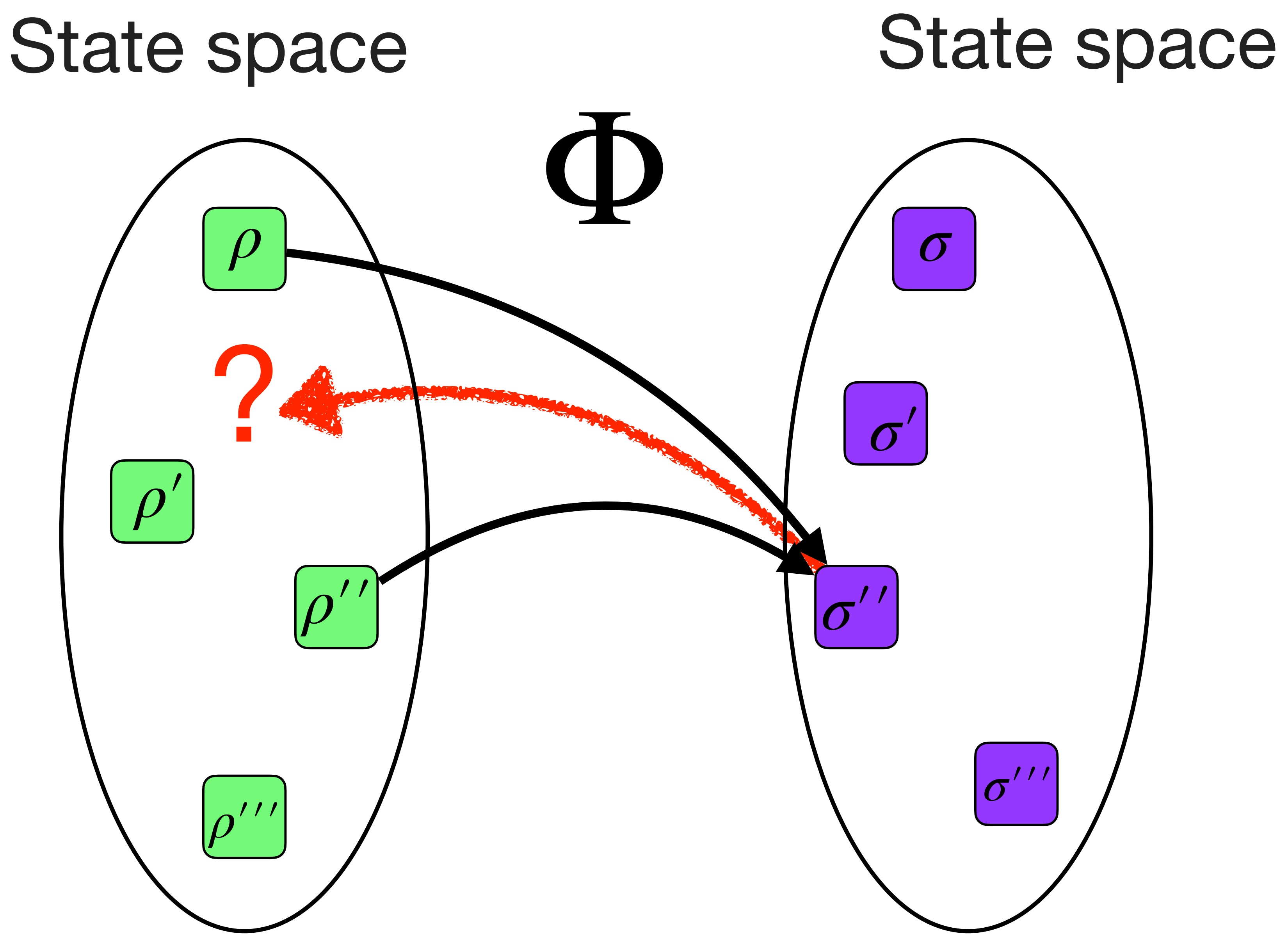


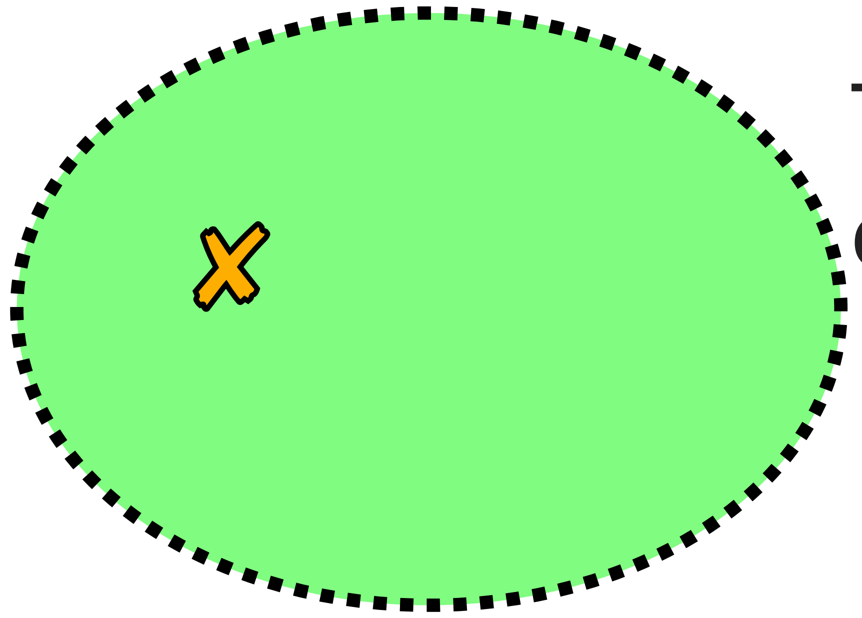
Second Problem: not injective



Reversing a channel is quite an ambiguous task.

There are many possible choices for a reverse channel.

How to choose one?



The set of possible reverse channels

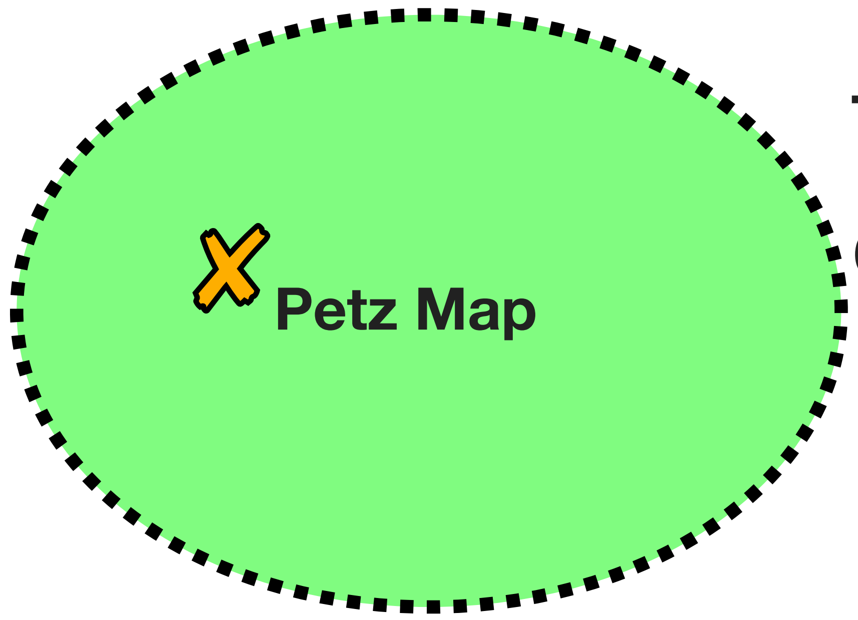
Looking for a canonical reverse channel

Reverse channels are used already in many different field!

- In **thermodynamics** are a fundamental tool for deriving fluctuation relations at the core of the discussion on the **arrow of time**.
- Reverse channels lies at the core of **error correction**.

The most commonly used reverse channel

Petz (recovery) **map**



The set of possible reverse channels.

Does there exist a framework explaining why the Petz map should be THE reverse channel?

D. Petz, Sufficient subalgebras and the relative entropy of states of a von neumann algebra, Comm. Math. Phys. 105, 123 (1986).

D. Petz, Sufficiency of channels over von Neumann algebras, The Quarterly Journal of Mathematics 39, 97 (1988).

Looking for a canonical reverse channel

Satosi Watanabe

The problem of reversing a channel is the problem of **retrodicting** a state: inferring the original state from the knowledge of the channel, (possibly) some prior information and the evolved state.

Being at time t_1 you want to retrodict the state present at time $t_0 < t_1$ knowing that at time t_1 your state is σ.

Channel: Φ
Evolved state: σ
Prior (Original Guess): π

→ Inference techniques → New Guess: π'

Canonical Statistical inference methods → **Petz** (recovery) **map** and **Bayes inspired reverse channel**

S. Watanabe, Symmetry of physical laws. part iii. prediction and retrodiction, Rev. Mod. Phys. 27, 179 (1955).
S. Watanabe, Conditional probabilities in physics, Progr. Theor. Phys. Suppl. E65, 135 (1965).
D. Petz, Sufficient subalgebras and the relative entropy of states of a von neumann algebra, Comm. Math. Phys. 105, 123 (1986).
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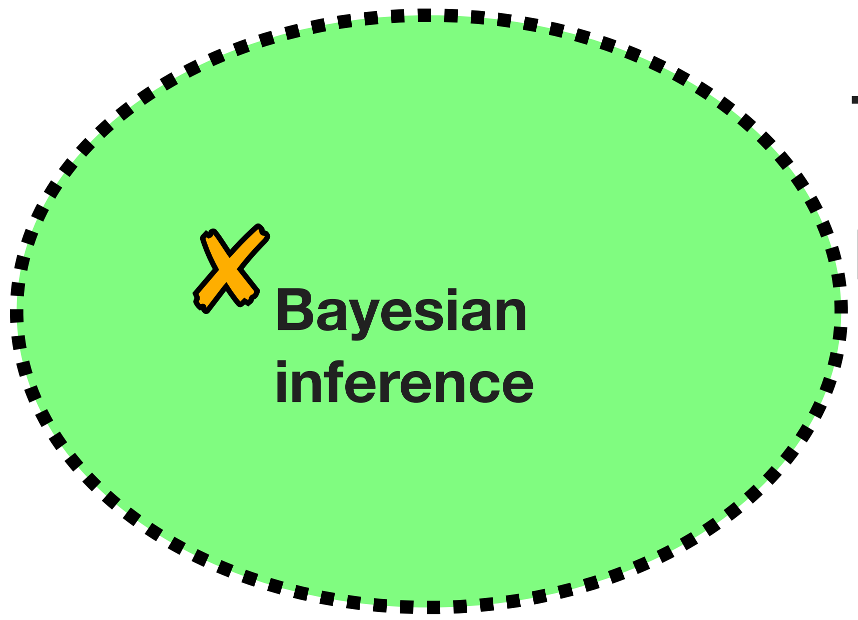
Buscemi, Francesco and Scarani, Valerio, Fluctuation Theorems from Bayesian Retrodiction, 10.1103/PhysRevE.103.052111
C. C. Aw, F. Buscemi, and V. Scarani, Fluctuation theorems with retrodiction rather than reverse processes, AVS Quantum Science 3, 045601 (2021), https://doi.org/10.1116/5.0060893.

Ambiguity again: why Bayesian inference?



The set of possible inference methods.

- Duality with maximum likelihood.
- Derivation from maximum entropy principles.
- Derivation from minimization of geometric distances (e.g. Kullback-Leibler).
- Derivation from principles of information goemetry (Amari)
- Derivation from geometric principles (Csizar).
- Consideration on properties of the convergence of subjective probability updates (e.g. Jeffrey, Bernando,…).
- …

Does there exist a framework explaining why Bayesian inference should be THE inference method?

It would be nice to tame the ambiguity.

To characterise the set of all the reasonable retrodiction channels.

Why Bayesian inference?

Is it possible to characterise all the reasonable retrodiction channels?

What makes Bayes so fundamental?

Is it possible to find a better retrodiction channel?

Stochastic maps

$$\Phi = \begin{pmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,n} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n,1} & \phi_{n,2} & \cdots & \phi_{n,n} \end{pmatrix}$$

