Causal Graph Rewriting

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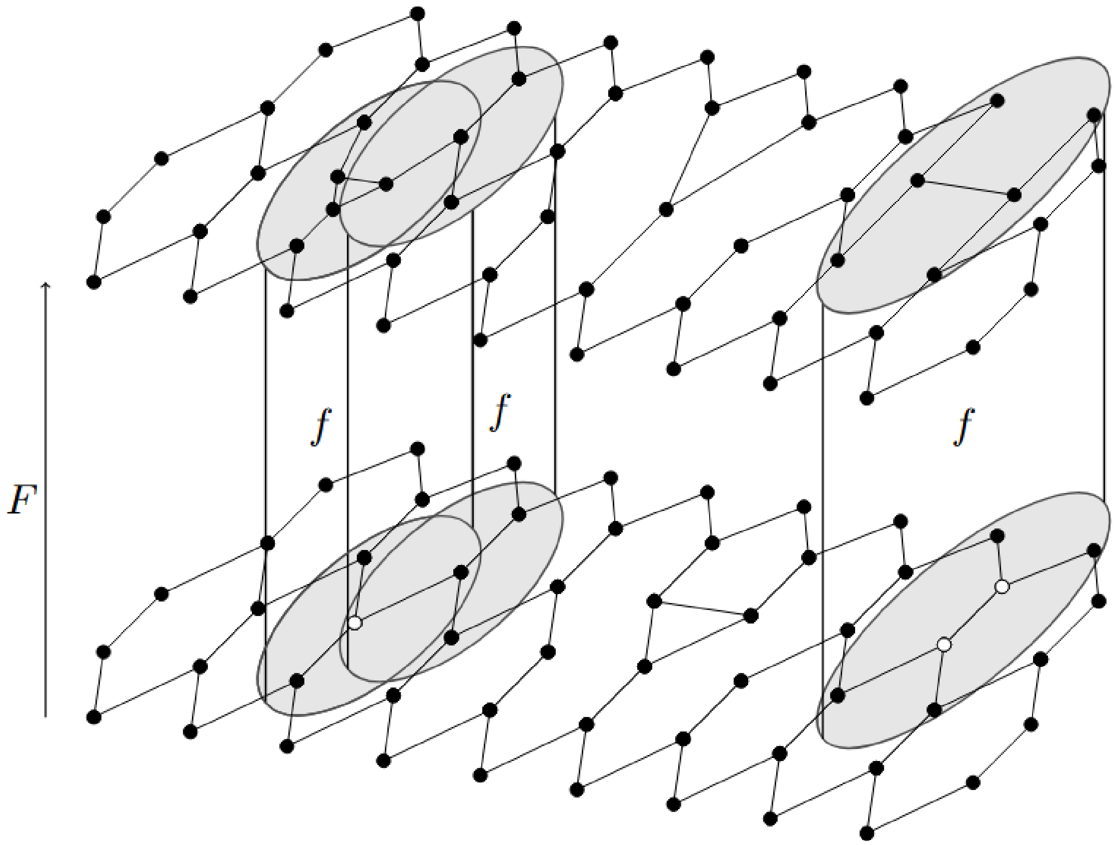
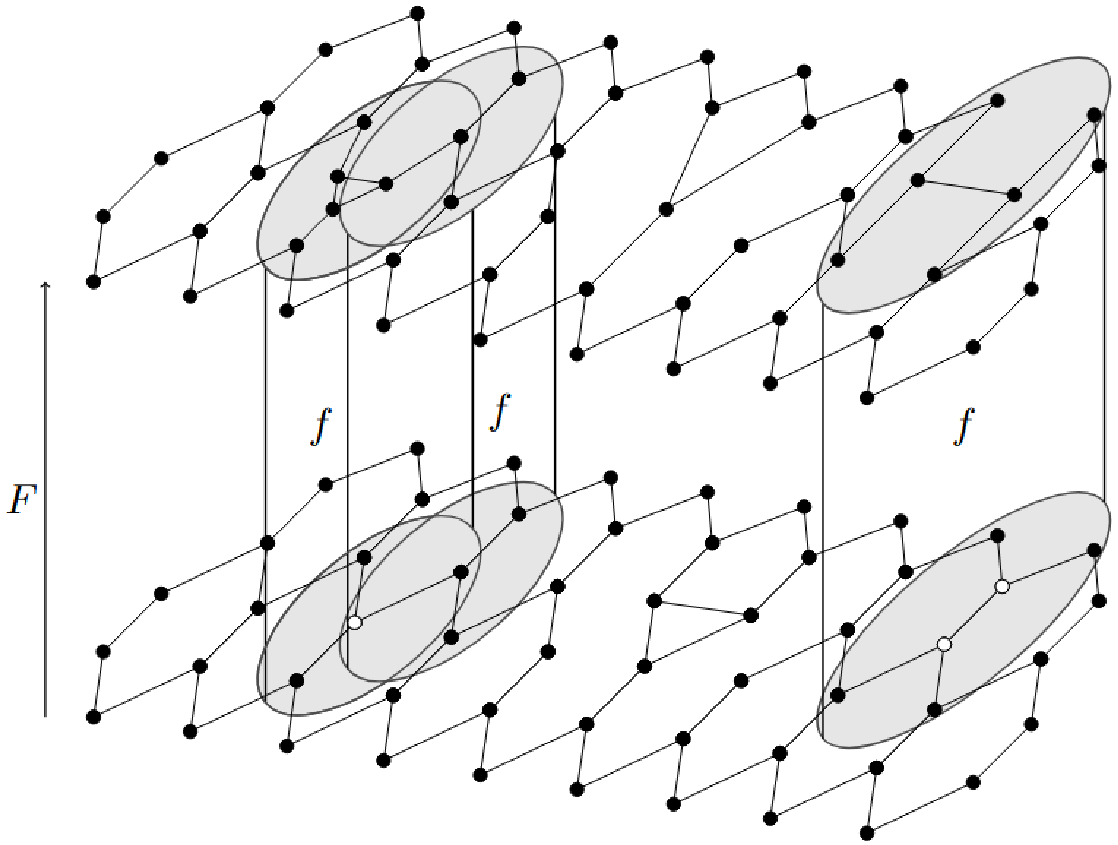
LMF Laboratoire Méthodes Formelles

