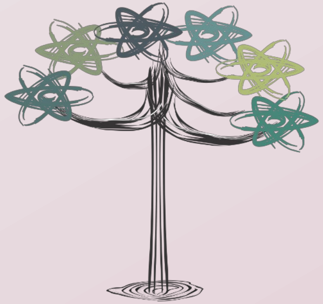
Thermalization via quantum walks

Alana Spak



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Summary

♦ Group in Brazil;

♦ Quantum walks;

♦ Recent progress (thermalization);

♦ Thermalization via quantum walks.

Curitiba, Paraná, Brasil

Research Group

Supervisor: Prof. Renato Angelo

- **Foundations of Quantum Mechanics**
- Quantum Information
- Quantum Thermodynamics
- Classical limit of Quantum Mechanics

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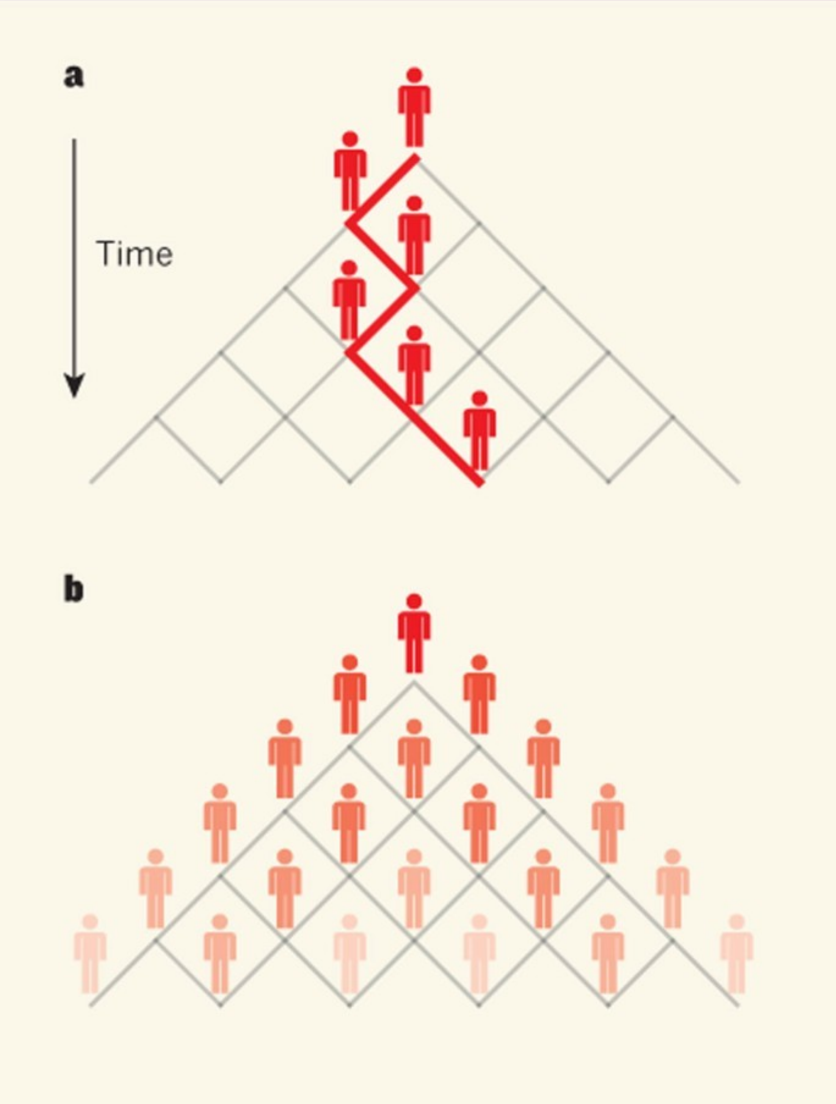
Thermalization in quantum systems????