$$\forall k \ \sum_{i=1}^{n} \phi_{i,k}$$

$$\forall k, i \ \phi_{i,k} \in [0, 1]$$

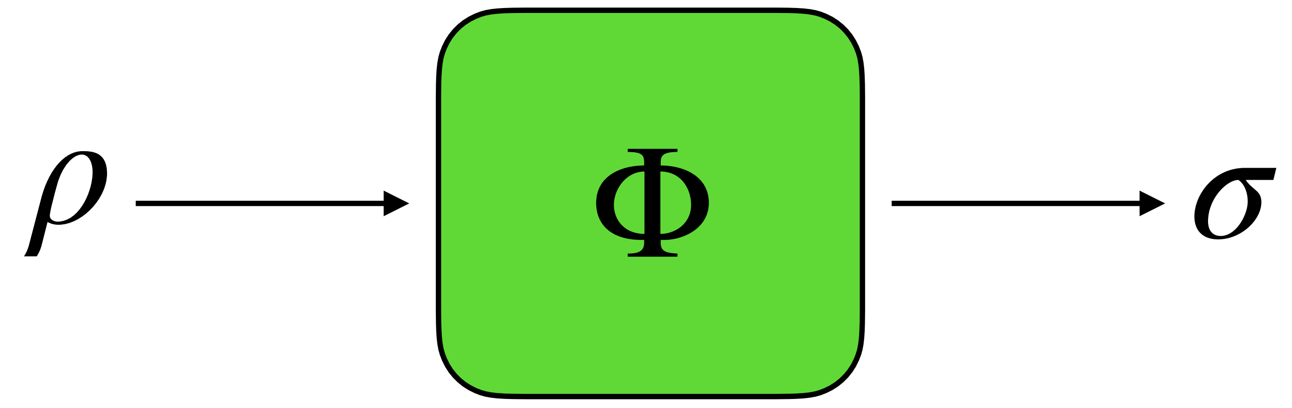
Left stochastic map

$$\rho = (\rho_1, \rho_2, \ldots, \rho_n)$$

$$\sum_{i=1}^{n} \rho_i = 1$$

$$\forall i \ \rho_i \in [0, 1]$$

Probability vector



$$\sigma = \Phi\rho$$

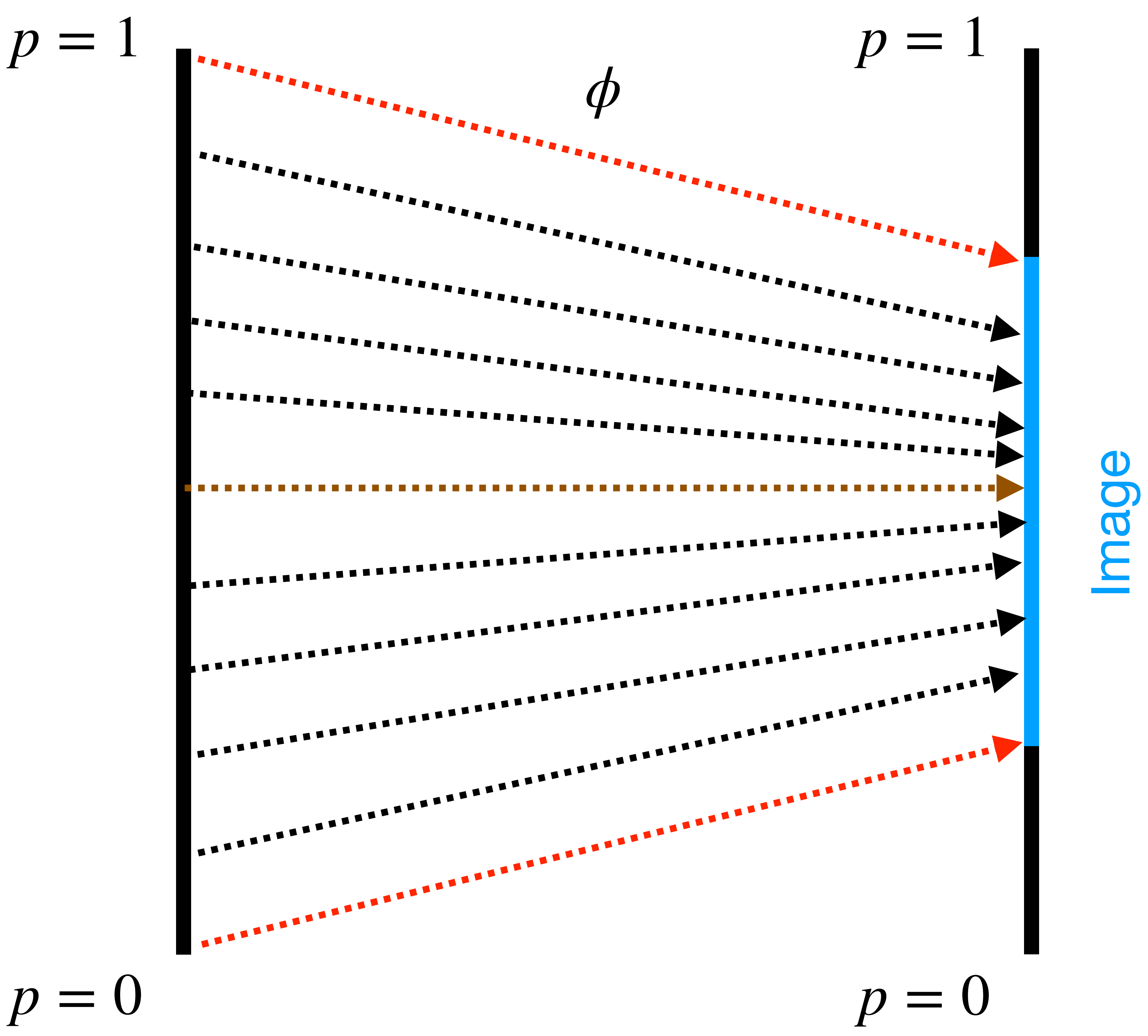
Example in 2 dimensions

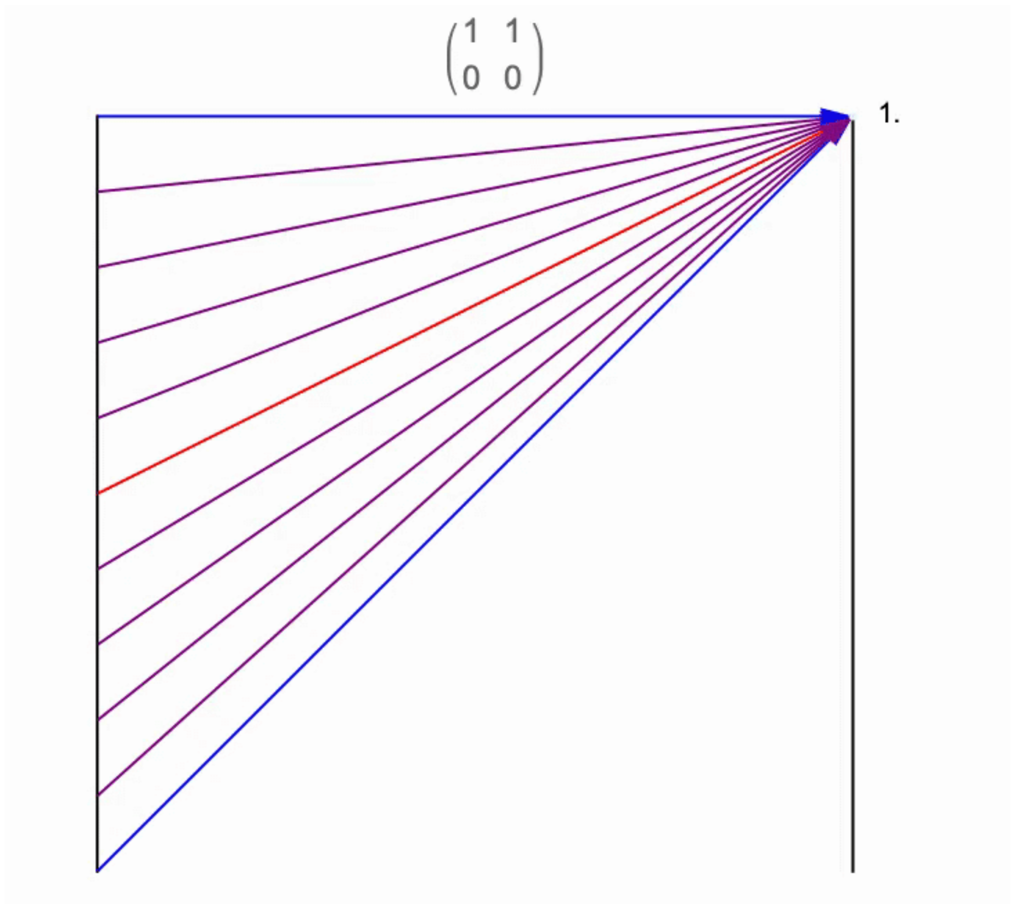
Stochastic map

$$\Phi = \begin{pmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{pmatrix}$$

Probability vector

$$\rho = (p, 1-p) \quad p \in [0,1]$$

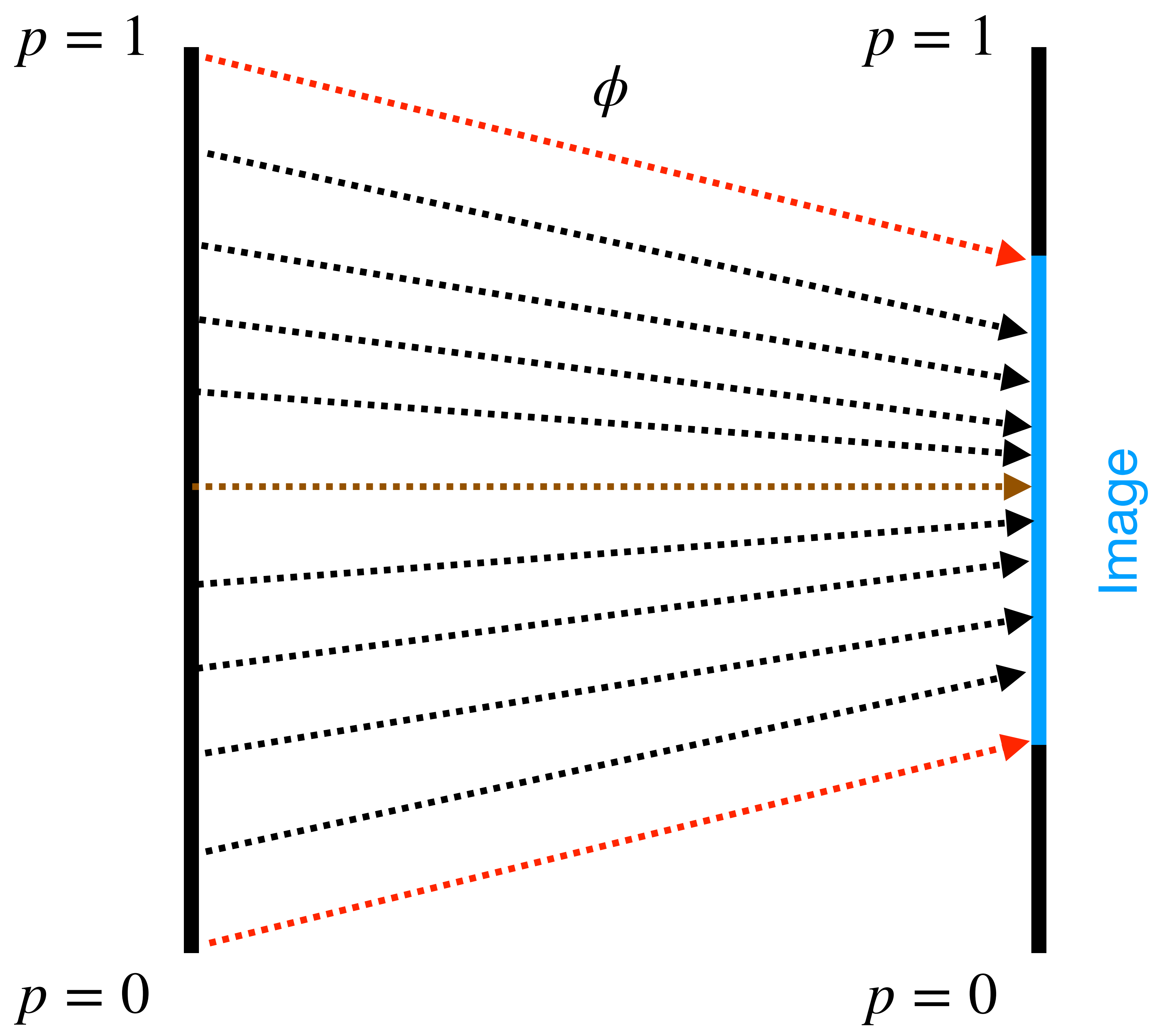




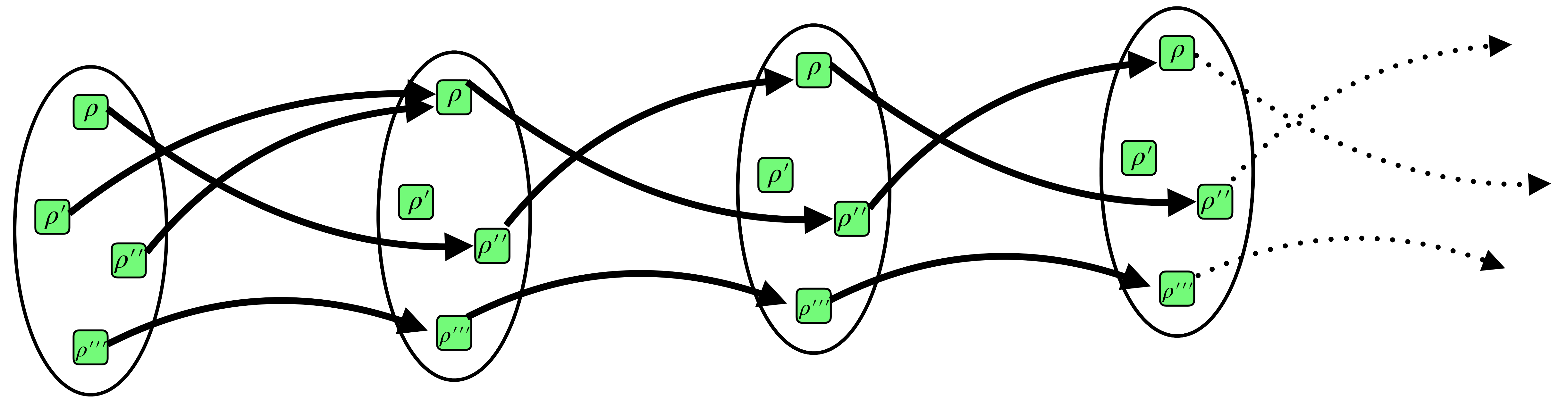
Reversing is connected to the idea of recovering the information about the initial state.

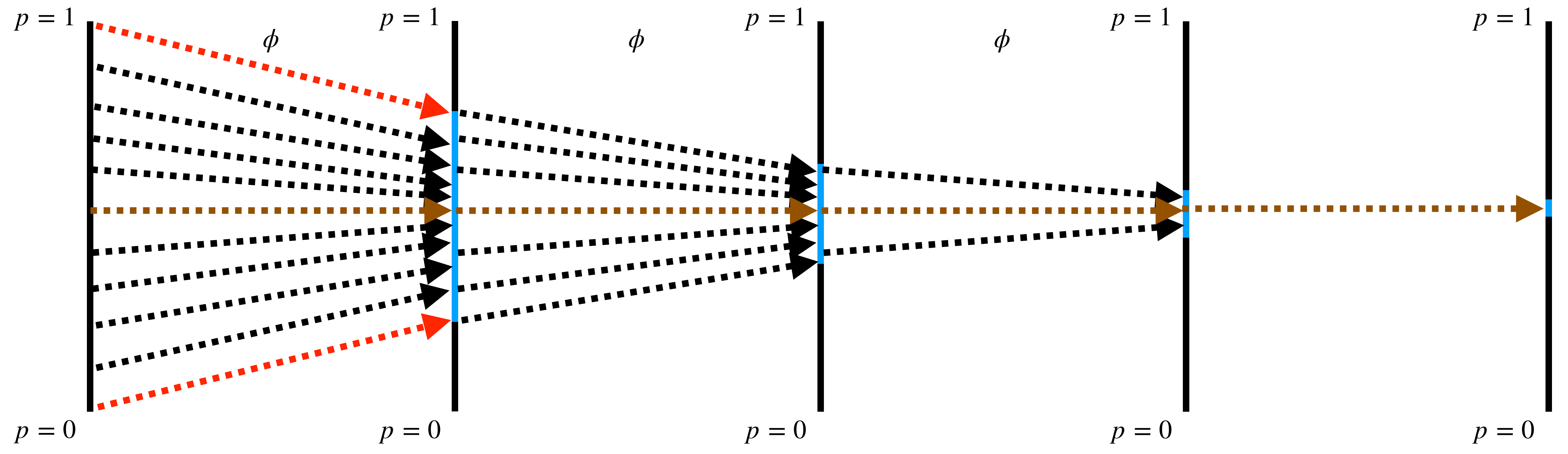
In the case of stochastic maps, states are probability vectors.

Loss of information for stochastic maps: contractivity



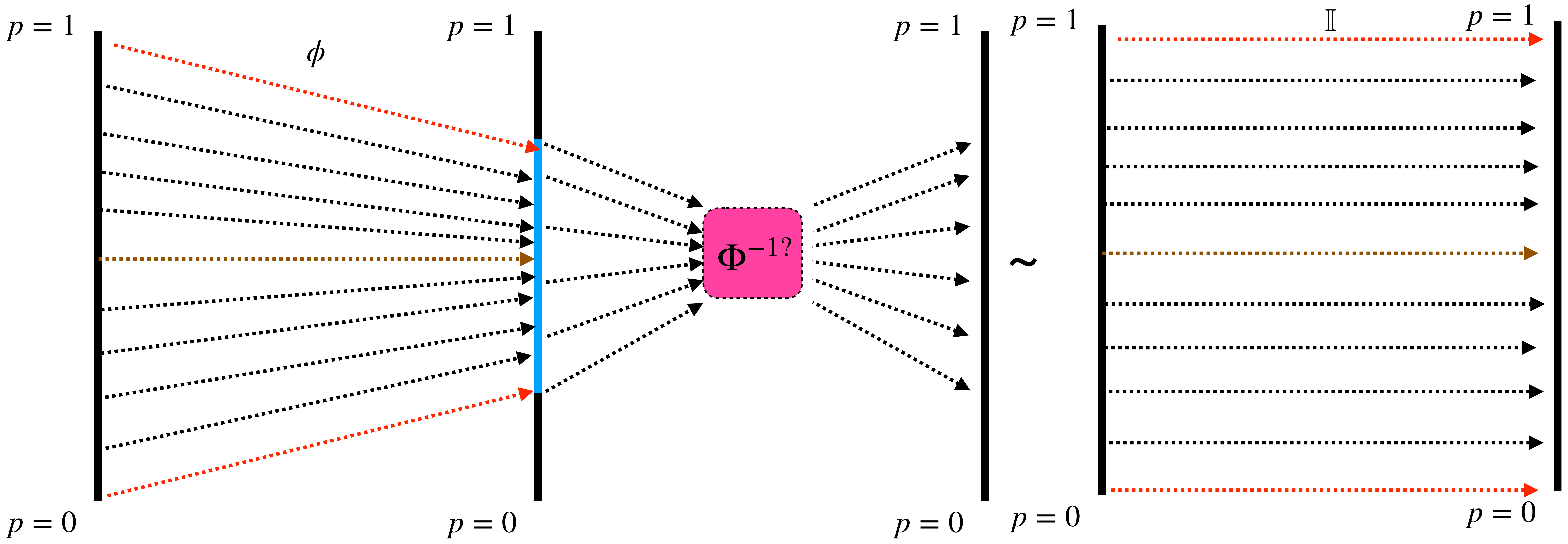
In the continuous case we find a new problem for the reversibility: channels are **contractive**.





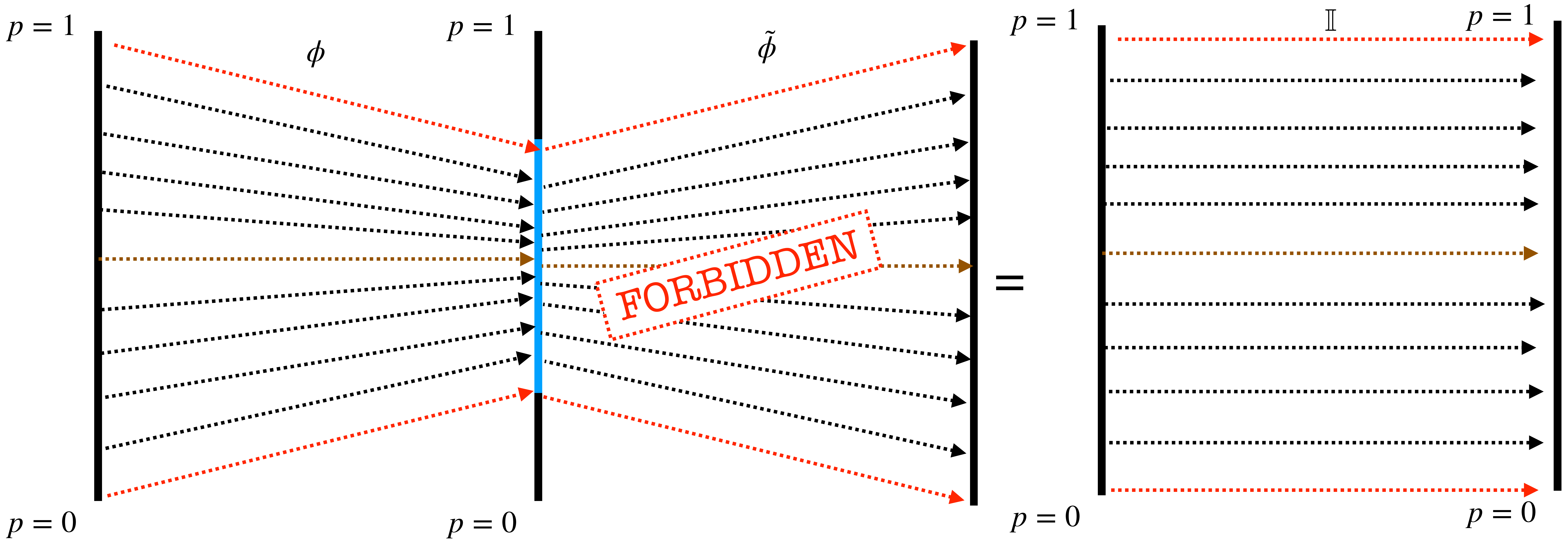
Contractivity is a property of channels. **Channels never expand.**