QuaCS

Introduction



Introduction



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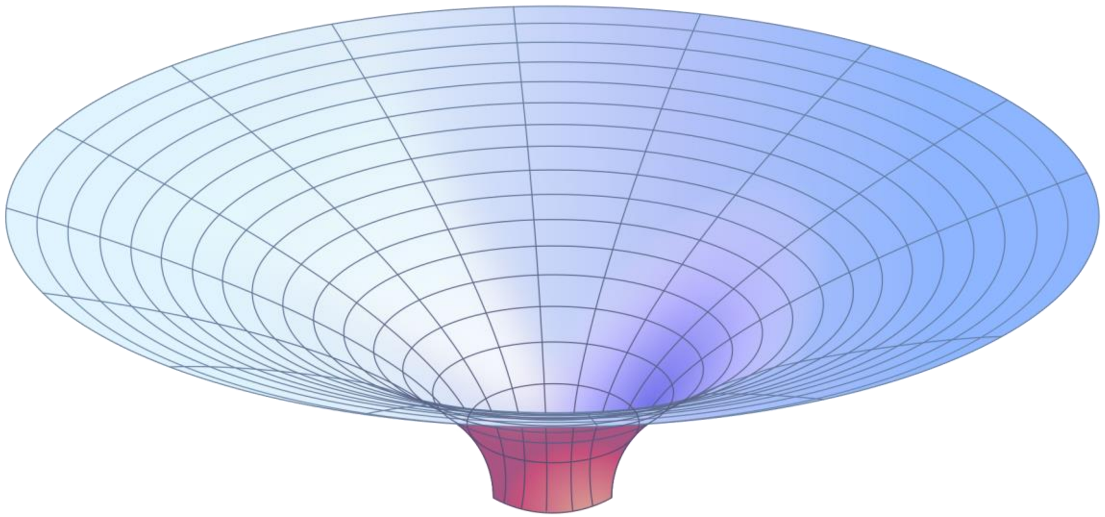
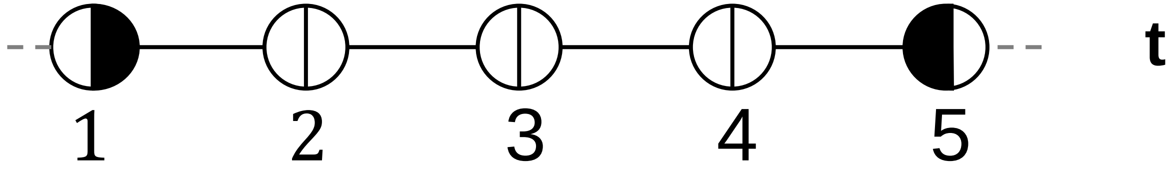


Dynamical systems and rewriting

Dynamical system (Synchronous)