quantum walks

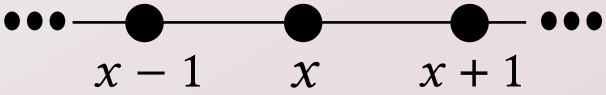
Quantum walks

Quantum walks

- ♦ 1D model
- ♦ continuous-time or discrete-time



Quantum walks



$$\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_s$$

$\mathcal{H}_s \rightarrow \{|\uparrow\rangle, |\downarrow\rangle\}$

$\mathcal{H}_p \rightarrow \{|x\rangle, x \in \mathbb{Z}\}$

State:

$$|\psi(t)\rangle = \sum_x (a_x(t)\,|\uparrow\rangle + b_x(t)\,|\downarrow\rangle) \otimes |x\rangle$$

$$a_x(t), b_x(t) \in \mathbb{C}$$

Quantum walks

Time evolution operator

$$U = D_p(\mathbb{I}_p \otimes C_s) \longrightarrow |\psi(t)\rangle = U^t |\psi(0)\rangle$$

where

$$D_p = \sum_x (|x-1\rangle\langle x| \otimes |\downarrow\rangle\langle\downarrow| + |x+1\rangle\langle x| \otimes |\uparrow\rangle\langle\uparrow|),$$

C_s : coin operator

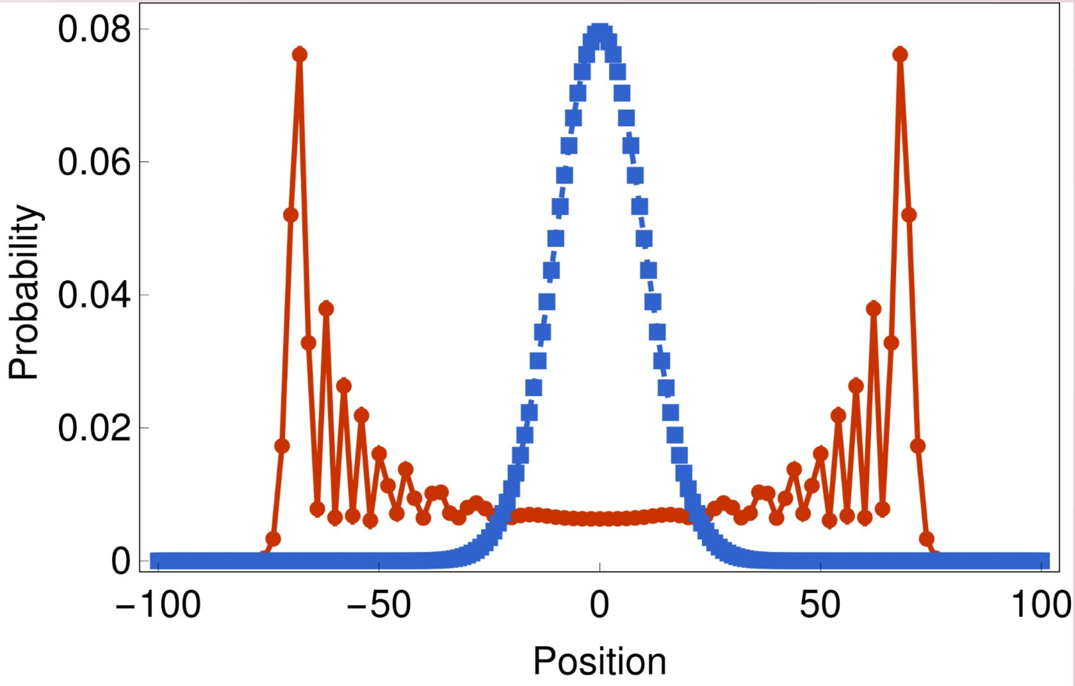
Quantum walks

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Dispersion:

♦ classical: $\sigma \sim \sqrt{t}$

♦ quantum: $\sigma \sim t$



$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle \otimes |\uparrow\rangle + |0\rangle \otimes |\downarrow\rangle\right)$$

Entanglement

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \ldots \otimes \mathcal{H}_N$$