Reversing a channel

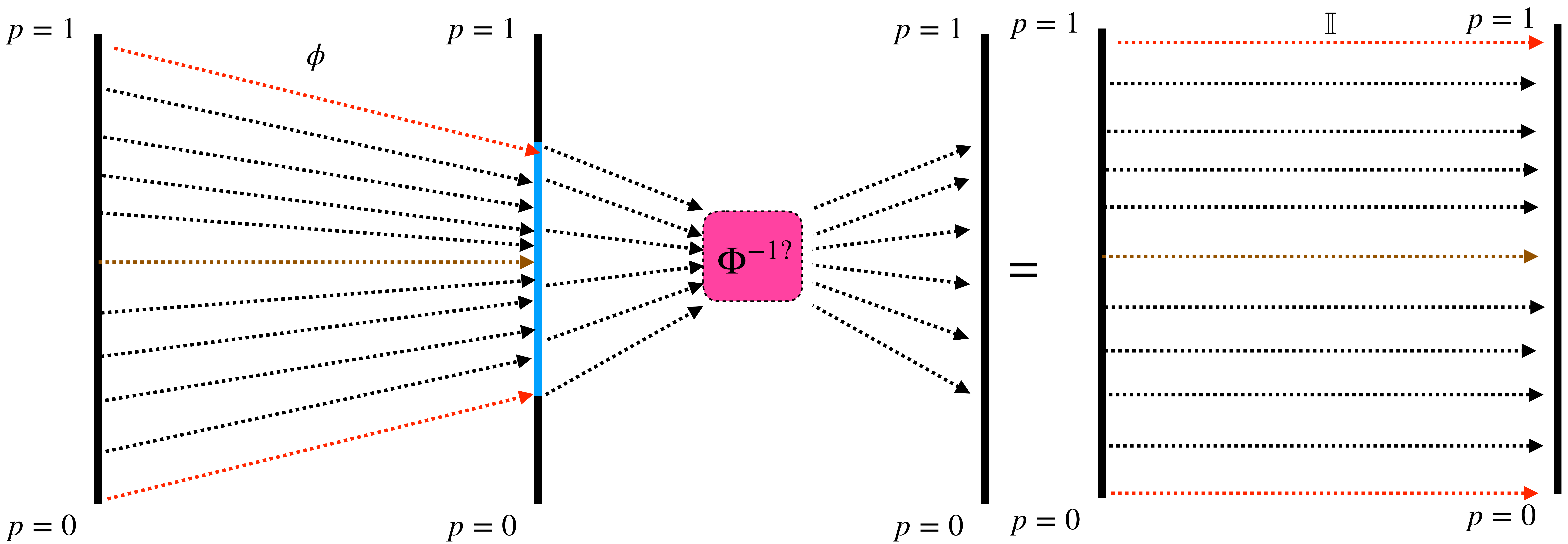


The central object to study is the forth-and-back channel

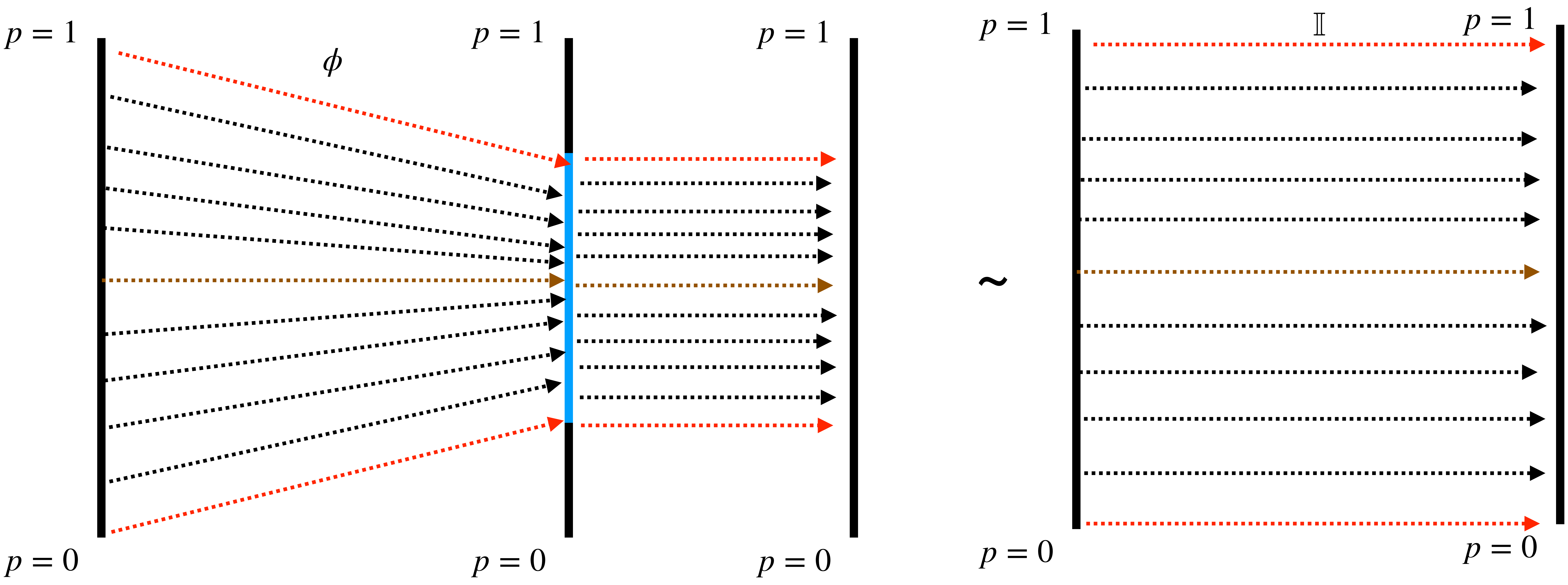
Reversing a channel



Reversing a channel

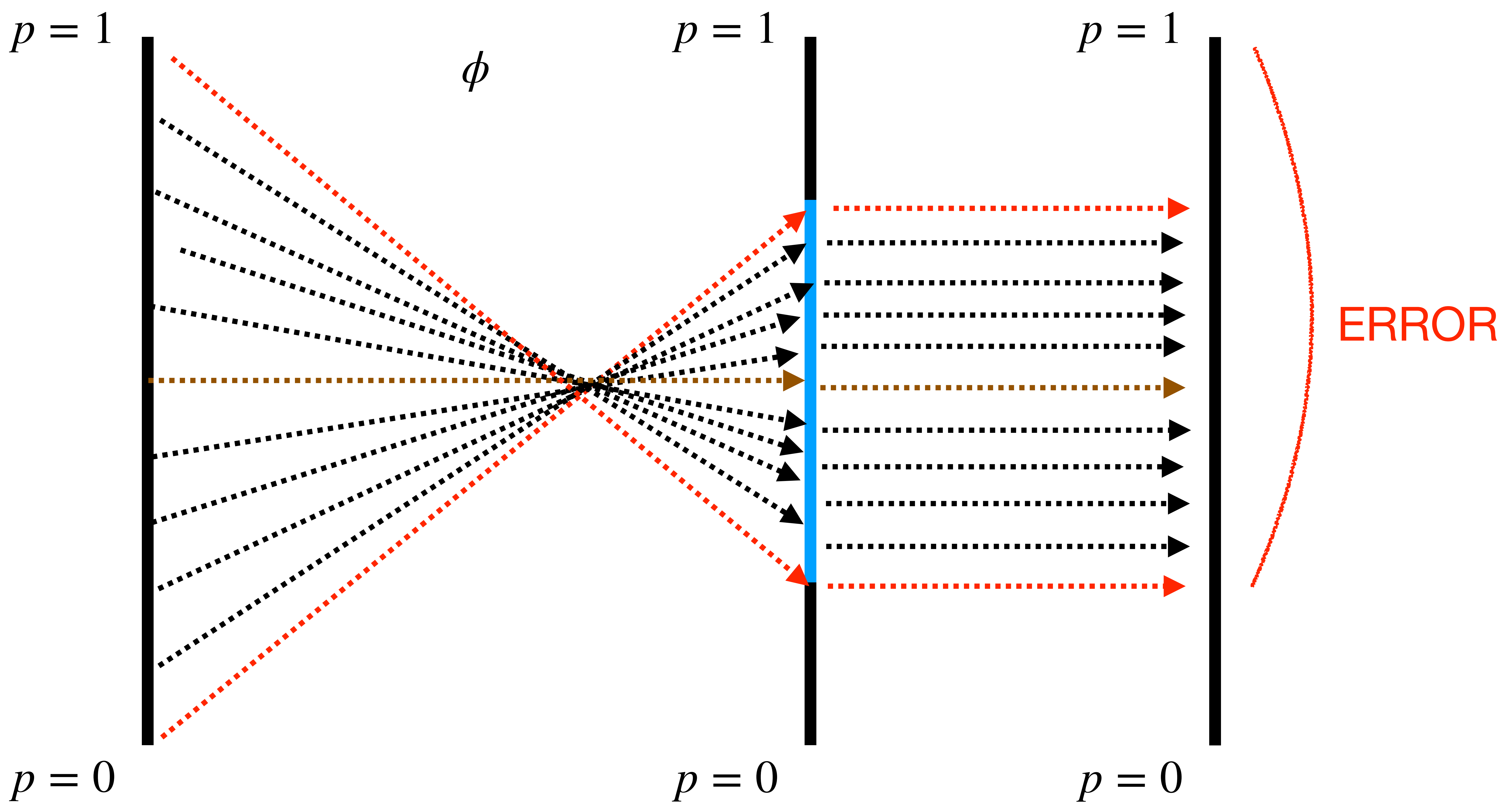


Reversing a channel



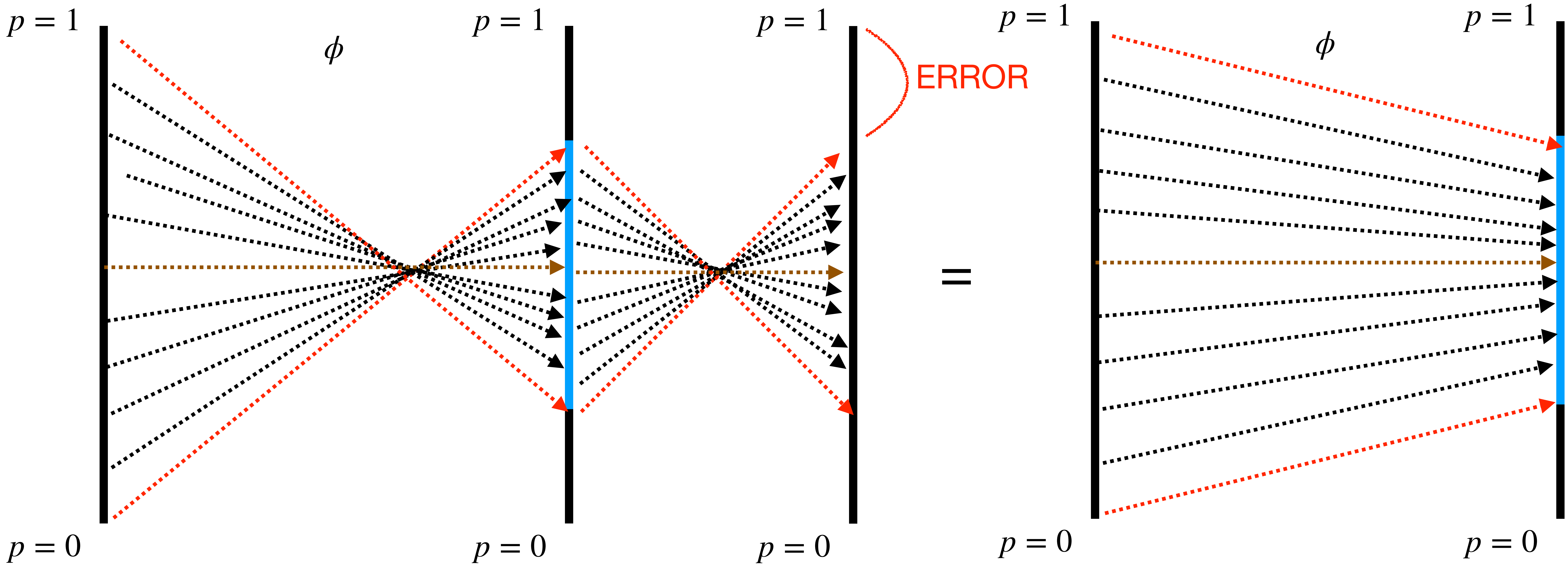
Reversing a channel

Flips are dangerous



Reversing a channel

Flip should be avoided going forth-and-back



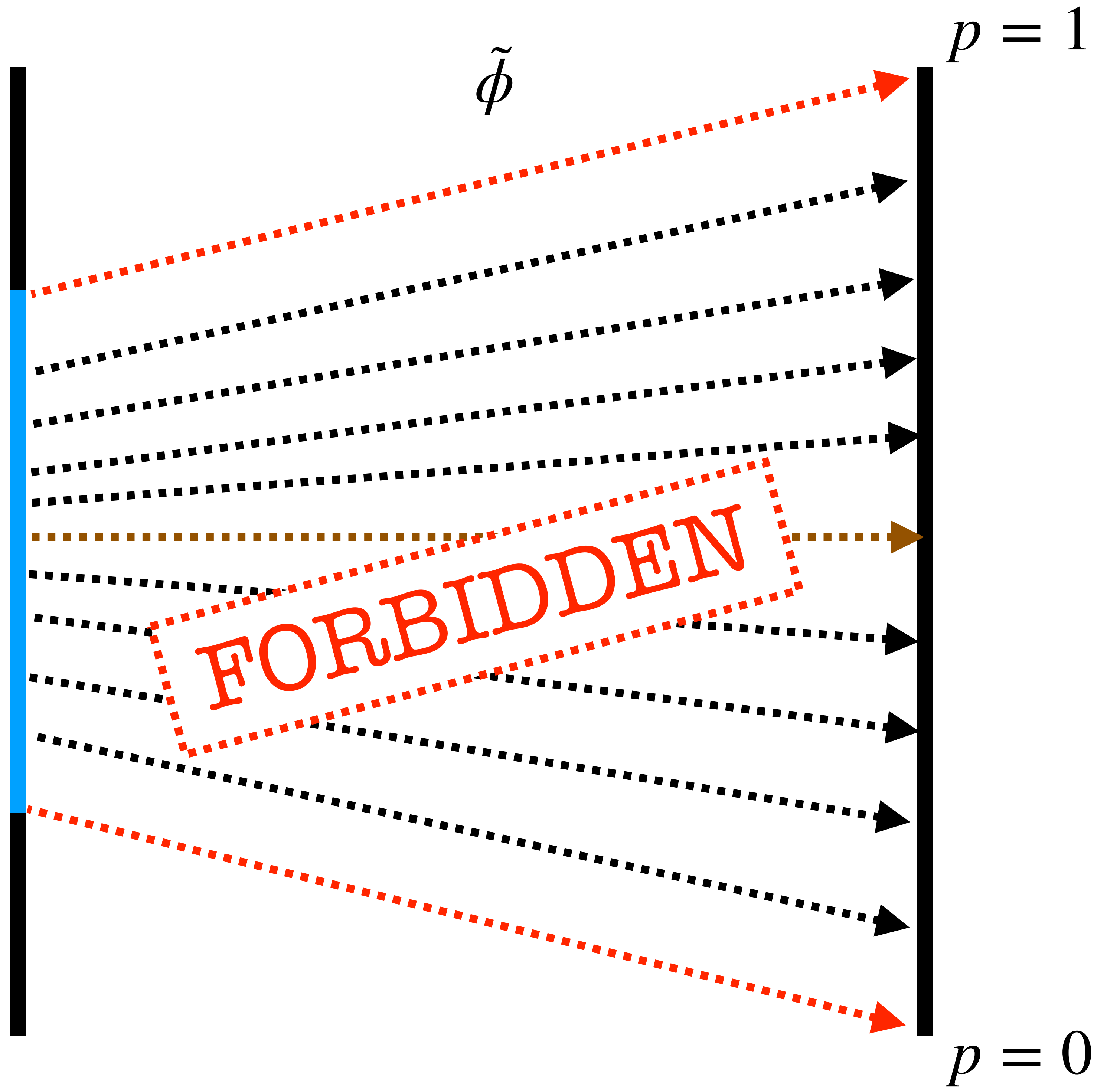
Petz and Bayes Reverse always take this in consideration!

First 3 principles for a reverse channel

1. **If the channel can be inverted just by simply inverting the arrow, just do it.** *“we already know how to take the inverse of a permutation or unitary channel”.*

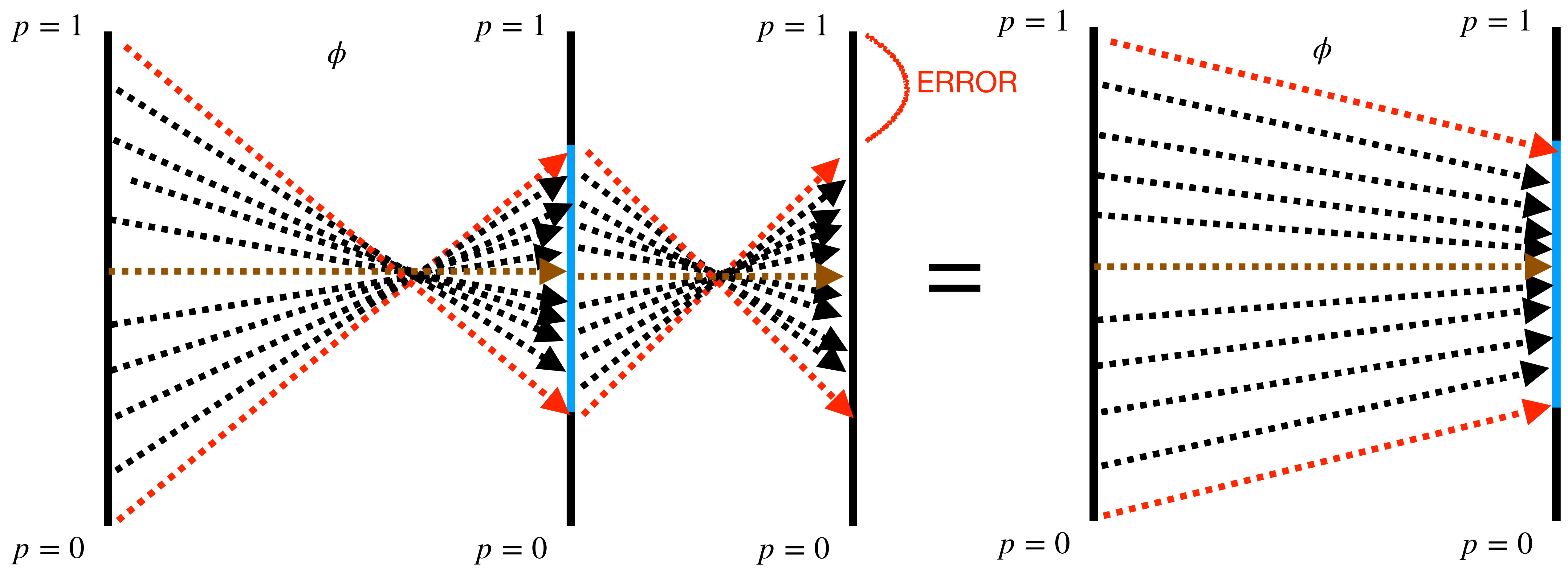
2. **The state retrieval channel should be physical.**

"It should be a meaningful retrieval map even in the single-shot scenario, not just in the full statistics case".