Dynamical systems and rewriting

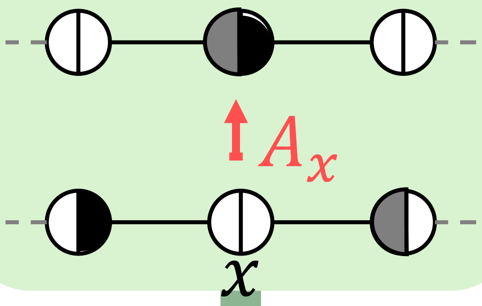
Dynamical system
(Synchronous)

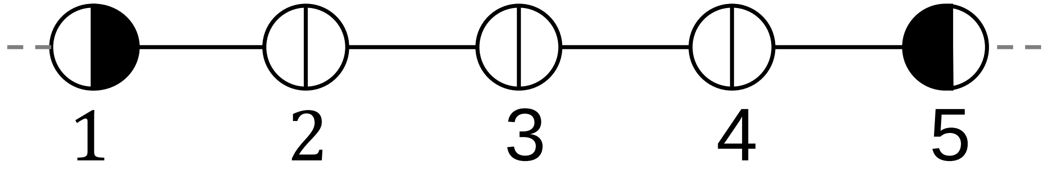


Dynamical systems and rewriting

Dynamical system
(Synchronous)

Local rule





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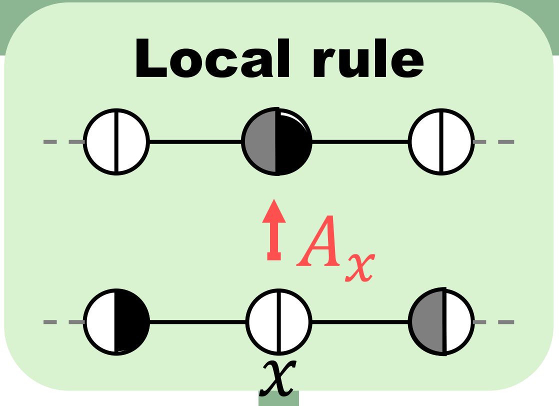

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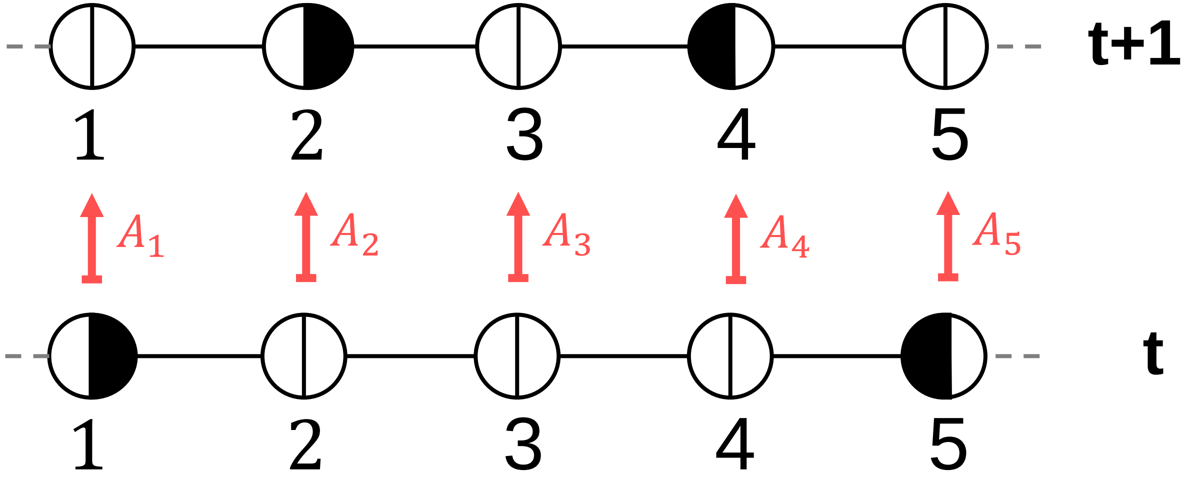

