Introduction to benchmarking and certification of quantum computers

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Plan for the talk

1. Overview: Benchmarking and certification

2. Some math: group twirls

3. Three important protocols:

    - randomized benchmarking
    - classical shadows
    - randomized compiling

1. Overview

The challenge:

How do we know our quantum computer is functioning correctly?

If it is, *how well* is it functioning?

Certification

The task of ensuring the correct functioning of a quantum device in terms of the accuracy of the output.

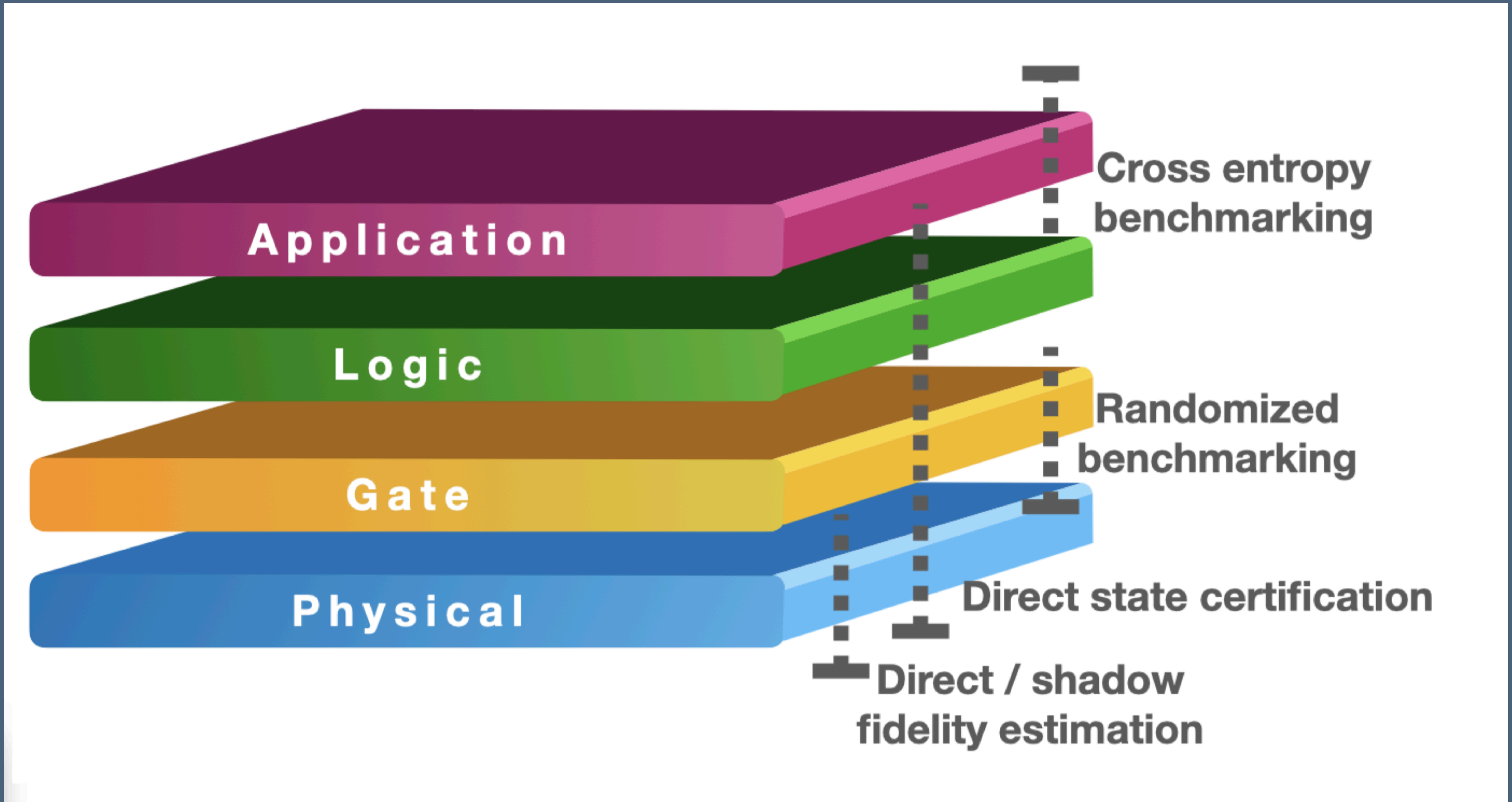
Benchmarking

The task of assigning a reproducible performance measure to a quantum device.

(Some of) the hurdles:

- Full quantum state or process tomography requires exponential resources
- Quantum computations cannot be efficiently simulated classically

Layers of abstraction



from Kliesch and Roth 2020, *Theory of quantum system certification*

States and channels (fixing notation)

States:

- Default meaning: density operators ρ (positive semi-definite, Hermitian operator with trace 1)
  - pure states go by $\psi = |\psi\rangle\langle\psi|$

Channels:

- Superoperators mapping states to states (Completely Positive, Trace-Preserving maps)
- Channels get curly letters $\mathcal{E}(\rho)$
  - Unitaries are non-curly. Example: A unitary channel acts as $\mathcal{U}(\rho) = U\rho U^\dagger$

What to estimate?