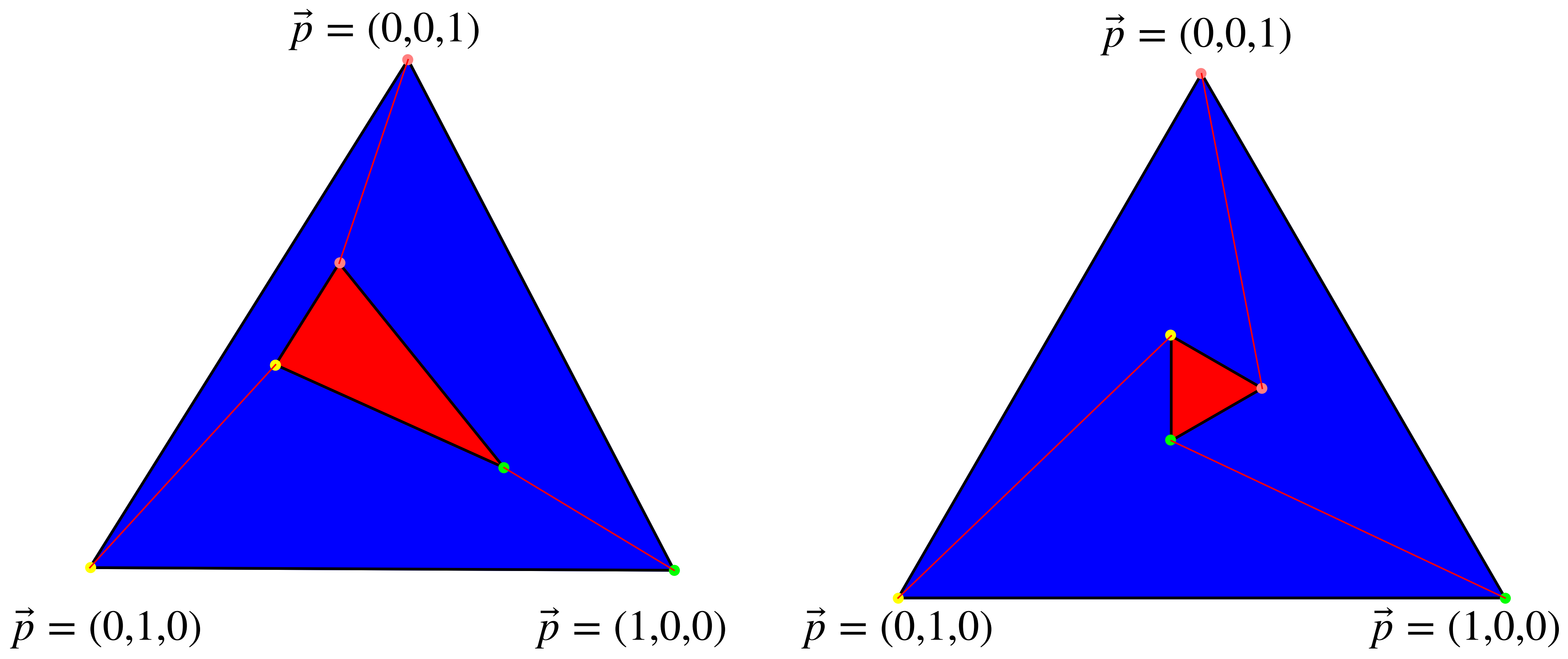


3. **All the eigenvalues of the back and forth map must be positive.**

"Every inversion (negative eigenvalues) or rotation (complex eigenvalues) ruins the retrieval."



Flips

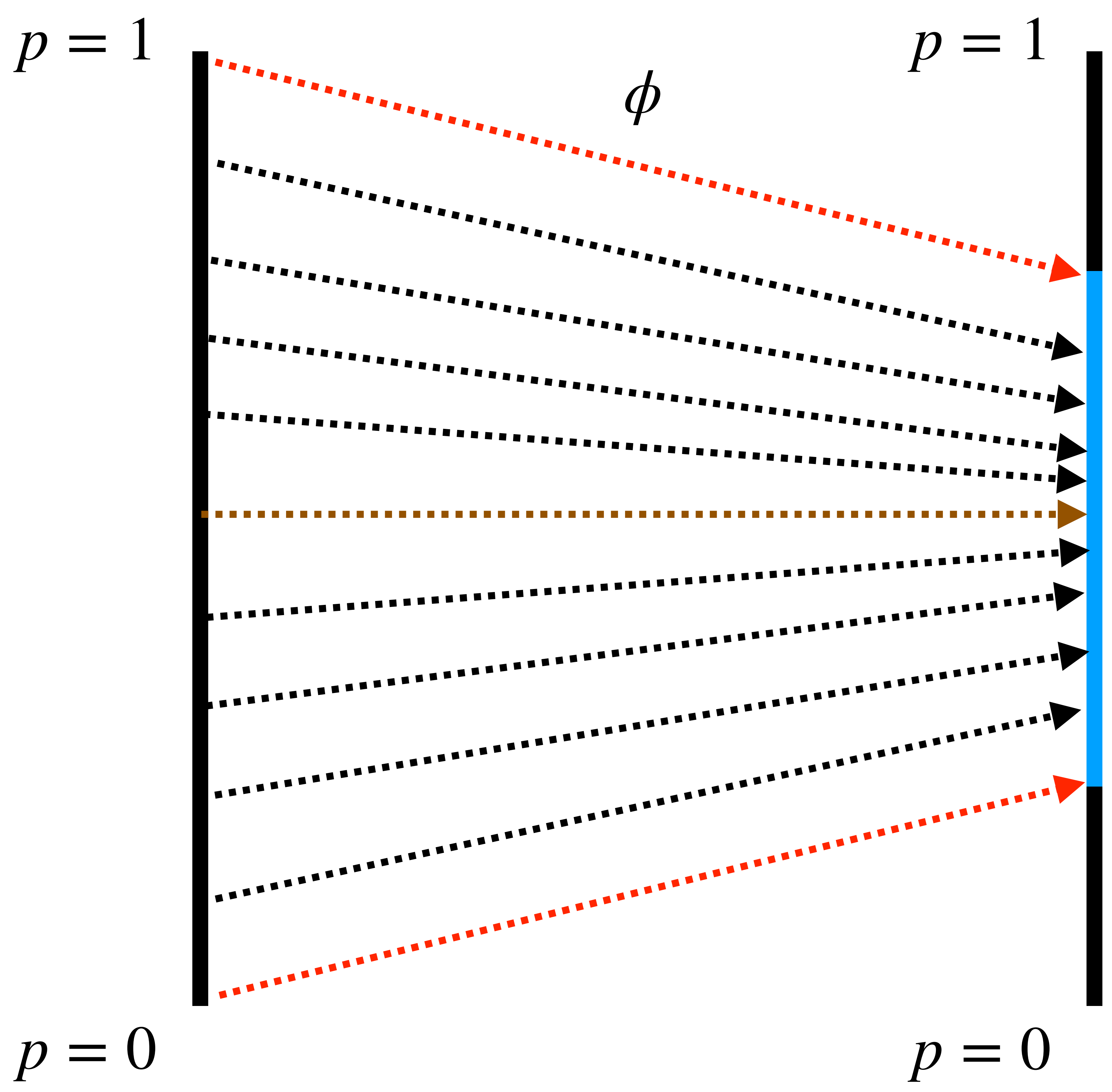


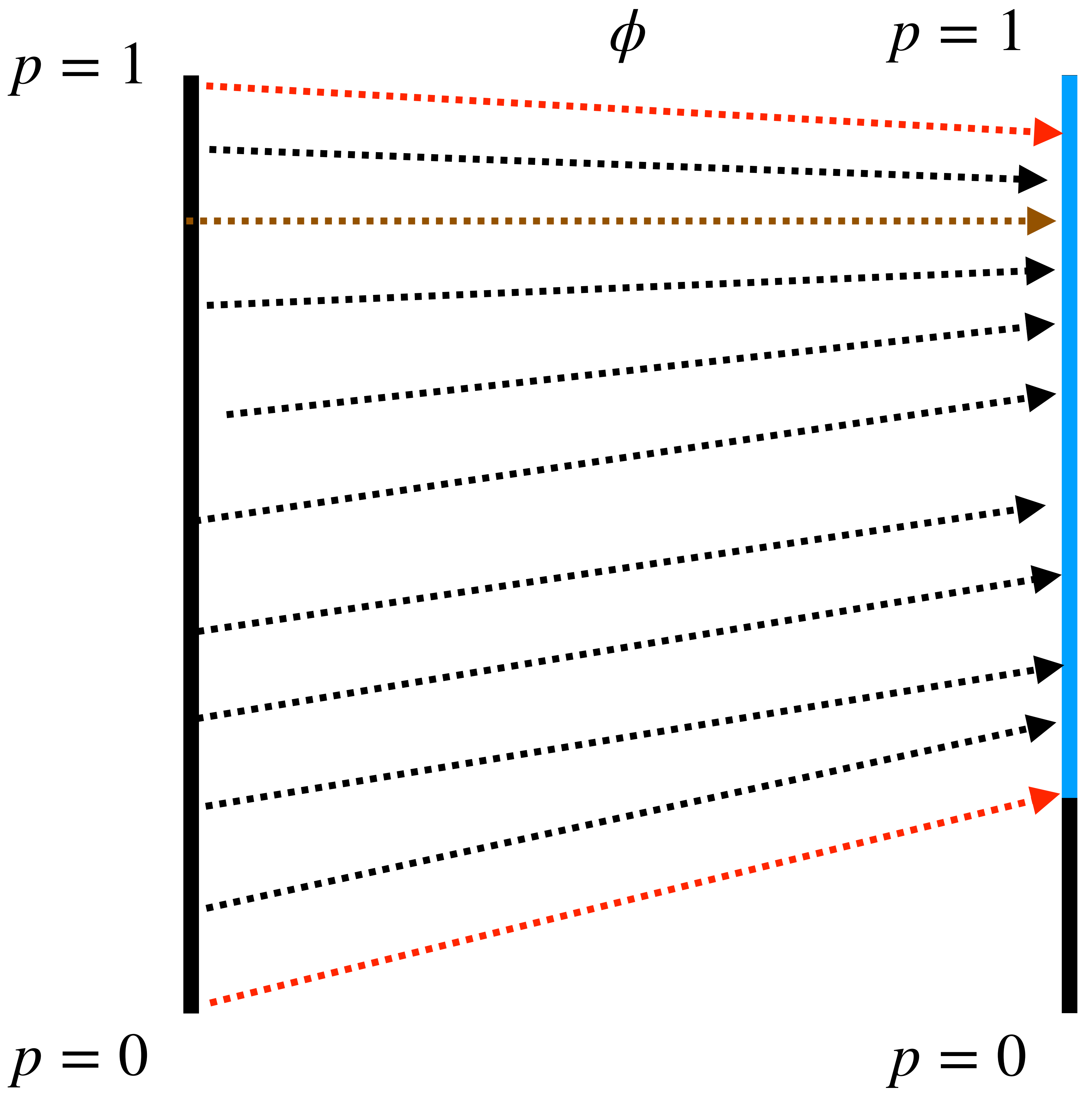
Rotations

Exploiting a general property of channels

An additional property of channels

Every channels always preserves at least one vector or state, they have at least one fixed point.





We want to exploit this property.

Stochastic channel:	Φ
Prior:	π

Basic ingredients

Exploiting our prior knowledge, last 2 principles for a reverse channel

4. **The fixed point of the forth-and-back channel must be the prior.** *"Since every channel has a fixed point, we take advantage of this property and we encode all our additional knowledge in it. We select a typical initial state that we want to always perfect recover".*

5. **The prior should be an equilibrium state for the back-and-forth channel.** *"The back-and-forth channel should be time symmetric on the fixed point".*

Taming the ambiguity: the space of retrieval channels

Stochastic channel: Φ

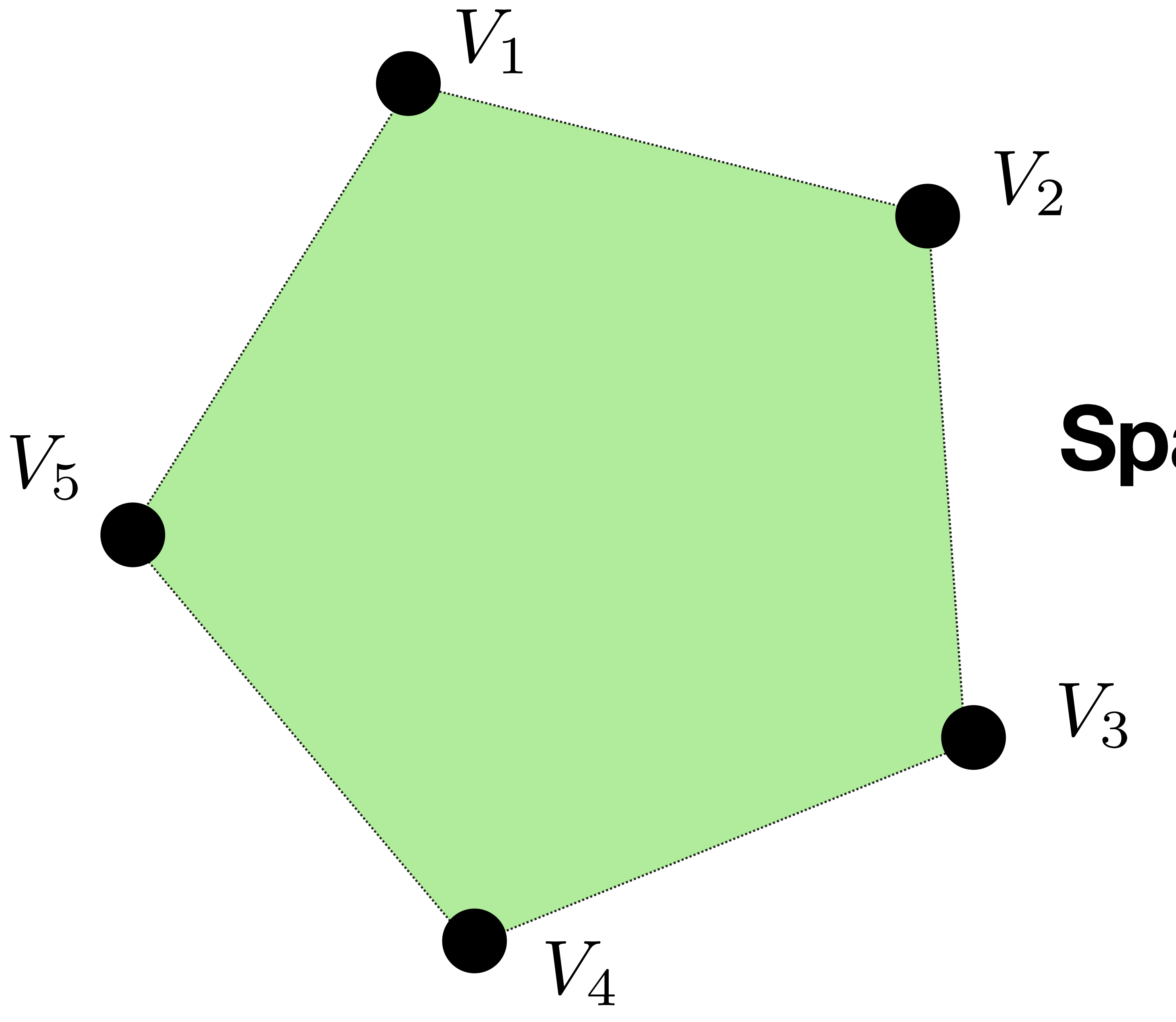
Prior: π

Basic ingredients

$\tilde{\Phi}$	Must be a (left-)stochastic matrix
$\tilde{\Phi} \circ \Phi$	The back-and-forth channel must have positive eigenvalues
$\tilde{\Phi} : \Phi\pi \to \pi$	$\tilde{\Phi}$ have this transition fixed.
$(\tilde{\Phi}\Phi)_{j,i}\pi_i = (\tilde{\Phi}\Phi)_{i,j}\pi_j$	The prior state is the equilibrium state for the back-and-forth channel

- Convex Set
- Finite set of vertices
- Algorithm for computing vertices by Jurkat and Ryser