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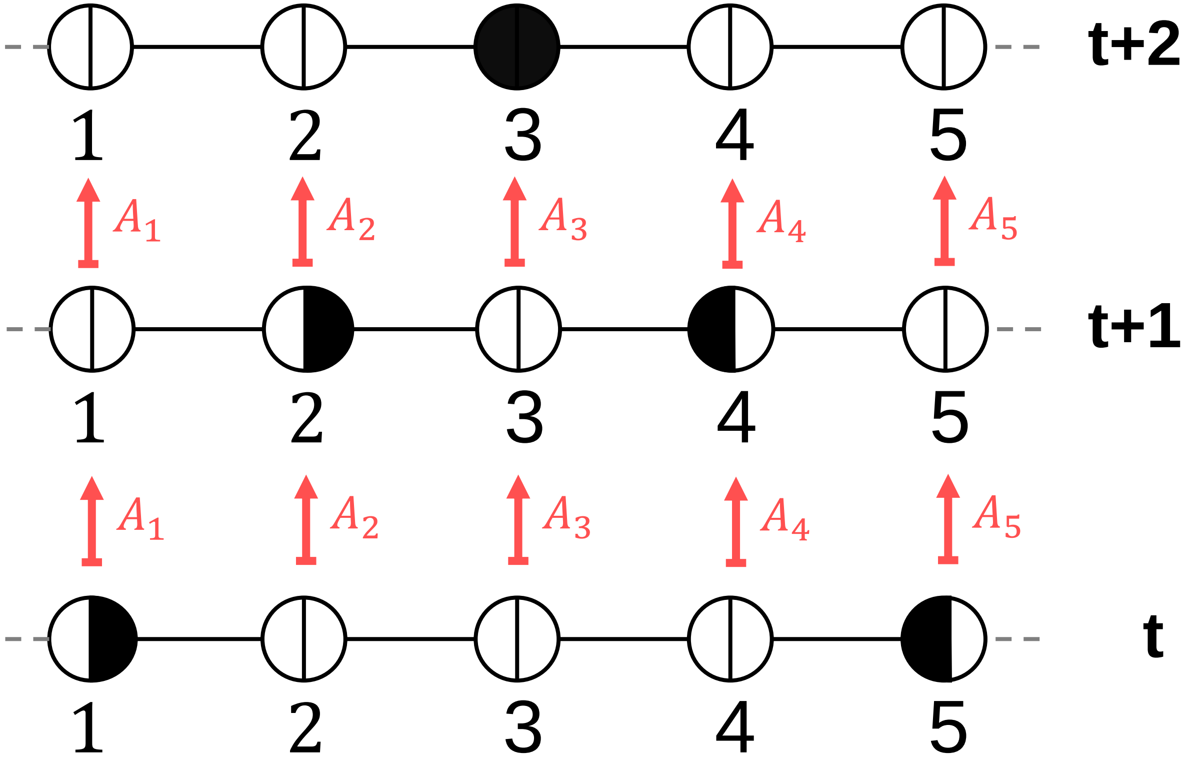

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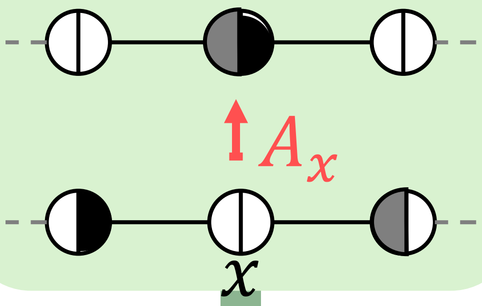

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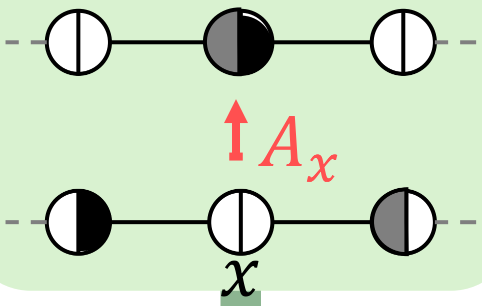