Separability: $|\psi\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$ $\longrightarrow$ **Entangled:** $|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$

Example:

$$|\psi\rangle = \frac{|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B}{\sqrt{2}}$$

$$S(\rho) = 0$$

$$S(\rho_A) = S(\rho_B) = 1$$

von Neumann entropy

$$S(\rho) = -\operatorname{Tr}(\rho \log \rho) = -\sum_i \lambda_i \log \lambda_i .$$

Pure state: $S(\rho) = 0$

$$S(\rho_A) = -\operatorname{Tr}(\rho_A \log \rho_A)$$

Mixed state: $S(\rho) = \log d$

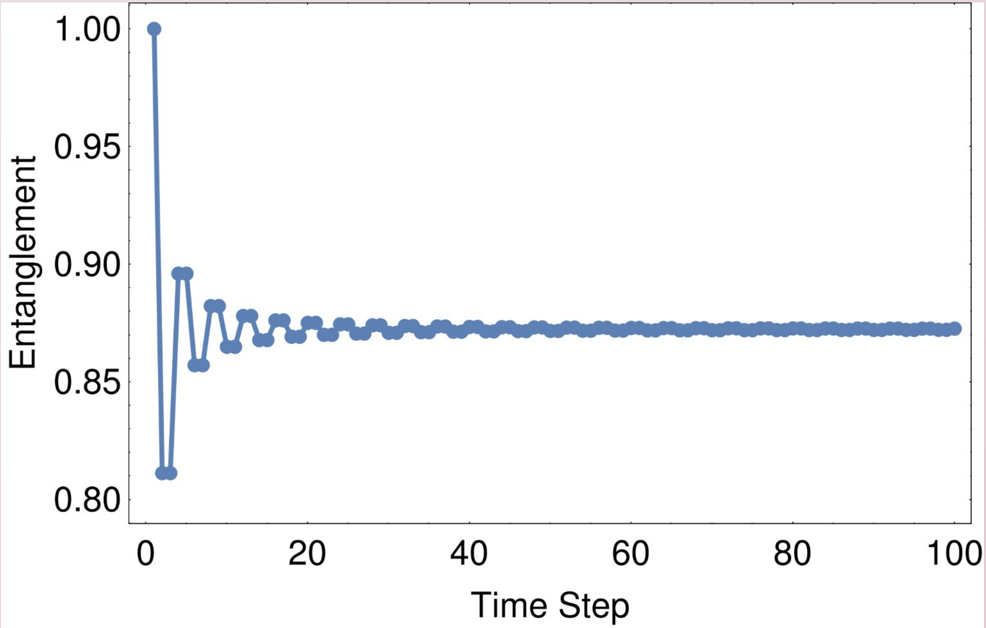
Quantum walks

$$\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_s$$

↓

entanglement

$$S(\rho_s) \simeq 0.872$$



Decoherence

- Quantum-to-classical transition;
- quantum effects: **system-environment** is minimal.

Recent progress

(Thermalization)

Thermalization

- Equilibrium: multipartite systems

  → quantum correlations (entanglement, discord,...)