1. **State preparations**:

   - State fidelity $$F(\rho, |\psi\rangle\langle\psi|) = \langle\psi|\rho|\psi\rangle = \mathrm{Tr}(|\psi\rangle\langle\psi|\,\rho)$$
   - Trace distance $$d(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1 \quad \text{with} \quad \|A\|_1 = \mathrm{Tr}\left(\sqrt{A^\dagger A}\right)$$

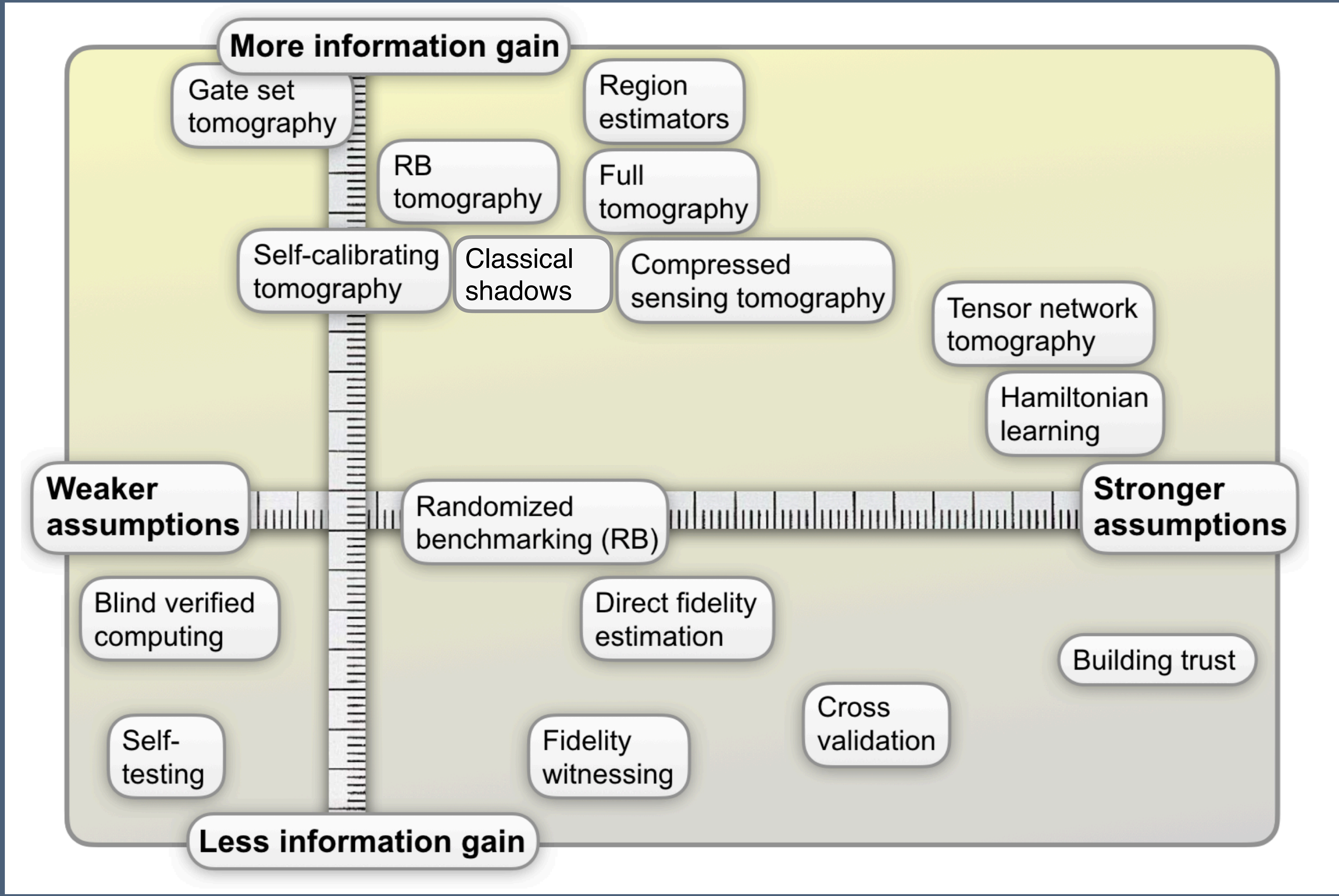
2. **Gates**:

   - Average gate fidelity $$F_{\mathrm{avg}}(\mathcal{E}, U) \equiv \int d\psi \, \langle\psi|U^\dagger \mathcal{E}(|\psi\rangle\langle\psi|)U|\psi\rangle = \int d\psi \, \mathrm{Tr}\left[\mathcal{U}(|\psi\rangle\langle\psi|)\mathcal{E}(|\psi\rangle\langle\psi|)\right]$$
   - Diamond distance $$d_\diamond(\mathcal{E},\mathcal{U}) = \frac{1}{2}\|\mathcal{E} - \mathcal{U}\|_\diamond = \frac{1}{2}\max_{\rho_{AB}}\|((\mathcal{E}_A - \mathcal{U}_A)\otimes\mathcal{I}_B)[\rho_{AB}]\|_1$$