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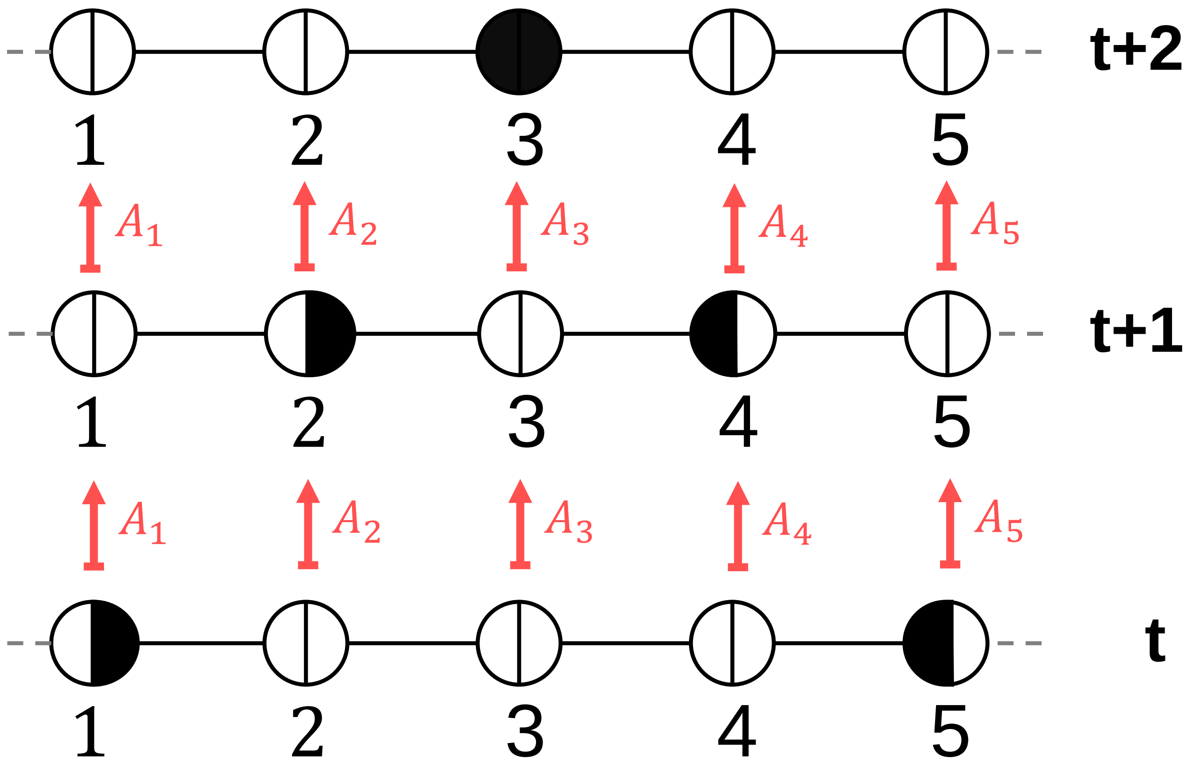
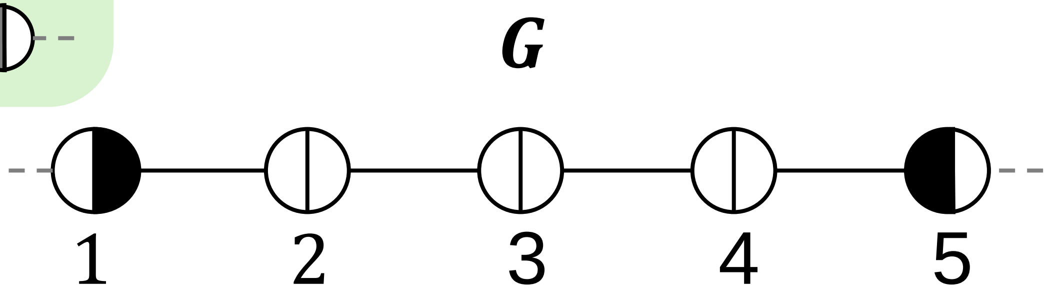


Naive asynchronous counterpart



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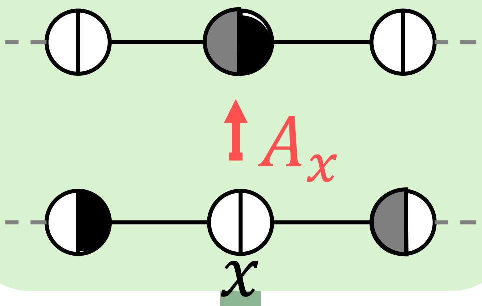