- Thermodynamic equilibrium: systems share the same temperature

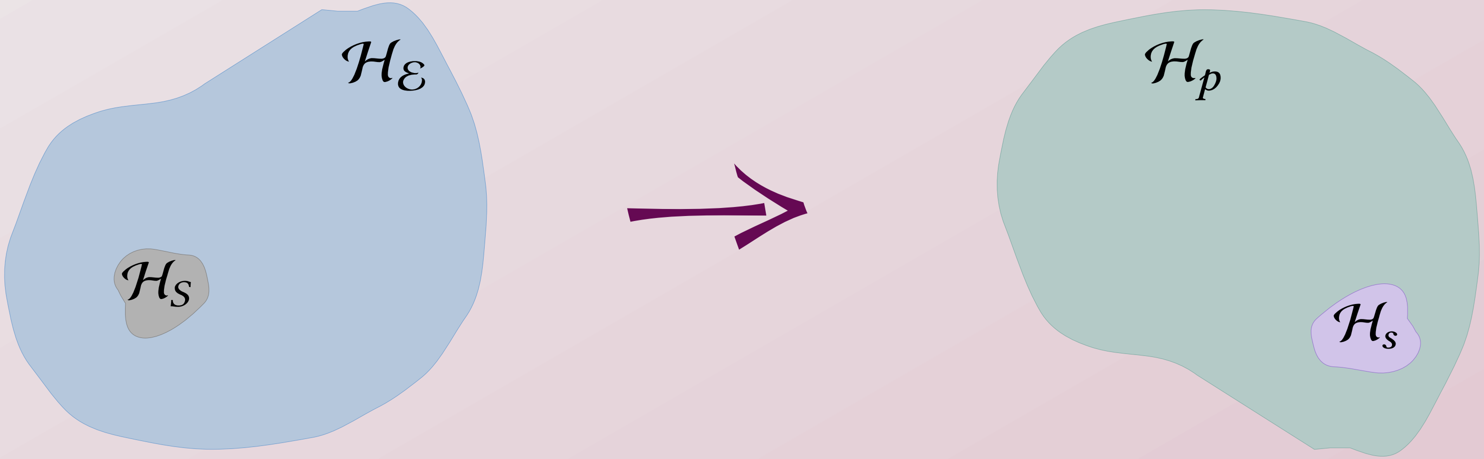
  → thermodynamics and information theory

- Isolated systems: thermalization occurs within subsystems

  → degrees of freedom

Thermalization

"Entanglement and the foundations of statistical mechanics"



$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_\mathcal{E}$$

$$\mathcal{H} = \mathcal{H}_s \otimes \mathcal{H}_p$$

(QWs)

Popescu, *et al.* **Nat. Phys.** (2) 2006.

Thermodynamic equilibrium and quantum walk

Thermodynamic equilibrium: between the degrees of freedom $\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_s$

If ρ_s reaches equilibrium $\longrightarrow$ *"Entanglement temperature"*

Romanelli, A. **PRA** (85) 2012.

Projects

Quantum walk in energy space

position $\longrightarrow$ energy

Thermalization: universal dynamic

$M_n = |n\rangle\langle n|$

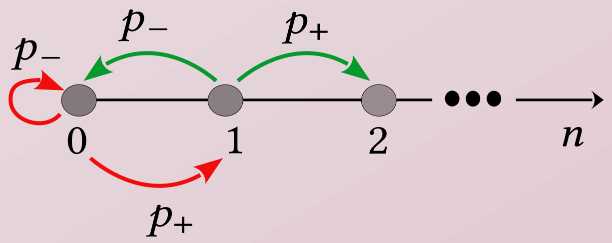
$\{\mathcal{E}_n, M_n\}$

energy scale

Quantum walk in energy space

Classical limit

$$\rho_0 = \sum_{m=0}^{\infty} \wp(m) M_m$$



Time evolution:

$$\phi(M_n) = \begin{cases} p_- M_n + p_+ M_{n+1} & \text{if } n = 0, \\ p_- M_{n-1} + p_+ M_{n+1} & \text{if } n > 0. \end{cases}$$