Landscape of protocols

Triple trade-off between

1. Information gain
2. Strength of assumptions
3. Resource requirements



from Eisert et al. 2020, *Quantum certification and benchmarking*

2. Group twirls

over the Clifford and Pauli group, unitary designs, and all that

Twirling a channel

Definition: average $\mathcal{U} \circ \mathcal{E} \circ \mathcal{U}^\dagger$ over $\mathcal{U}$ drawn from some group G

- After rewriting and specifying the group to the unitary group $\mathrm{U}(d)$:

$$\bar{\mathcal{E}}(\rho) = \int_{U(d)} U^\dagger \mathcal{E}\left(U \rho U^\dagger\right) U \, \mathrm{d}\mu_{\mathrm{Haar}}(U)$$

- This is an example of a Haar integral over the unitary group
  - We can solve those!

Introduction to Haar Measure Tools in Quantum Information: A Beginner's Tutorial

Antonio Anna Mele

Twirling over the unitary group

- We can solve integrals over the unitary group by exploiting *Schur-Weyl duality*, the rules are commonly called *Weingarten calculus*

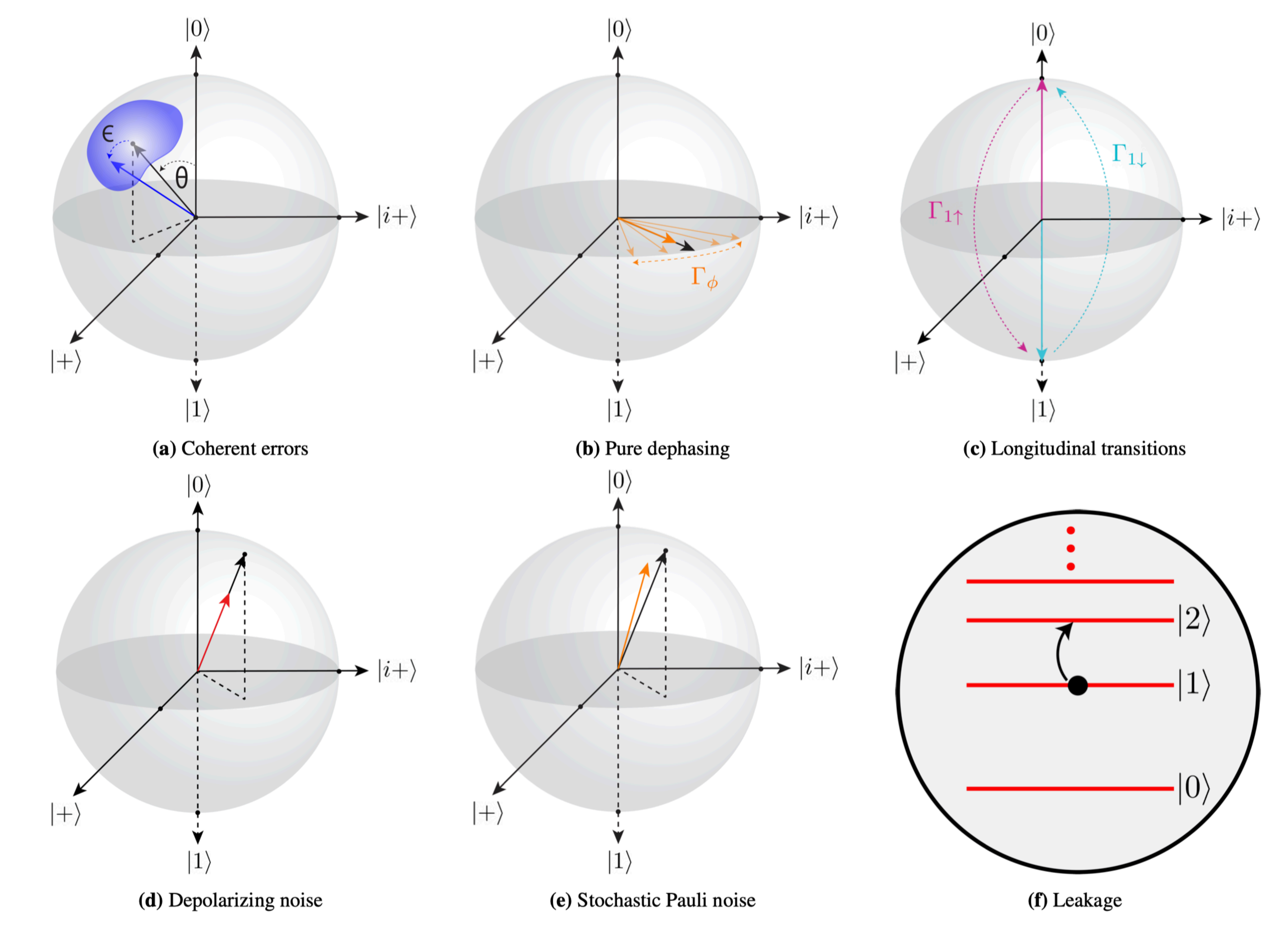
- Read Antonio's tutorial, it's great

- All we need for the purpose of this talk:

$$\bar{\mathcal{E}}(\rho) = \int_{\mathrm{U}(d)} U^\dagger \mathcal{E}\left(U \rho U^\dagger\right) U \mathrm{d}\mu_{\mathrm{Haar}}(U)$$
$$= p_{\mathcal{E}} \rho + (1 - p_{\mathcal{E}}) \frac{\mathbb{I}}{d}$$