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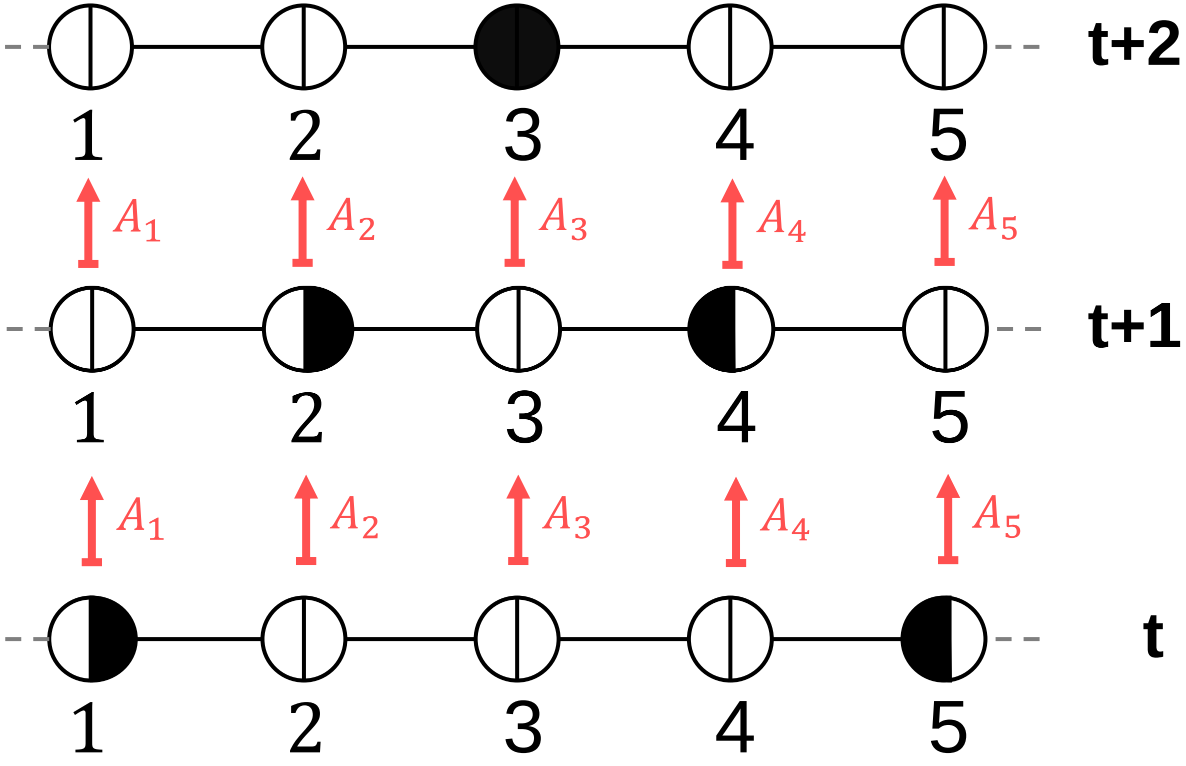
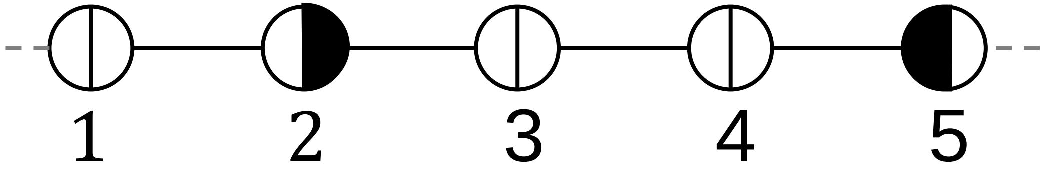


Naive asynchronous counterpart

$A_2 G$



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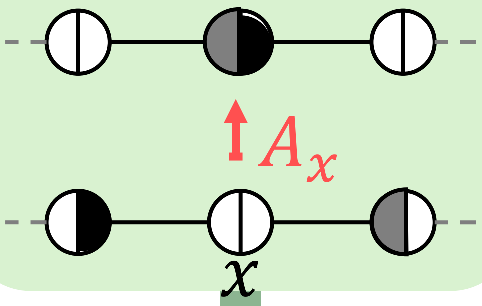