- The forth-and-back channel is a positive semidefinite matrix



Space of retrieval channels

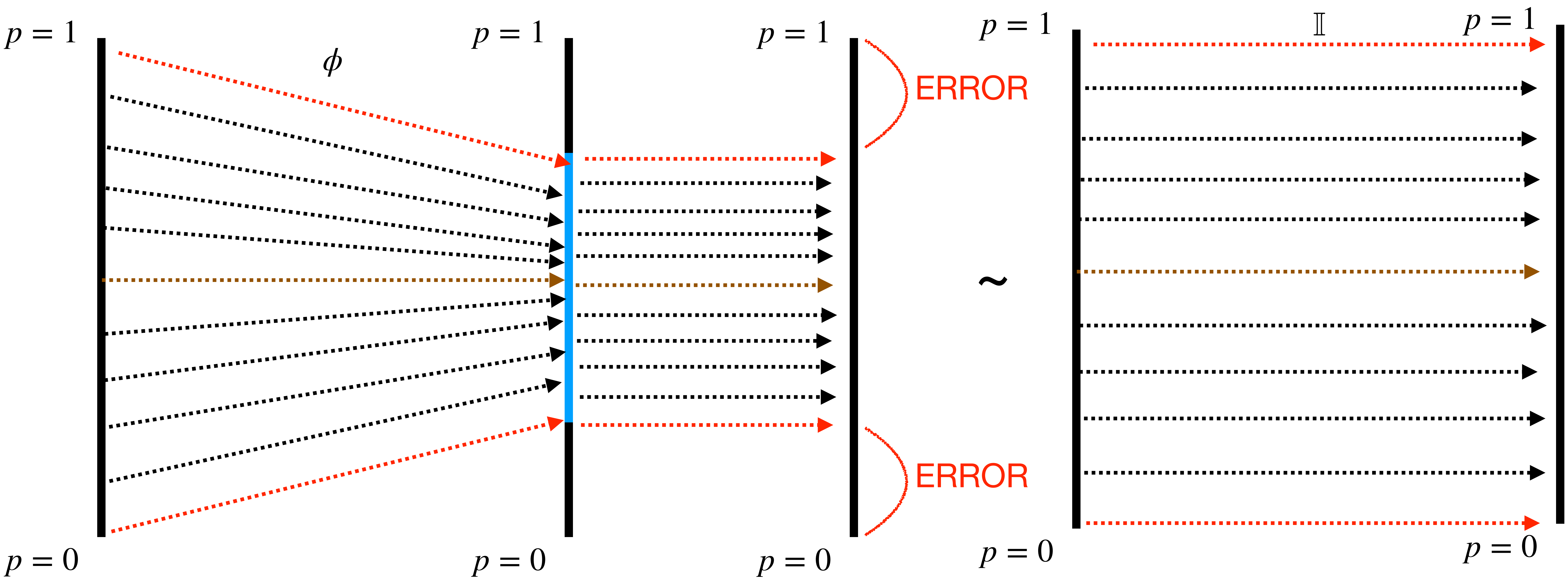
Each reverse channel is completely characterised by the vector of coefficients.
This is a probability vector.
Each reverse channel is characterised by a probability vector.

$$\tilde{\Phi} \leftrightarrow \vec{\lambda}^{\tilde{\Phi}}$$

Finding the optimal retrieval channel

Reversing a channel

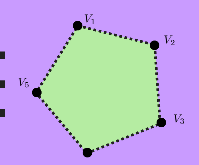
IDEA: Contract least possible

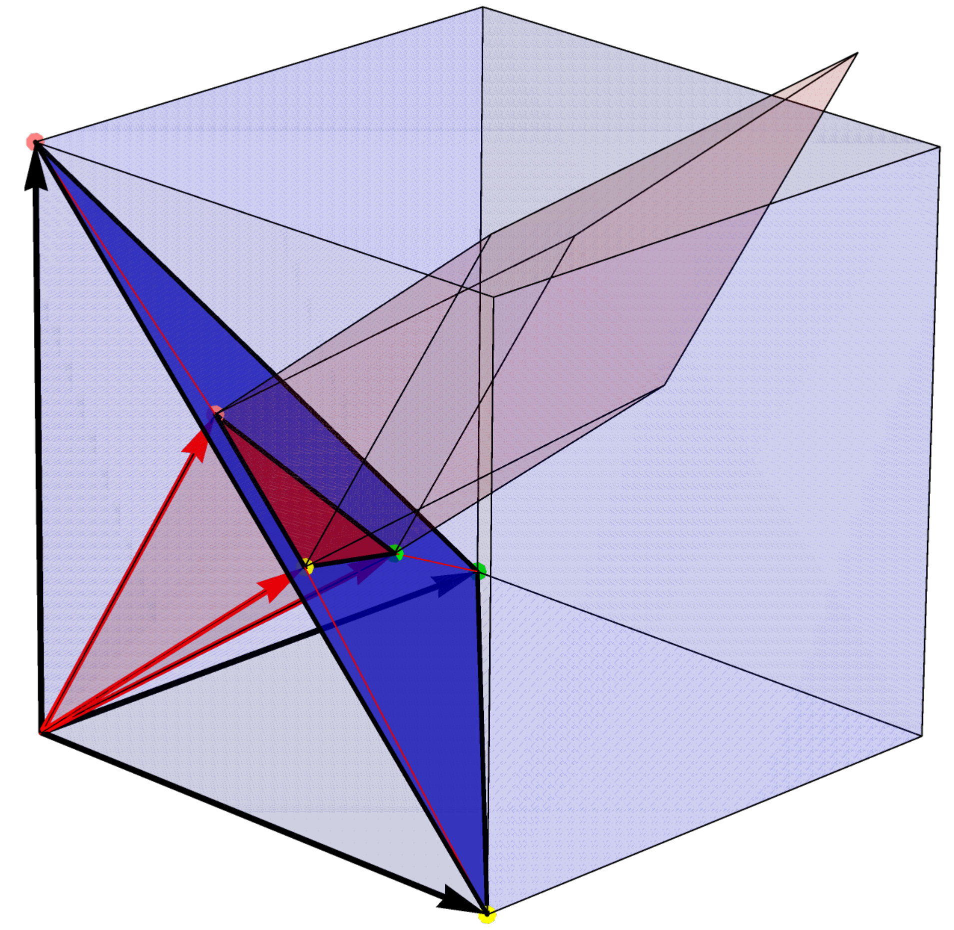


IDEA: minimal contraction

Optimisation criterion: **The optimal retrieval map is the one that maximise the determinant of the forth-and-back channel.**

$$\tilde{\Phi}_O = \max_{\tilde{\Phi} \in} \det \tilde{\Phi}\Phi$$





Second reason for choosing this optimisation criterion

$$D(\tilde{\Phi}\Phi \| \mathbb{1}) = \mathrm{Tr}[\mathbb{1}(\log \mathbb{1} - \log \tilde{\Phi}\Phi)] =$$
$$= -\mathrm{Tr}[\log \tilde{\Phi}\Phi] = \log \det(\tilde{\Phi}\Phi)^{-1}$$

Practical reason


Encyclopedia of Optimization pp 3375–3380 | Cite as

Semidefinite Programming and Determinant Maximization

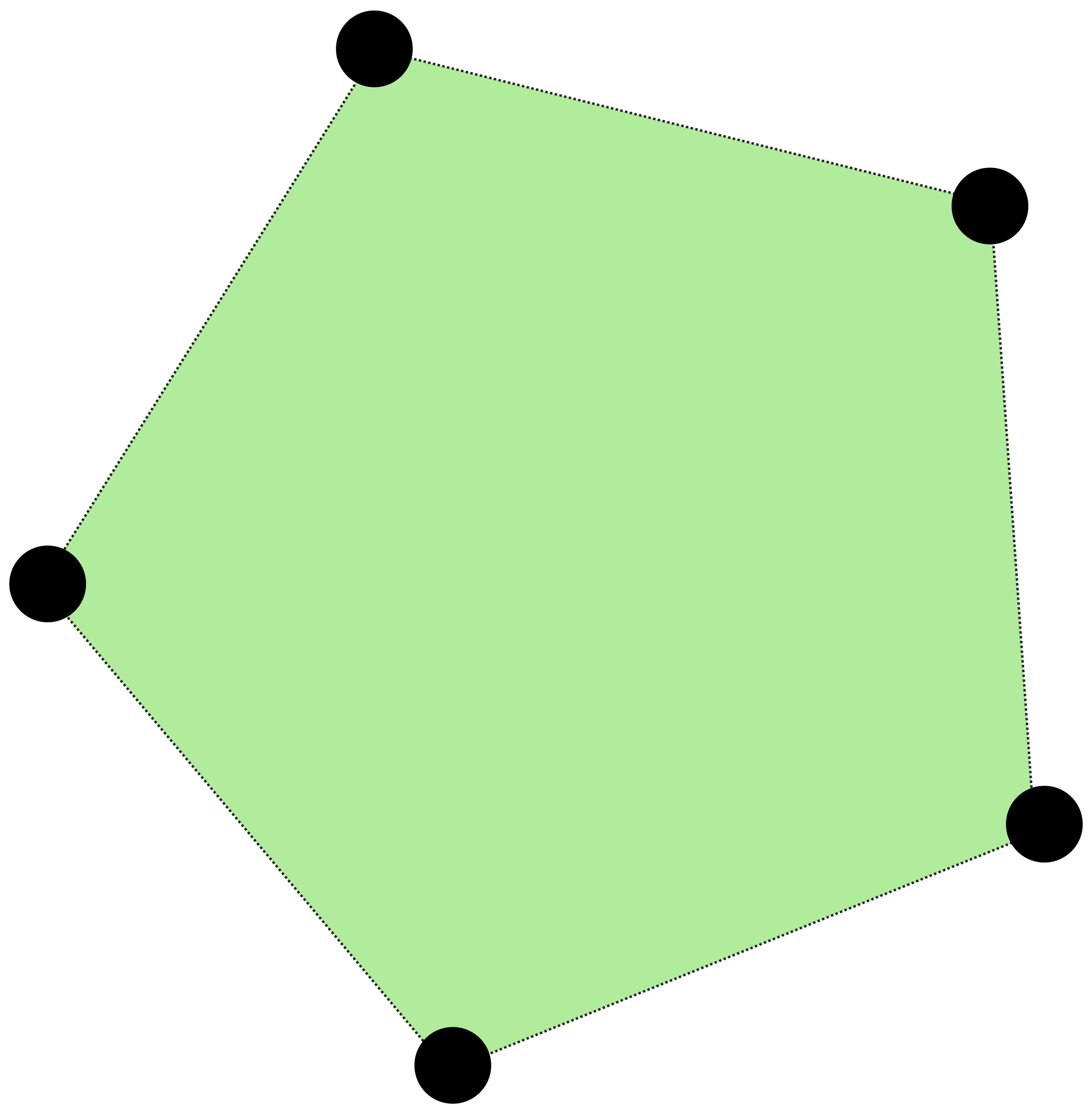
Lieven Vandenberghe, Stephen Boyd & Shao-Po Wu

Optimal retrieval channels

Stochastic channel: Φ

Prior: π

Basic ingredients

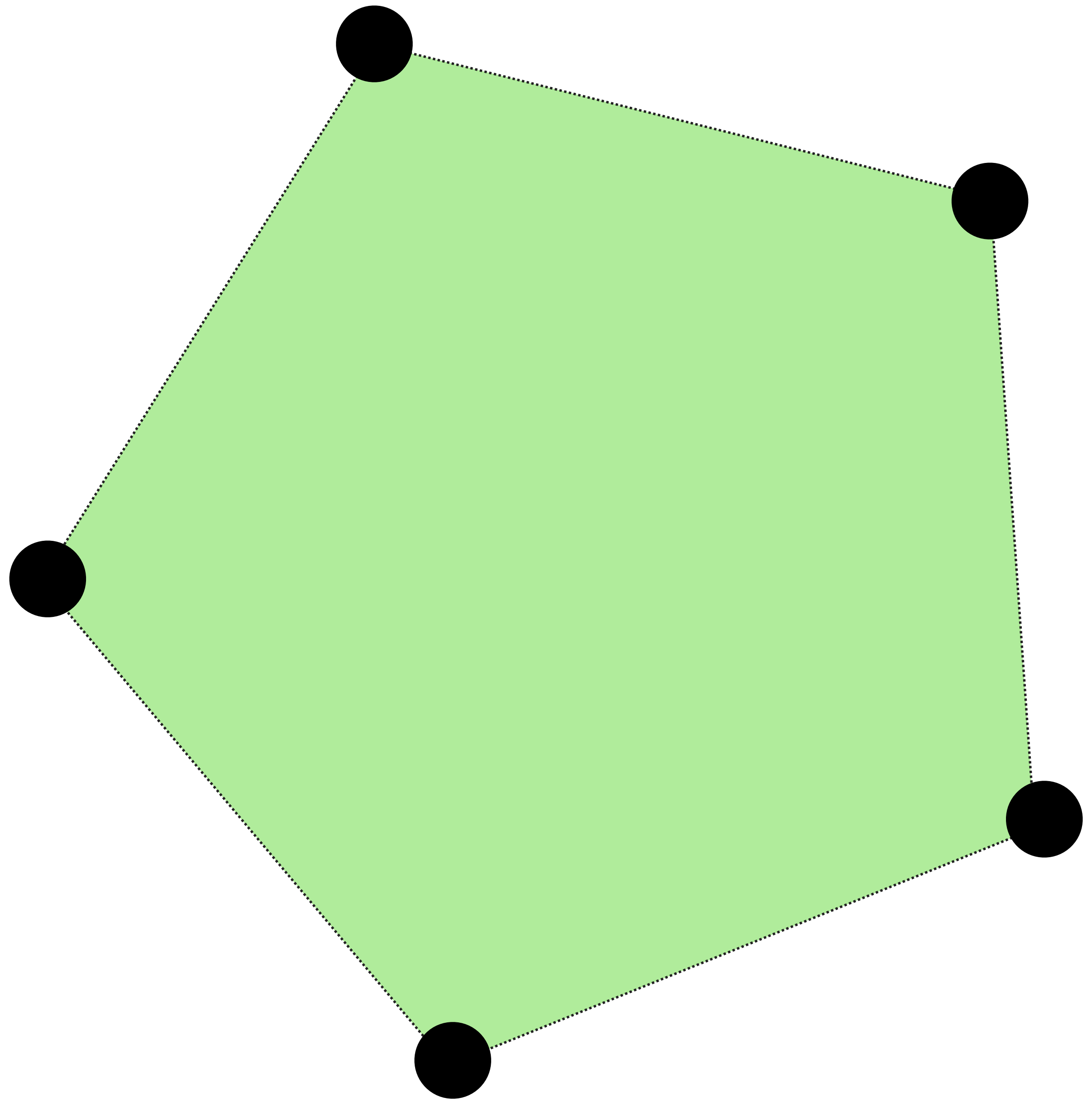


Space of retrieval channels $\tilde{\Phi} : \Phi\pi \rightarrow \pi$

Stochastic channel: Φ

Prior: π

Basic ingredients



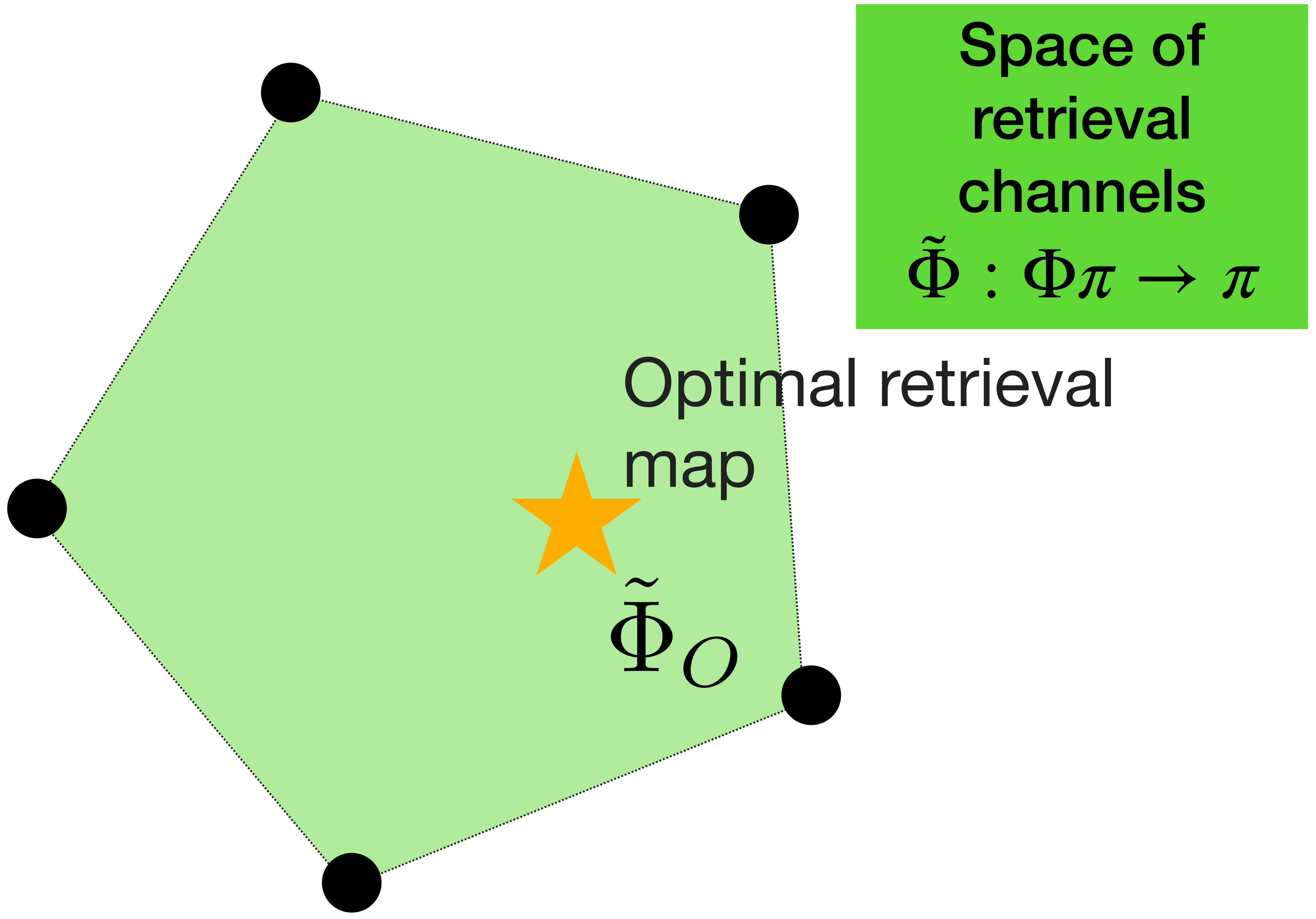
Space of retrieval channels
$\tilde{\Phi} : \Phi\pi \to \pi$