↓

non-unital map

Quantum walk in energy space

Classical limit

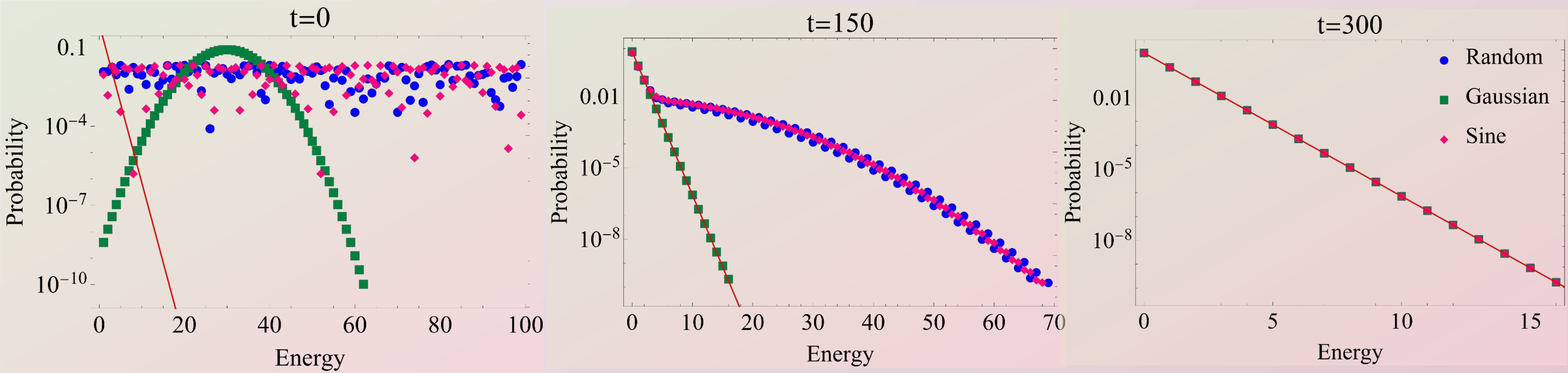
$$p_{t+1}(n) = \mathrm{Tr}\left[M_n \rho_{t+1}\right]$$

Recurrence relation

$$p_{t+1}(0) = p_- \Big[p_t(0) + p_t(1)\Big],$$

$$p_{t+1}(n) = p_-\, p_t(n+1) + p_+\, p_t(n-1).$$

Quantum walk in energy space



$$T = 1.38\,\mathcal{E}$$

$p_- = 0.8$ and $p_+ = 0.2$

Quantum walk in energy space

Analytical study

(i) Steady state: $\vec{p} = \vec{p}\,\mathcal{P},$

(ii) Normalization: $\sum_{i=1}^{N} p_i = 1.$

(Markov process)

$$\mathcal{P} = \begin{pmatrix} p_- & p_+ & 0 & 0 & 0 & \dots \\ p_- & 0 & p_+ & 0 & 0 & \\ 0 & p_- & 0 & p_+ & 0 & \\ 0 & 0 & p_- & 0 & p_+ & \\ 0 & 0 & 0 & p_- & 0 & \\ \vdots & & & & & \ddots \end{pmatrix}$$

$$\vec{p} = \begin{bmatrix} p_0 & p_1 & p_2 & \dots & p_n & \dots \end{bmatrix}$$

Quantum walk in energy space

Analytical study

(i) Steady state: $\vec{p} = \vec{p}\,\mathcal{P} \quad \longrightarrow \quad p_n = \left(\frac{p_+}{p_-}\right)^n p_0$

(ii) Normalization: $\sum_{n=1}^{N} p_n = 1 \quad \longrightarrow \quad p_0 = \frac{1}{\sum_{n=0}^{\infty}\left(\frac{p_+}{p_-}\right)^n}$

Quantum walk in energy space

Analytical study

$$p_\beta = \frac{e^{-\beta E_n}}{\sum_{n=0}^{\infty} e^{-\beta E_n}}$$

$$p_n \equiv p_\beta$$

$$\beta = \frac{n}{E_n} \ln\left(\frac{p_+}{p_-}\right)$$

Condition: $E_n = n\mathcal{E}$

$$T = \frac{1}{\beta} = \mathcal{E} \ln\left(\frac{p_-}{p_+}\right)$$

Takeaway messages

♦ Condition for thermalization: one system is large enough to induce decoherence;

♦ Quantum walks: promising model to explore the topic;

♦ For quantum walk in energy space, we are able to determine the tempertature in the classical limit.

CANA

- Co-supervisor: Giuseppe di Molfetta

Plastic quantum walks + Twist + Kerr non-linearities

limit of continous-time discrete space and continous space-time

quadratic term in the energy spectrum acts as an effective mass

How is it going to behave?

Acknowledgements

CAPES

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