‣ Twirling a channel over the unitary group turns it into a depolarizing channel

Side note: different types of noise



from Hashim et al 2024, *A practical introduction to benchmarking and characterization of quantum computers*

Clifford twirls

1. The Clifford group

- Clifford unitaries map Pauli operators to other Pauli operators (up to a phase) under conjugation:

$$\mathrm{Cl}(n) = \{V \in \mathrm{U}(2^n) \mid VPV^\dagger \in \mathrm{P}(n) \text{ for all } P \in \mathrm{P}(n)\}$$

- Swiss Army knife of quantum information:
  - Basis of quantum error-correcting codes (stabilizer formalism)
  - We can efficiently simulate them classically (Gottesman-Knill theorem)
  - They are a unitary 2-design
    - A what?

Clifford twirls

2. Unitary designs

- The twirl is a special case of expressions like $\int_{U(d)} (U)^{\otimes t} A \left(U^\dagger\right)^{\otimes t} \mathrm{d}\mu_{\mathrm{Haar}}(U)$
  - a.k.a. t-th moments
- A **unitary t-design** is a finite set for which the average is equivalent to the Haar integral over the unitary group:

D is a unitary t-design iff $$\frac{1}{|D|} \sum_{U_i \in D} (U_i)^{\otimes t} A \left(U_i^\dagger\right)^{\otimes t} = \int_{U(d)} (U)^{\otimes t} A \left(U^\dagger\right)^{\otimes t} \mathrm{d}\mu_{\mathrm{Haar}}(U)$$

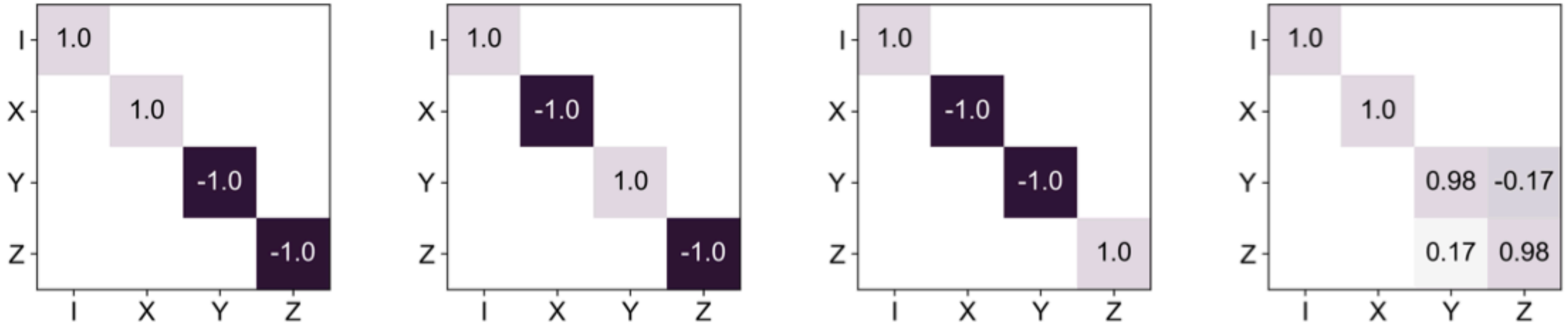
- The Clifford group is a unitary 2-design (for qubits even a 3-design)

Representing quantum channels

Pauli transfer matrices (PTMs)

- Vectorization: turn $d \times d$ density matrices into a length-d^2 vector $|\rho\rangle\rangle = \sum_{P_i \in \mathcal{P}_n} \mathrm{Tr}\,[P_i \rho]\,|i\rangle\rangle$

➡ Channels are represented as $d^2 \times d^2$ matrices $(M_{\mathcal{E}})_{ij} = \langle\langle i|M_{\mathcal{E}}|j\rangle\rangle = \mathrm{Tr}\,[P_i \mathcal{E}(P_j)]$