Run the optimisation algorithm…

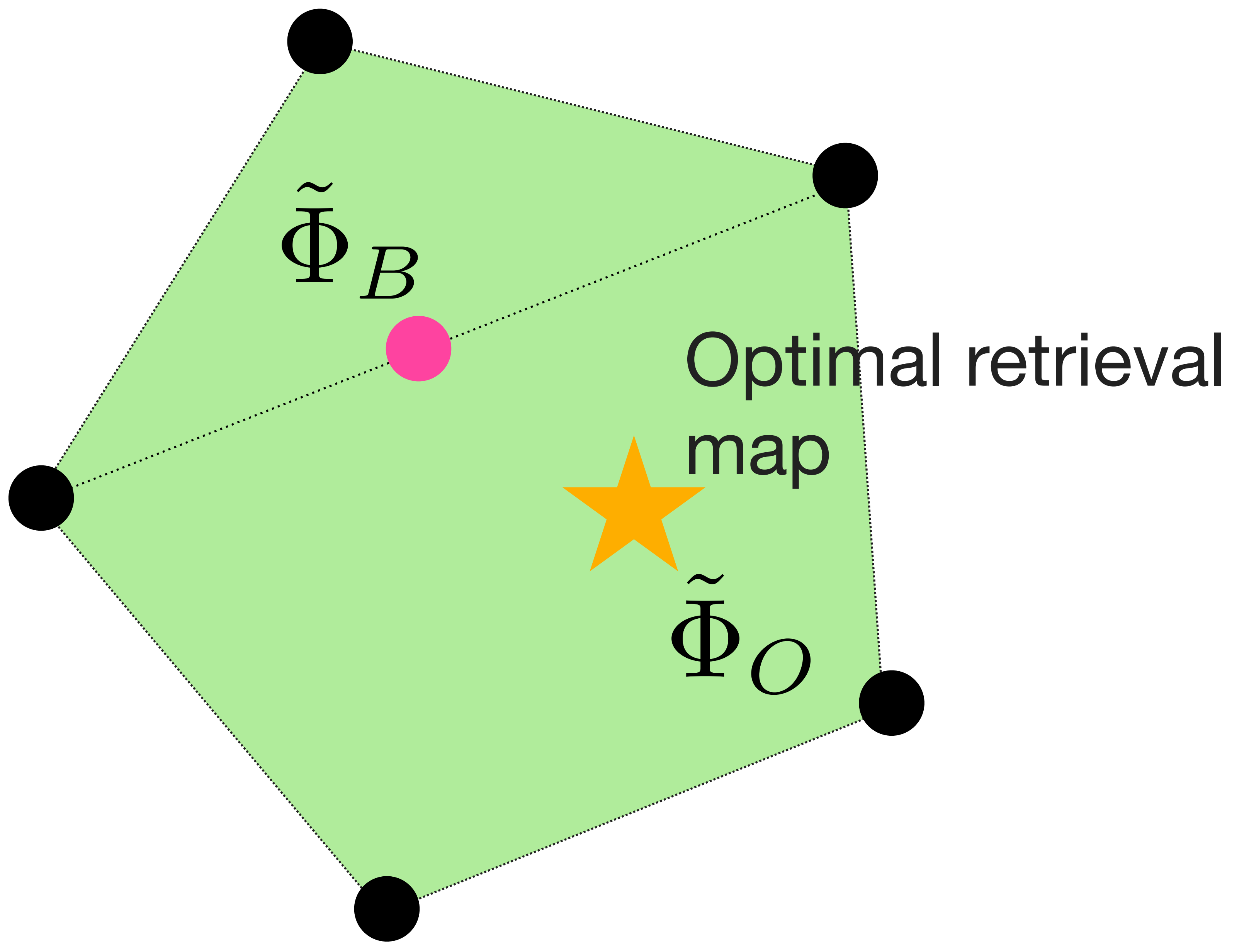
Stochastic channel: Φ

Prior: π

Basic ingredients

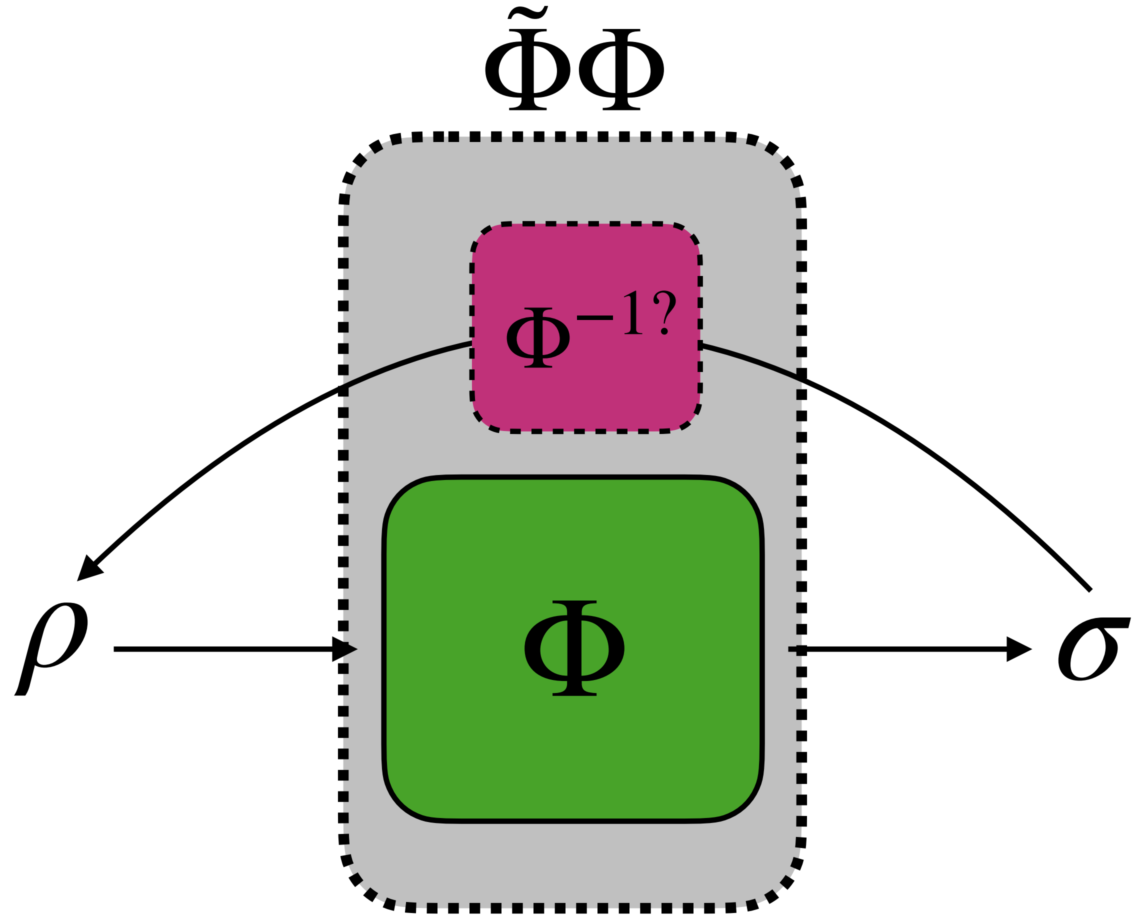


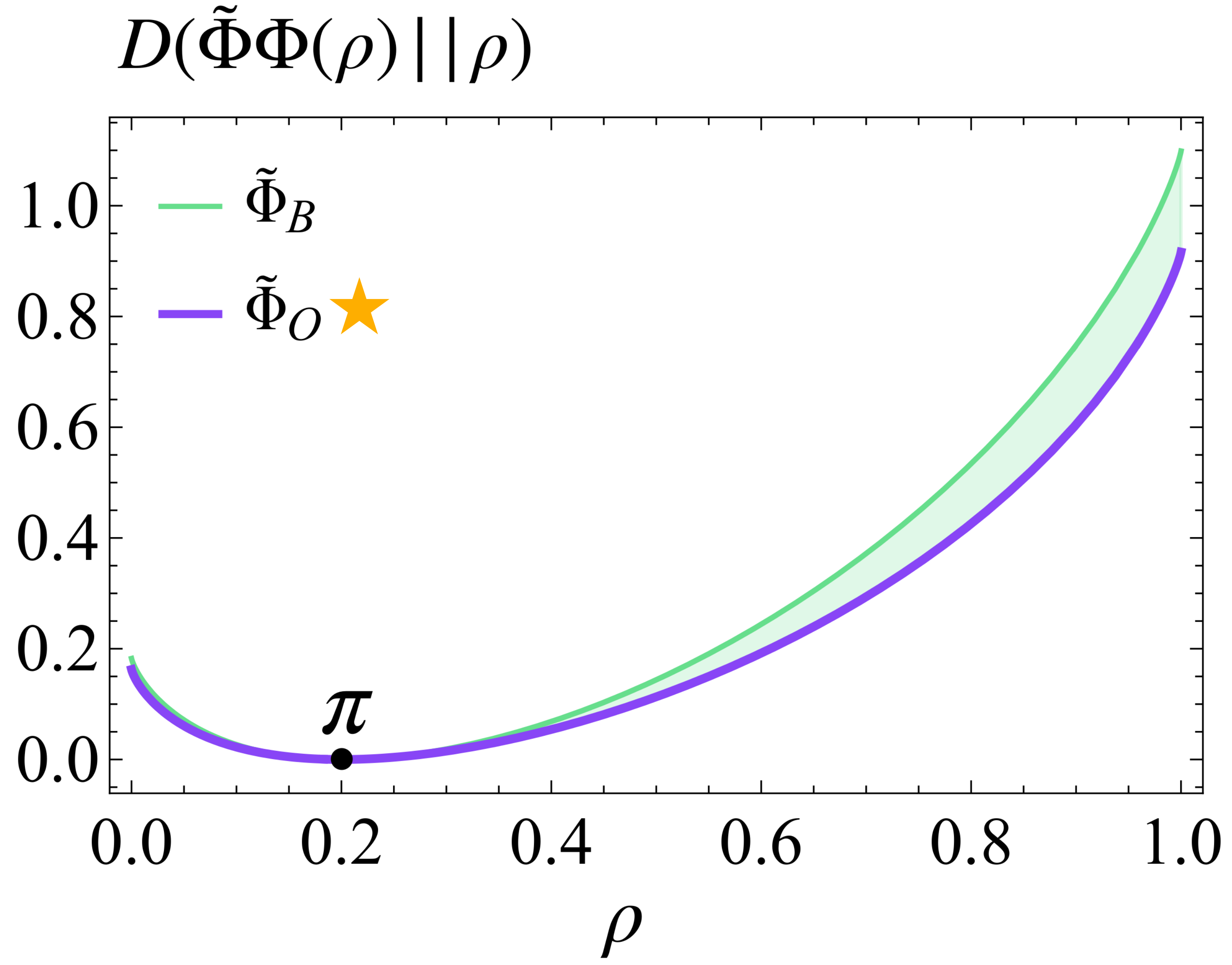
The optimal retrieval map is found!

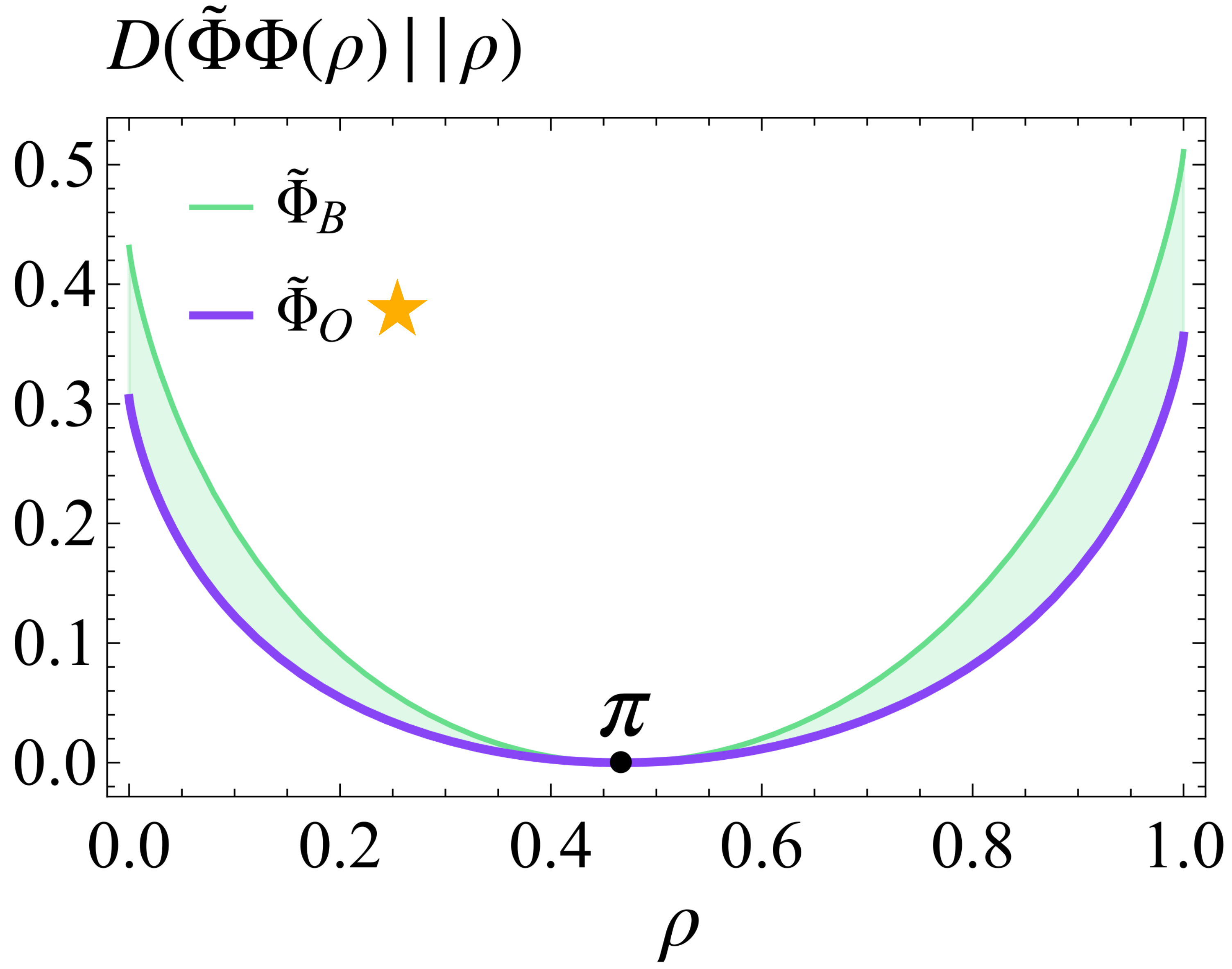


Bayes inverse and the optimal retrieval map are not the same!
Petz map and the optimal retrieval map are not the same!