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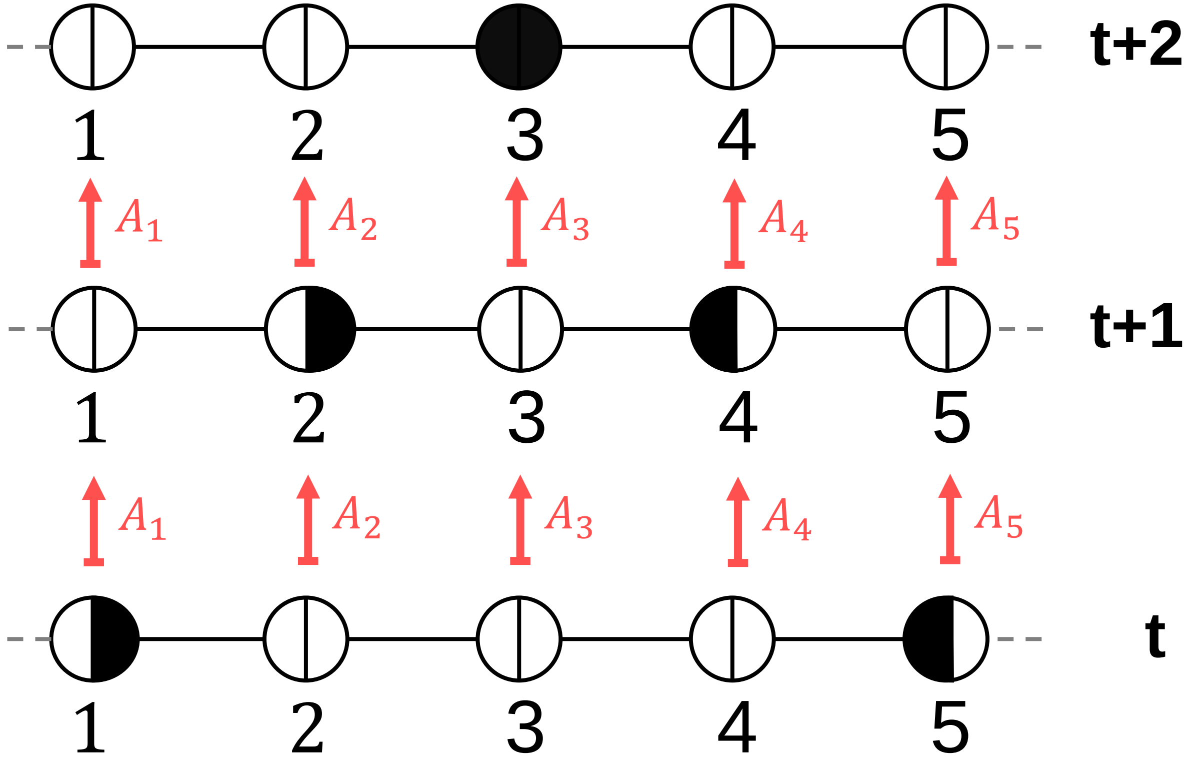

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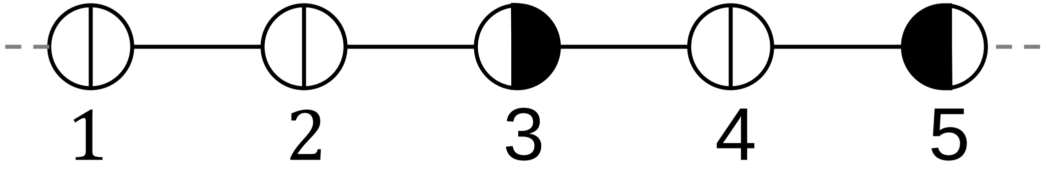
$A_3 A_2 G$



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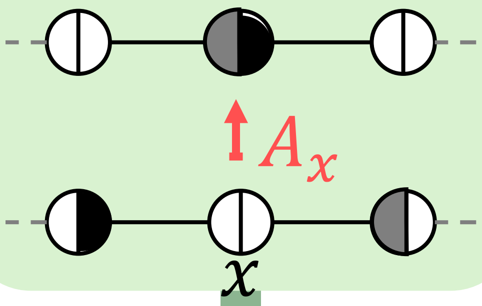

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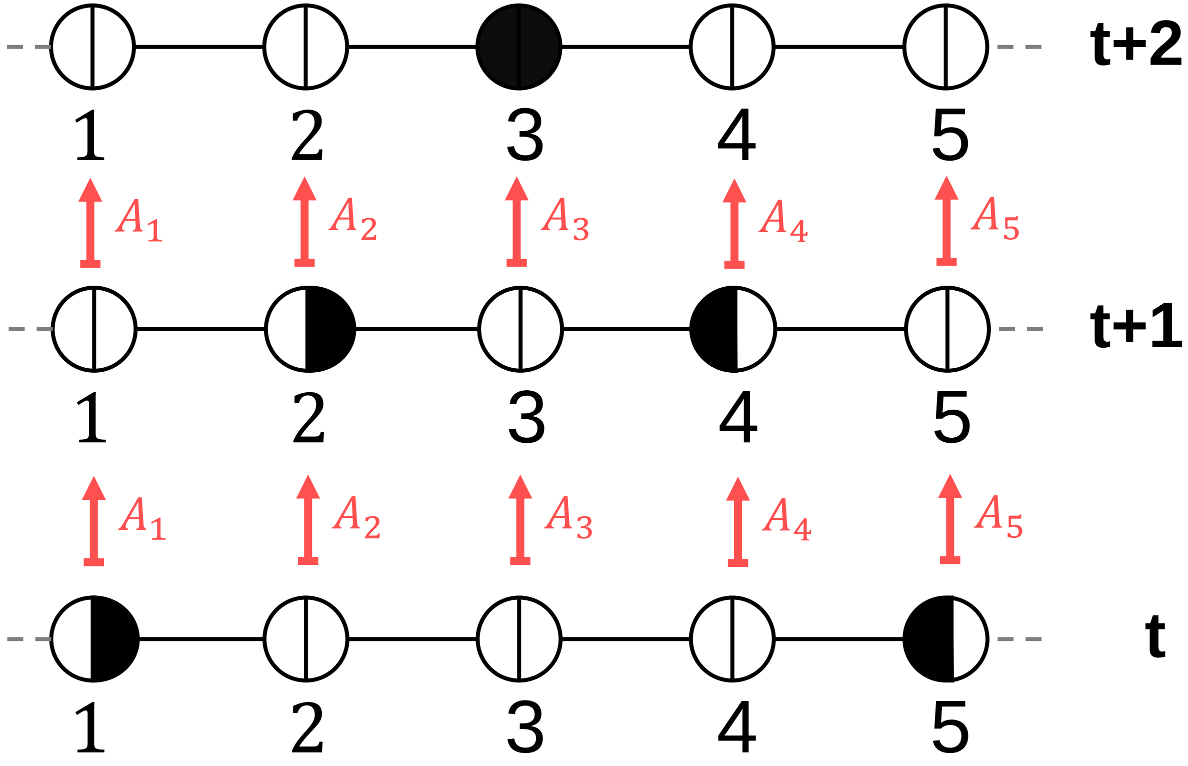