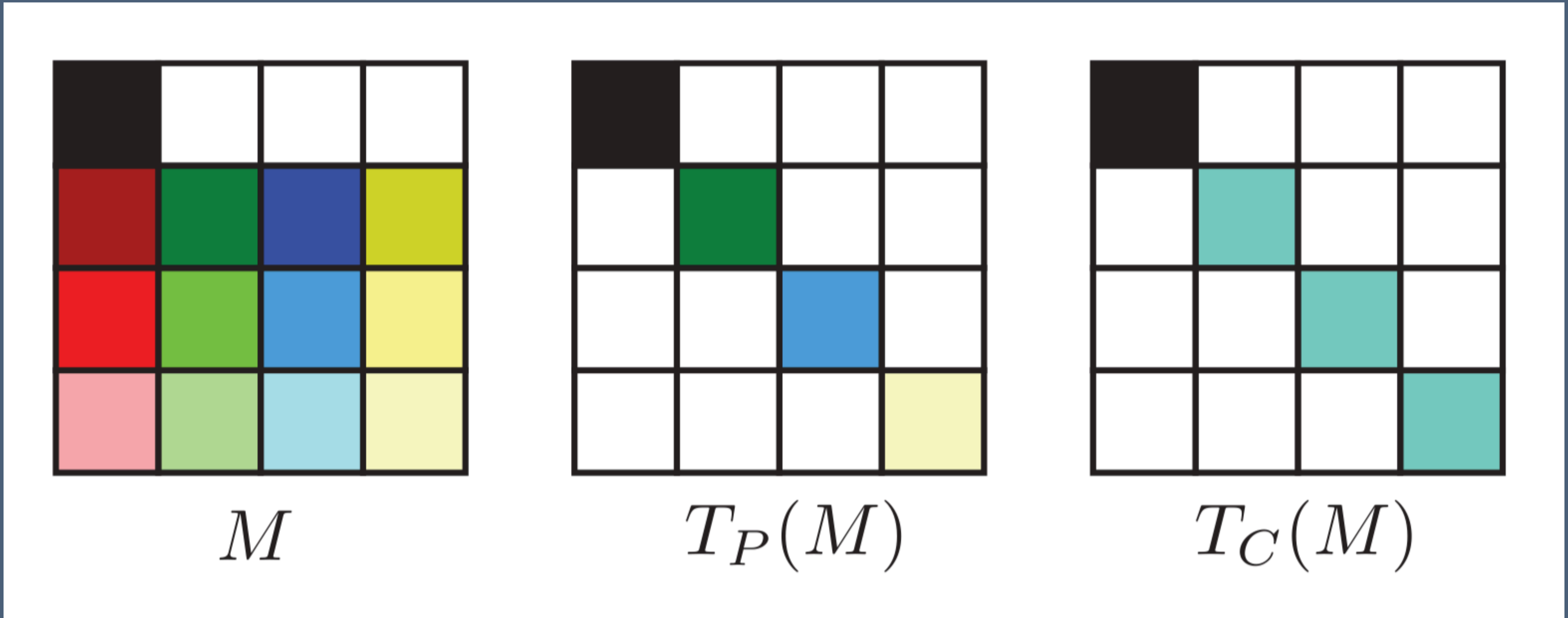


(a) PTM for Pauli X **(b)** PTM for Pauli Y **(c)** PTM for Pauli Z **(d)** PTM for $R_x(10^\circ)$

from Hashim et al 2024, *A practical introduction to benchmarking and characterization of quantum computers*

Clifford vs. Pauli twirling

PTM of a single-qubit channel | Pauli twirl | Clifford twirl

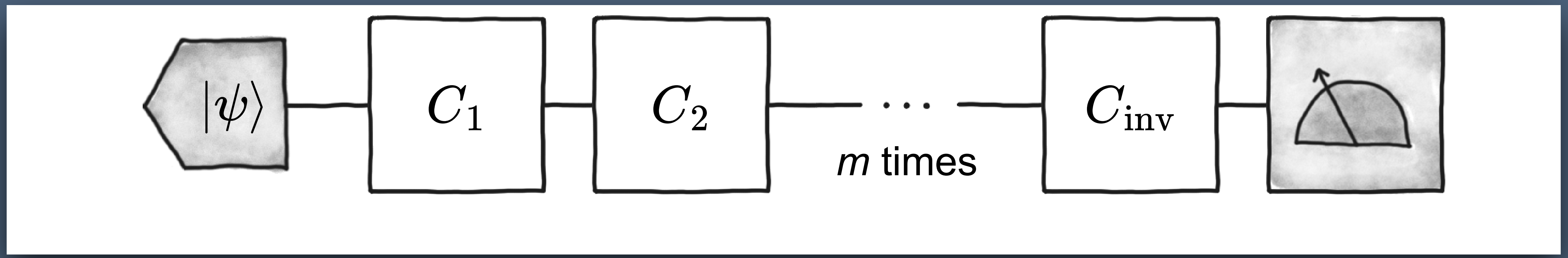


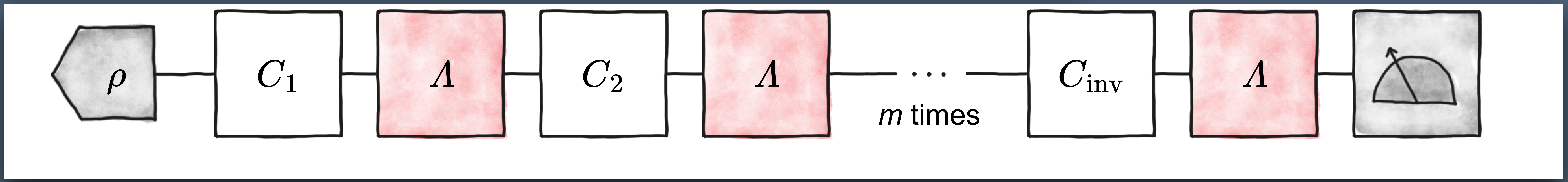
from Hashim et al 2024, *A practical introduction to benchmarking and characterization of quantum computers*