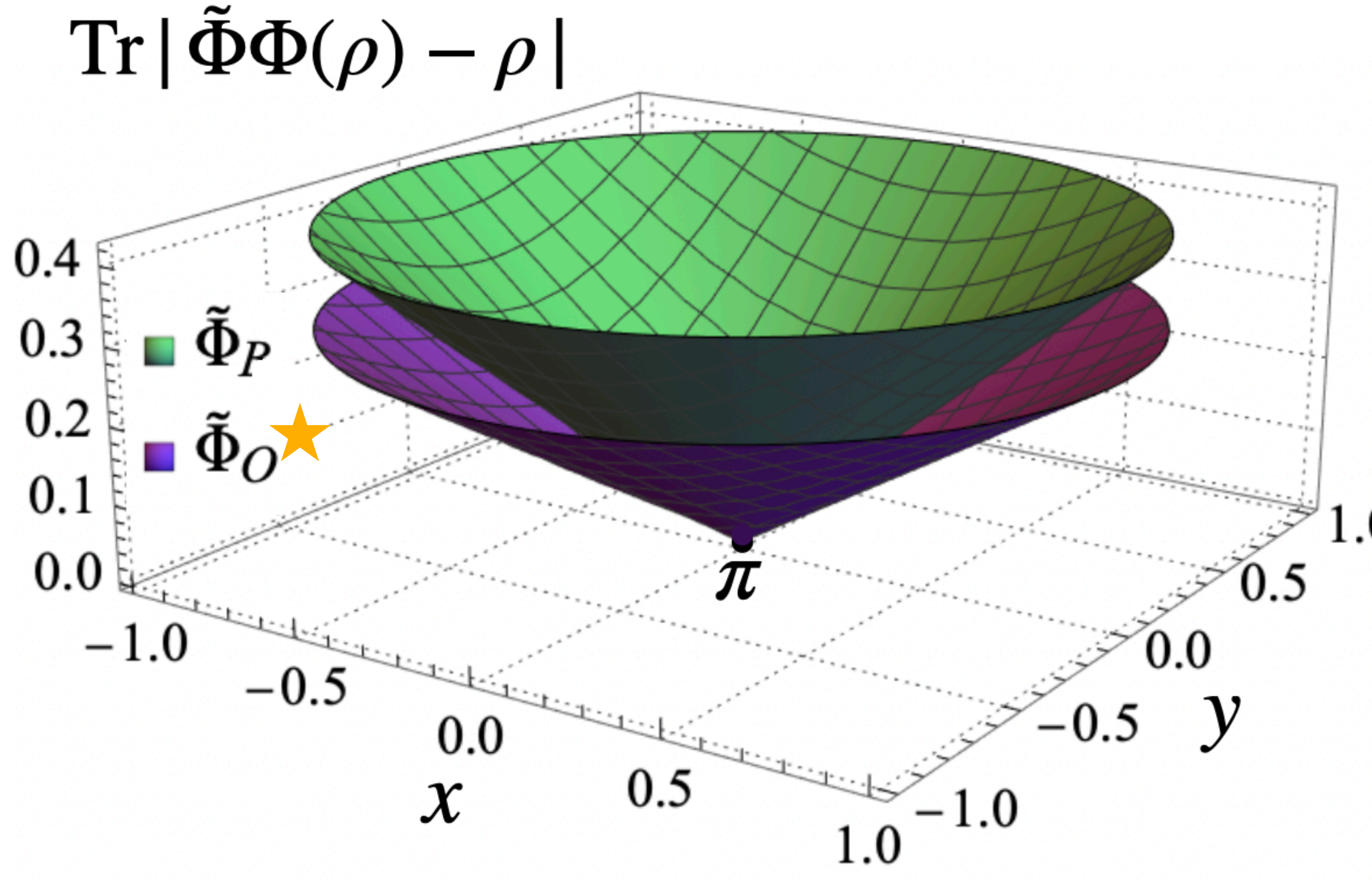
Comparison of the state retrieval with Bayes and Petz







Stochastic maps and Bayes



Quantum channels and Petz

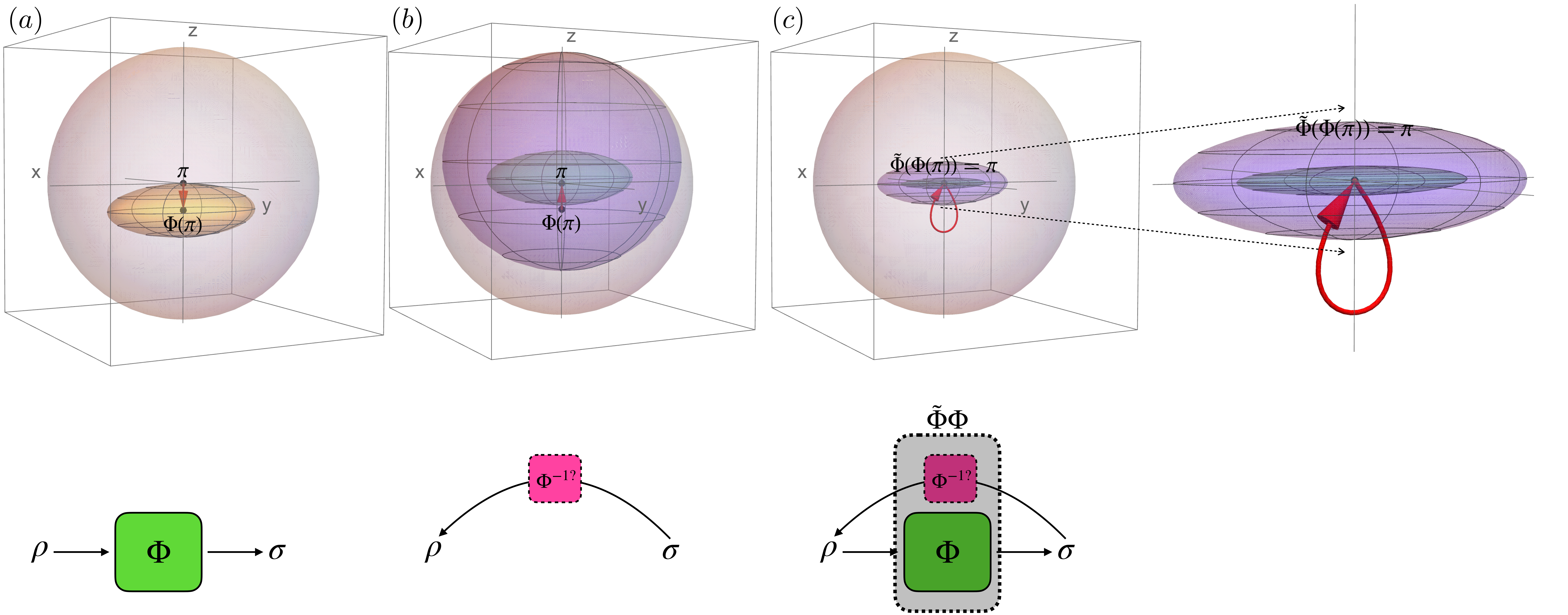
Comparison of the optimal state retrieval with Petz

Stochastic channel:	Φ
Prior:	π

Basic ingredients

$\tilde{\Phi}_P$

$\tilde{\Phi}_O$ ★

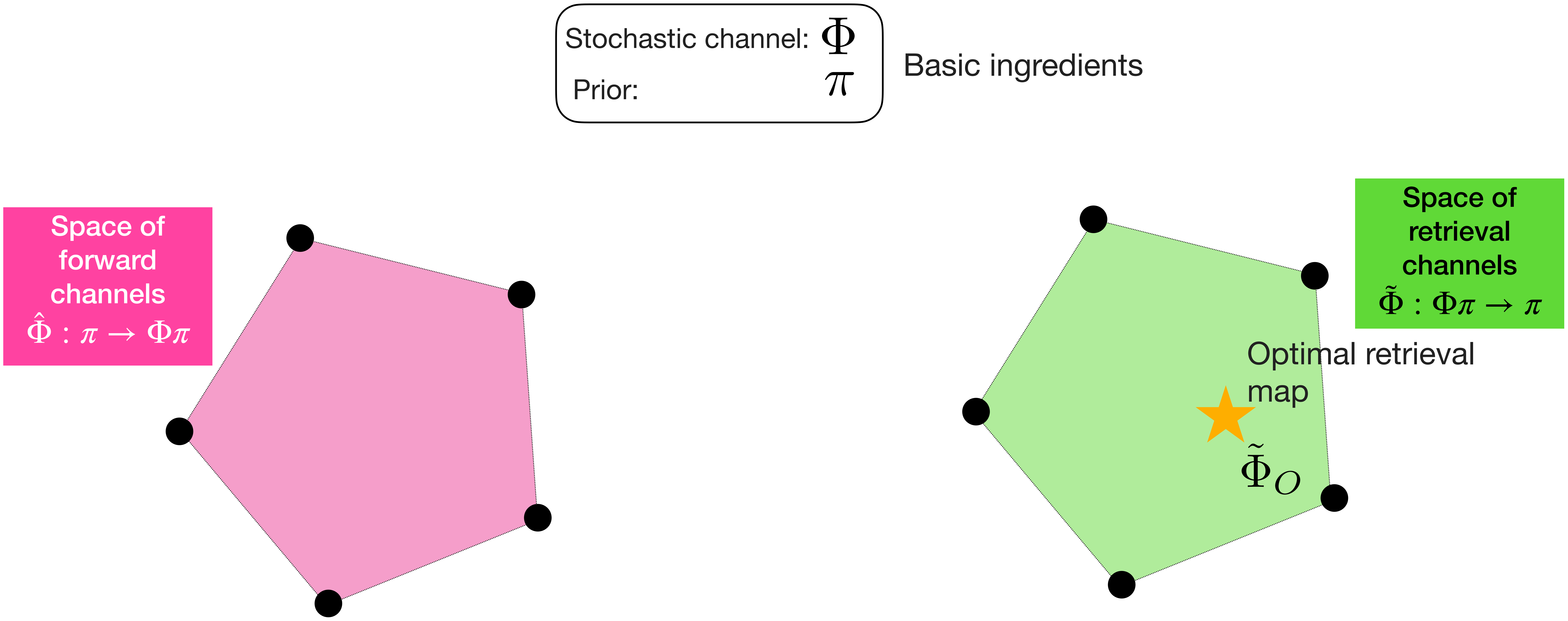


General analytical results are possible

Theorem. *Whenever a transformation* Φ *has positive spectrum and it is detailed balance with respect to the prior state (meaning that* $\Phi\,\mathbb{J}_\pi = \mathbb{J}_\pi\,\Phi^\dagger$*) the optimal state retrieval is given by the identity map.*

Where does Bayes come from?

Can we add a property that shrinks the whole space of retrieval maps to a single point?

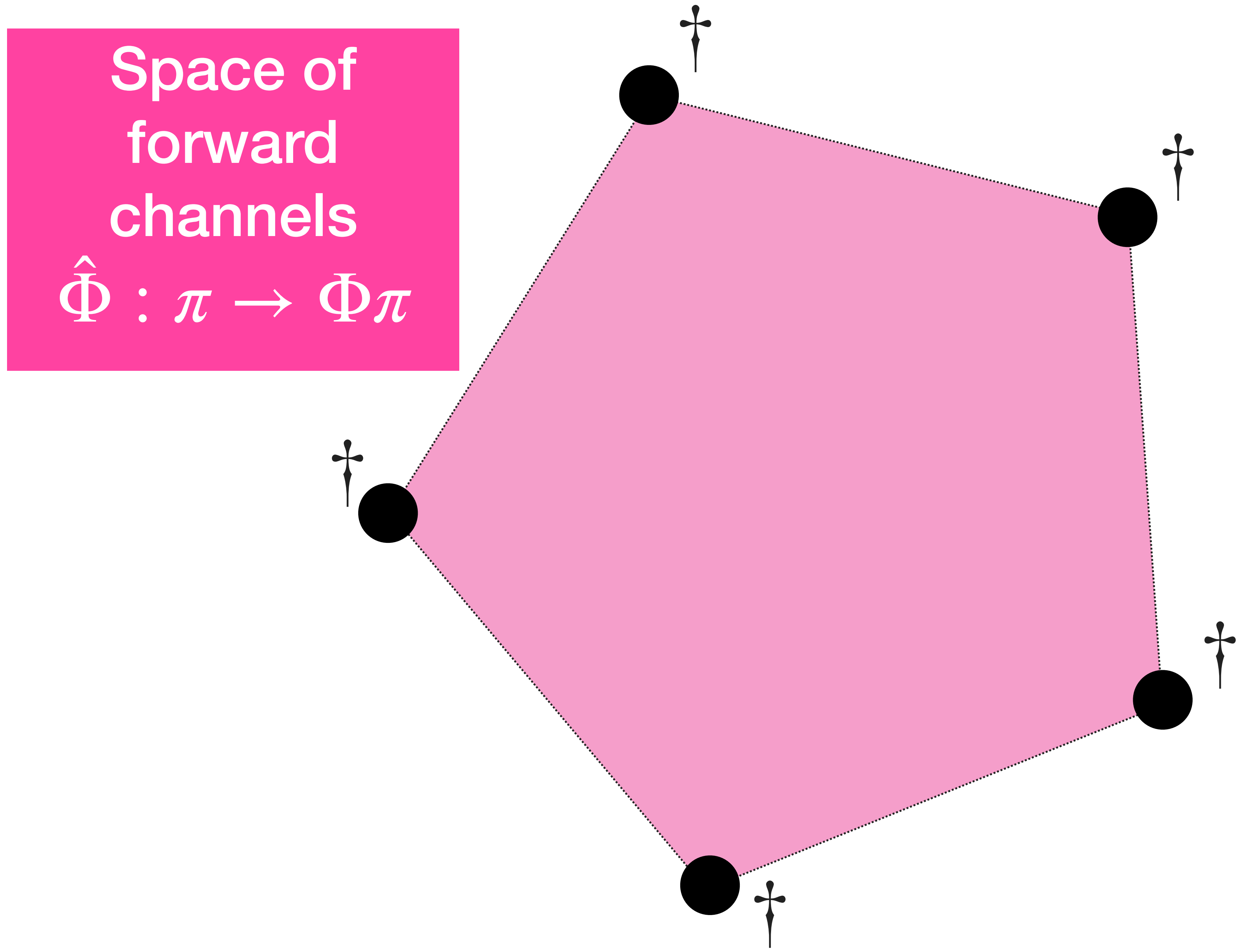

Stochastic channel: Φ
Prior: π
Basic ingredients
Space of forward channels $\hat{\Phi} : \pi \to \Phi\pi$
Space of retrieval channels $\tilde{\Phi} : \Phi\pi \to \pi$
Optimal retrieval map
$\tilde{\Phi}_O$

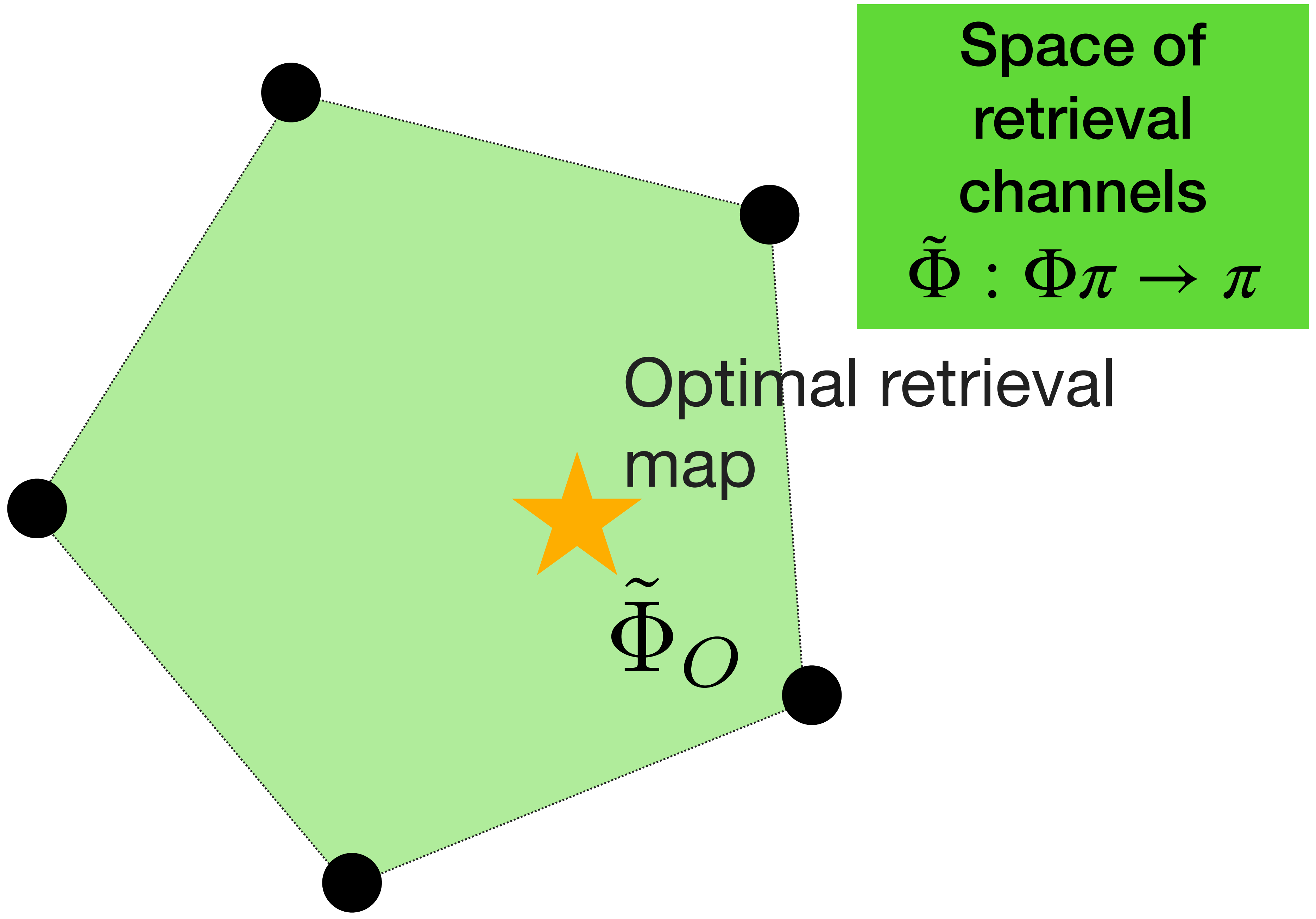
Stochastic channel: Φ

Prior: π

Basic ingredients

The vertices are the self-adjoints!