3. Twirling in action

Randomized benchmarking (RB)

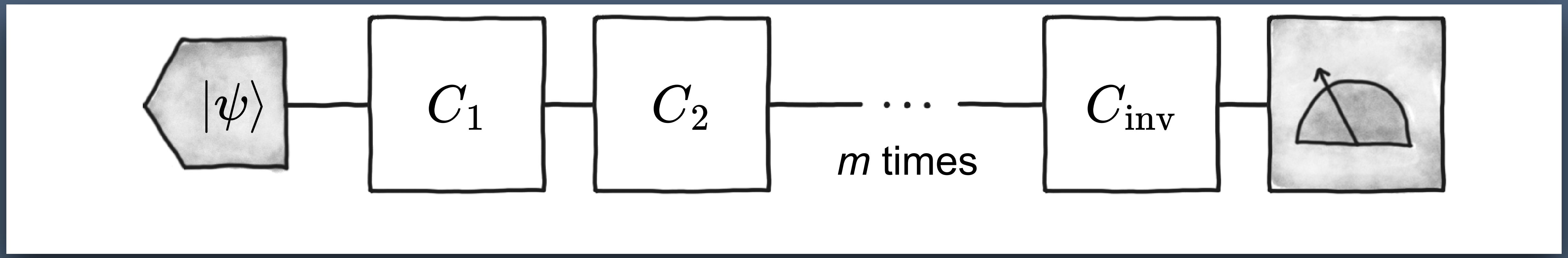
- **Goal**: Estimate a performance measure for quantum gate implementations
- RB solves two challenges:
  1. Efficiency
  2. SPAM (state preparation and measurement) error robustness
- How?





Drawings credit: https://github.com/wilkensJ/drawio-library

The standard RB protocol



1. Choose a sequence $(C_1, \ldots, C_m)$ of uniformly random Clifford gates.

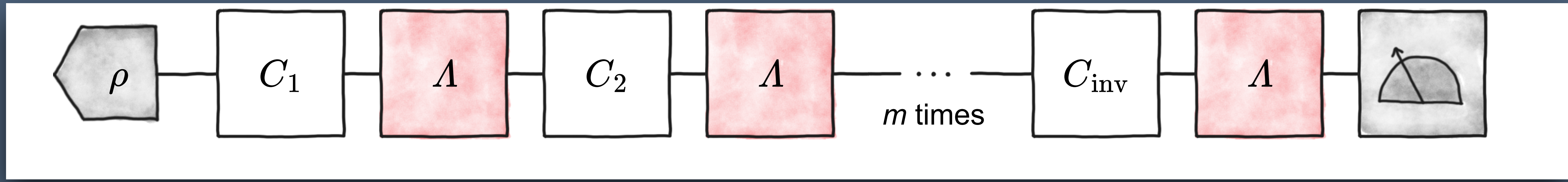
2. On the quantum computer, apply the sequence followed by its inverse C_{inv} to an initial state ρ_ψ, resulting in the output state ρ_{out}.

3. Estimate the survival probability $s_{m,\mathbf{C}} = \mathrm{Tr}\left[E_\psi \rho_{\text{out}}\right]$ by repeatedly performing step 2 and measuring the POVM element E_ψ.

4. Repeat steps 1—3 *N* times for independently drawn sequence and calculate the average.

5. Repeat steps 1—4 for different sequence lengths *m* and fit the resulting data to the exponential decay $s_m = Ap^m + B$.

Standard RB: analysis



- Total channel of the noisy random sequence:

$$\mathcal{S}(\rho) = \Lambda \circ \mathcal{C}_1^\dagger \circ \mathcal{C}_2^\dagger \cdots \mathcal{C}_m^\dagger \circ \Lambda \circ \mathcal{C}_m \circ \Lambda \circ \mathcal{C}_{m-1} \cdots \Lambda \circ \mathcal{C}_1(\rho)$$

- Evaluate the twirl:

$$\frac{1}{|\,\mathrm{Cl}(d)|} \sum_{C_m \in \mathrm{Cl}(d)} \mathcal{C}_m^\dagger \circ \Lambda \circ \mathcal{C}_m(\rho) = \int_{U(d)} \mathcal{U}^\dagger \circ \Lambda \circ \mathcal{U}(\rho)\, \mathrm{d}\mu_{\mathrm{Haar}}(U)$$

$$= \mathcal{D}_p(\rho) = p\rho + (1-p)\frac{\mathbb{I}}{d}$$

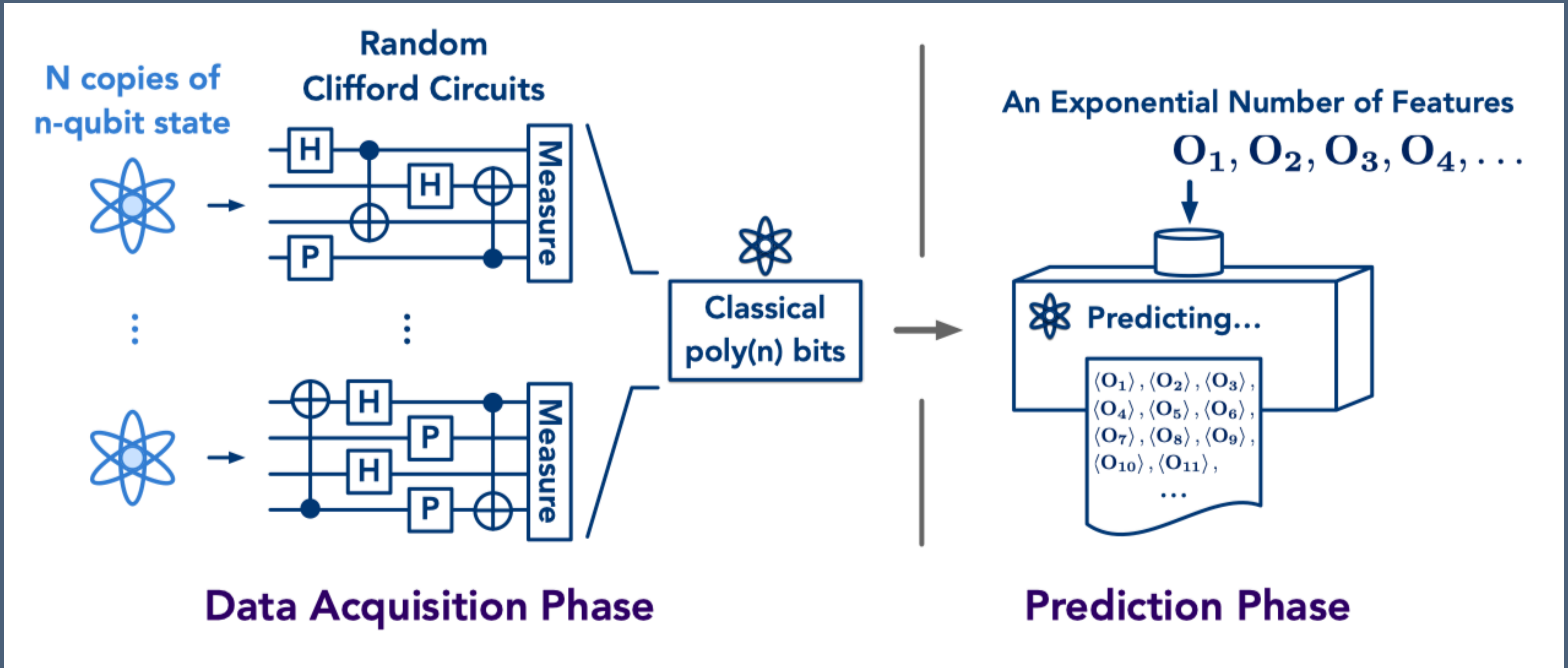
- The full channel is proportional to the power of a depolarizing channel, with a decay parameter p that directly relates to the average gate fidelity:

$$F_{\mathrm{avg}}(\Lambda) = F_{\mathrm{avg}}(\mathcal{D}_p) = p + \frac{1-p}{d}$$

Classical shadows

- **Goal:** Estimate expectation values of an unknown quantum state

… but efficiently!



from Huang and Kueng 2019, *Predicting Features of Quantum Systems from Very Few Measurements*

Classical shadows: Global Clifford protocol

Quantum part:

1. Apply a random Clifford to the state: $\rho \mapsto C \rho C^\dagger$
2. Perform a computational-basis measurement, resulting in $|\hat{x}\rangle \in \{0,1\}^n$

Classical part:

1. Apply the inverse of the Clifford in classical memory: $C^\dagger |\hat{x}\rangle\langle\hat{x}| C$
2. Calculate the classical snapshot $\hat{\rho} = \mathcal{M}^{-1}\left(C^\dagger |\hat{x}\rangle\langle\hat{x}| C\right)$

➡ Repeat *N* times, estimate functions of ρ via averages (median-of-means) over the snapshots.