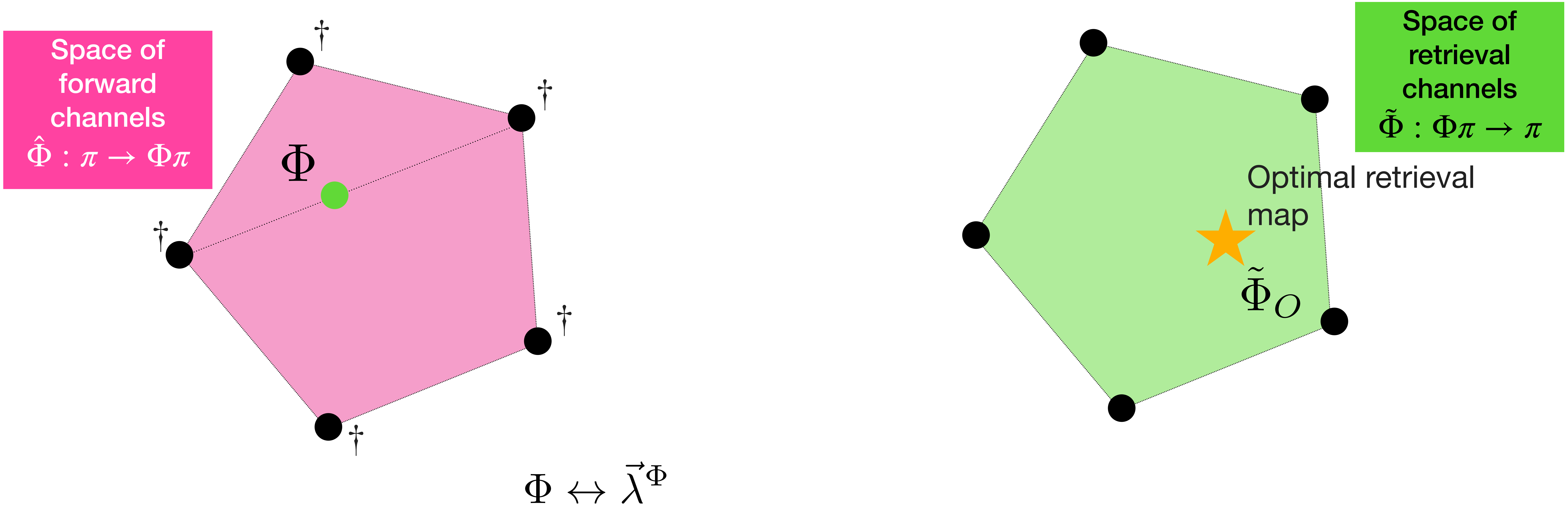


Stochastic channel: Φ

Prior: π

Basic ingredients

The vertices are the self-adjoints!

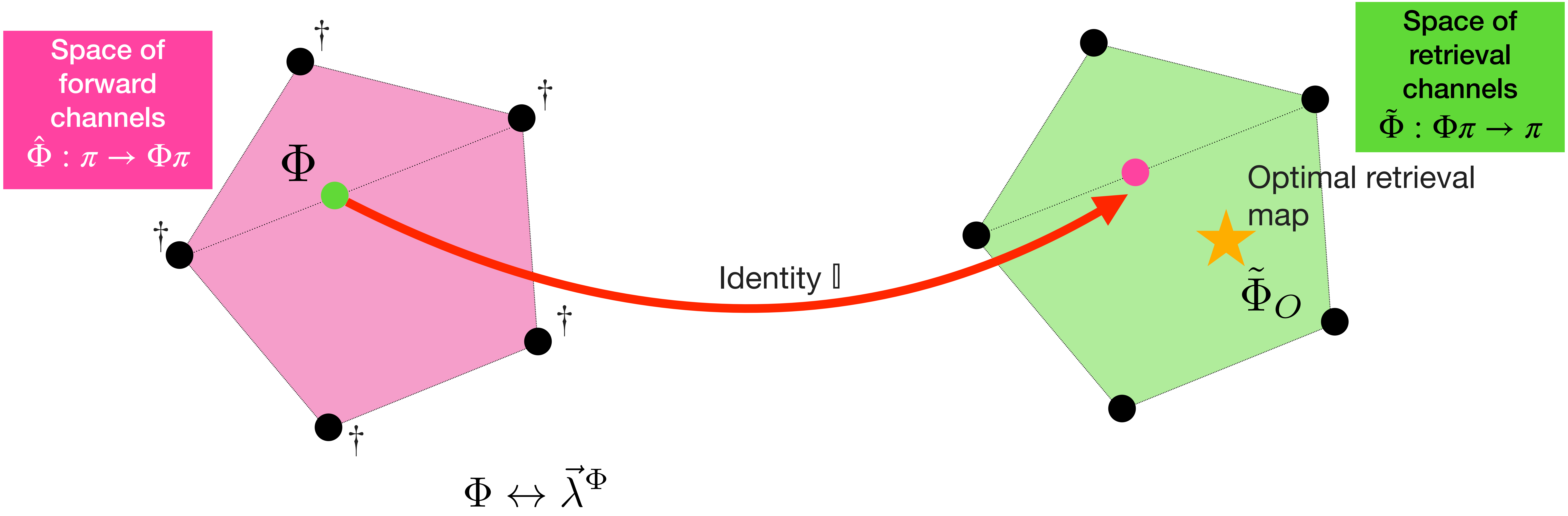


Reversion as a linear transformation between these two spaces

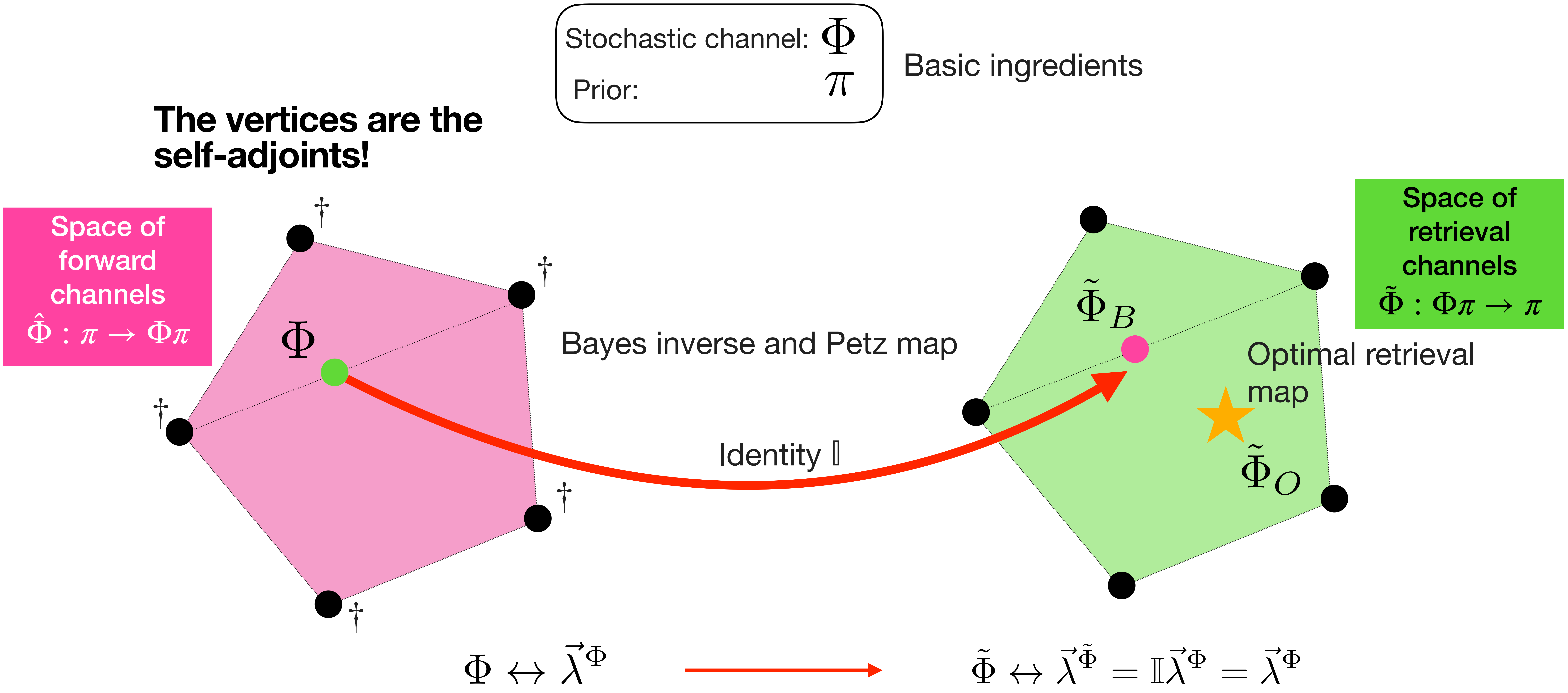
Stochastic channel:	Φ
Prior:	π

Basic ingredients

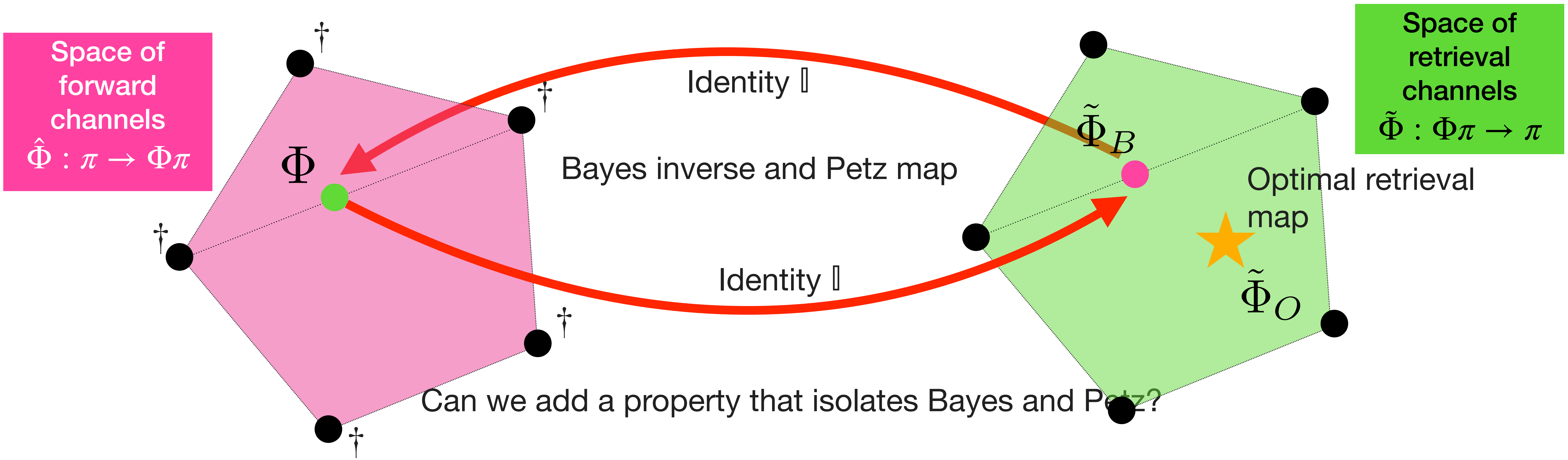
The vertices are the self-adjoints!



Reversion as a linear transformation between these two spaces



Reversion as a linear transformation between these two spaces



Can we add a property that isolates Bayes and Petz?

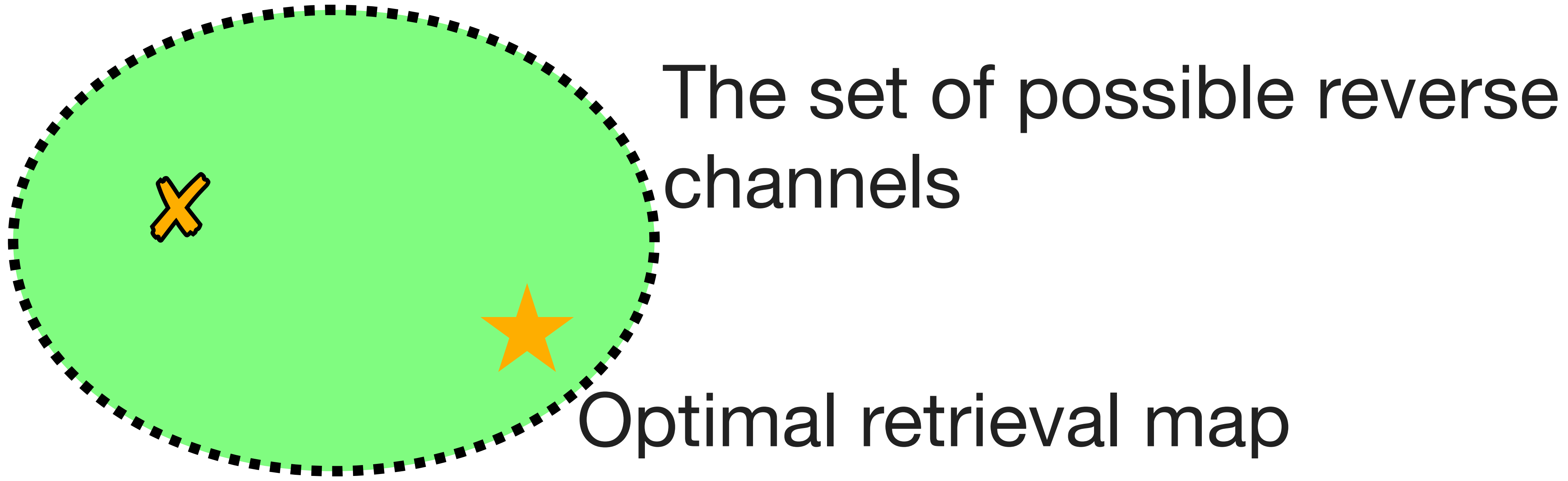
Maybe.

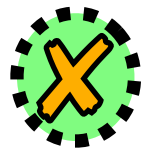
Numerical evidence of a sufficient 6th property (involutivity) that isolates Bayes and Petz.

Simplicity is a nice criterion

Conclusions

- We dealt with the ambiguity in defining a reverse channel by characterising a set of admissible retrieval channels.
- We gave a computable criterion for choosing the optimal retrieval channel.
- The “canonical” reverse channel (Bayes inspired) is an admissible retrieval channels.
- The Bayes and Petz maps maybe can be isolated by asking for involutivity.





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Thank you

University of Strathclyde Glasgow

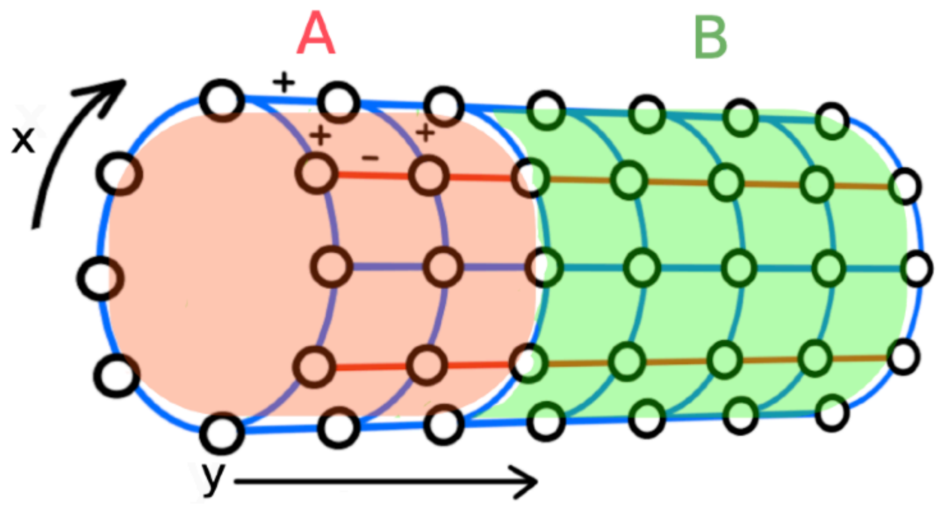
PhD in Condensed Matter, Glasgow, Scotland, Strathclyde University

Supervisor Luca Tagliacozzo

Entanglement in many-body systems:

Tools that I learned:

- Ising Chains, Entanglement growth, Tensor networks, DMRG, Conformal field theories



Tools that I created:

- Package for simulation of Fermionic Systems in Julia

F_utilities.pkg

GitHub

ICFO

(?)

Postdoc in Quantum Information group Barcelona, Spain, ICFO

Supervisor Antonio Acín

Many body system certification:

- Certification of bound on energies and other observables of many body systems with SDP and DMRG.

Undecidability theory:

- Undecidability of the membership problem in resource theories. Techniques for proving undecidability. Algorithmic information theory.

Information Geometry:

- Fisher Information, contractivity of channels, non-Markovianity, Reverse Csizar theorem, error correction.

Foundation of probability theory:

- Bayes Theorem, Maximum entropy methods, derivation of probability, De Finetti subjectivism.

PERIMETER INSTITUTE

Postdoc in Quantum Foundation group Waterloo, ON, Canada, Perimeter Institute

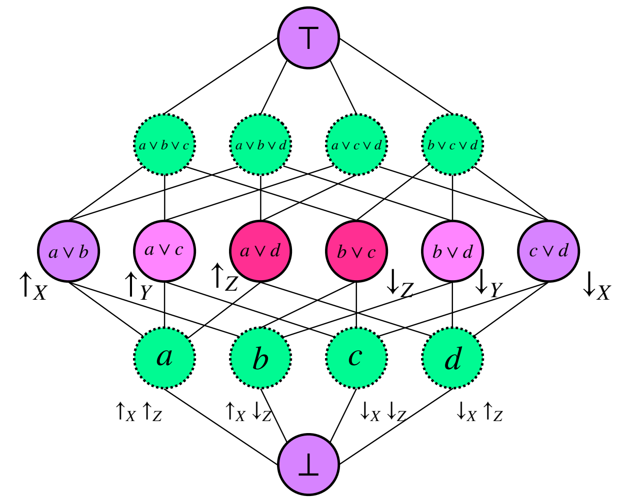
Supervisor Robert Spekkens

Symmetries and Local tomography:

- Unitary representation of groups, failure of local tomography in classical, quantum and post-quantum theories and GPT.

Logic, lattices, quasi-probability:

- Reconstruction of probability and quasi-probability from logic, Inaccessibility Hypothesis, epistemic restrictions. Reconstruction of quantum mechanics from logic.



Correlations in time and space:

- State over times, quantum inference, Wigner Friends, time reversal in quantum and classical mechanics.