Classical shadows: analysis

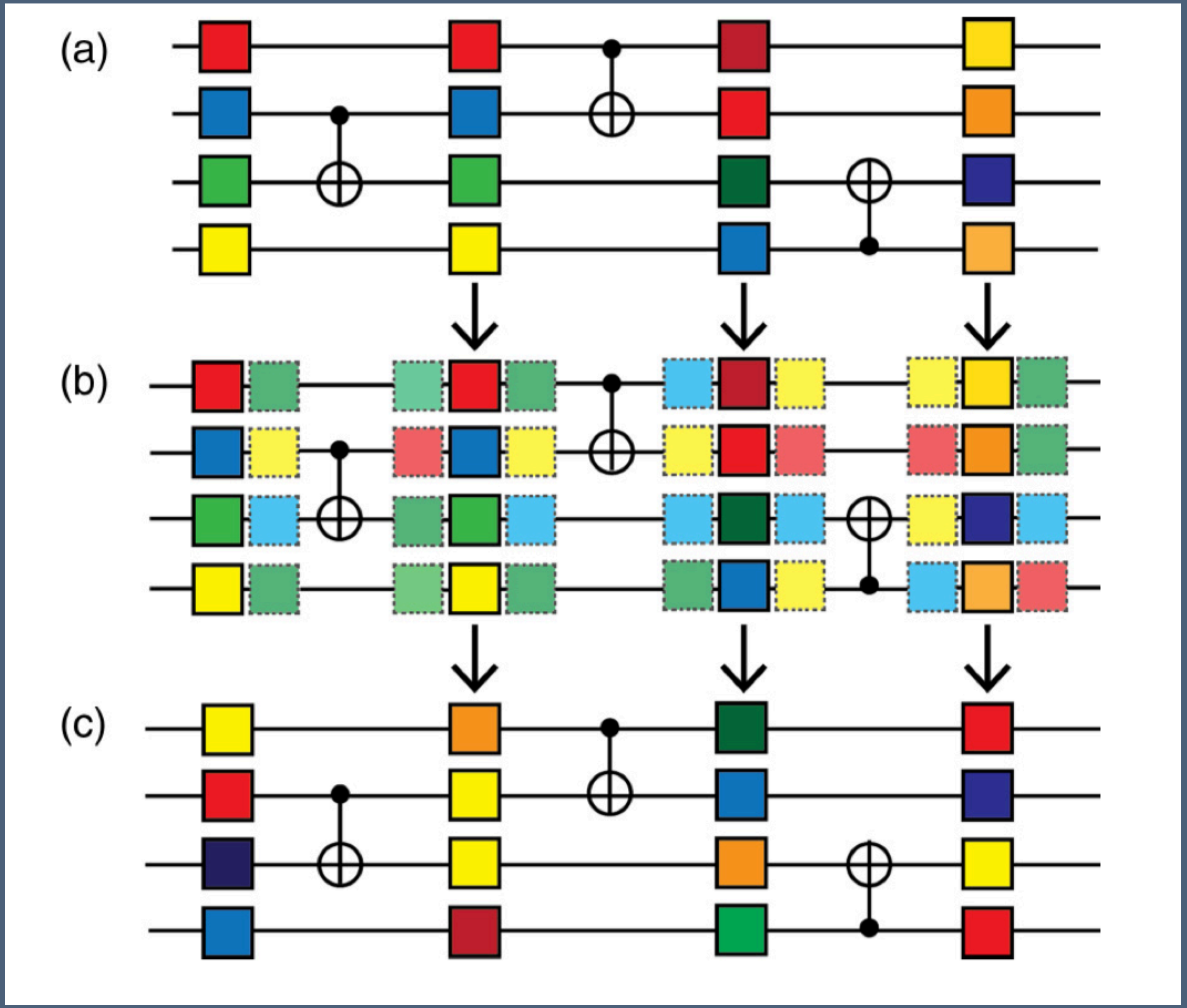
- Goal: estimate many expectation values $\langle\!\langle O|\rho\rangle\!\rangle$
- Insert a prepare-and-measure channel $\sum_x |A_x\rangle\!\rangle\langle\!\langle E_x| = \mathbb{I}$: $\langle\!\langle O|\rho\rangle\!\rangle = \sum_x \langle\!\langle O|A_x\rangle\!\rangle\langle\!\langle E_x|\rho\rangle\!\rangle$
  - Here: computational-basis measurement channel $\mathcal{M}_Z = \sum_{z\in\{0,1\}^n} |z\rangle\!\rangle\langle\!\langle z|$
  - Add the random unitary and its inverse:

$$\begin{aligned}\langle\!\langle O|\rho\rangle\!\rangle &= \langle\!\langle O|\mathcal{M}^{-1}\mathcal{M}(\rho)\rangle\!\rangle \\ &= \mathbb{E}_{U\in G} \sum_{z\in\{0,1\}^n} \langle\!\langle O|\mathcal{M}^{-1}\mathcal{U}^\dagger|z\rangle\!\rangle\langle\!\langle z|\mathcal{U}|\rho\rangle\!\rangle\end{aligned}$$

➡ $\mathcal{M}$ is just the twirl of the measurement channel $\mathcal{M}_Z$! Easy to calculate and invert

Randomized compiling

- **Goal**: Tailor noise to a specific form



Write circuit as sequence of “easy” and “hard gates”

Sandwich the easy gates between randomly drawn Paulis

Compile the Paulis into “dressed” easy gates

from Wallman and Emerson 2016, *Noise tailoring for scalable quantum computation via randomized compiling*

Outlook & Questions

- Many open questions, practically relevant challenges
- Literature recommendations to learn more:
  - Eisert et al., Quantum certification and benchmarking, Nat Rev Phys **2**, 382 (2020)
  - Hashim et al., A Practical Introduction to Benchmarking and Characterization of Quantum Computers, arXiv:2408.12064
  - Silva and Greplova, Hands-on Introduction to Randomized Benchmarking, arXiv:2410.08683
  - Kliesch and Roth, Theory of quantum system certification: a tutorial, PRX Quantum **2**, 010201 (2021)

Thank you for your attention!

References

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