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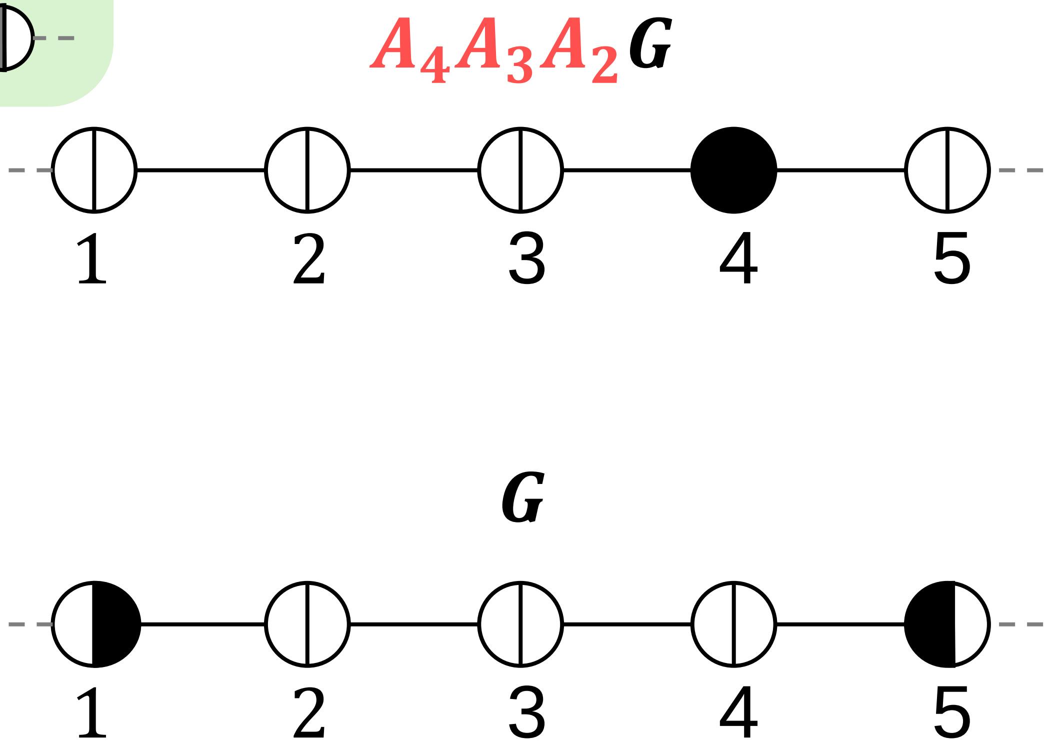

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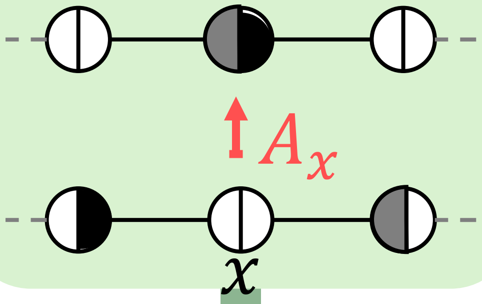

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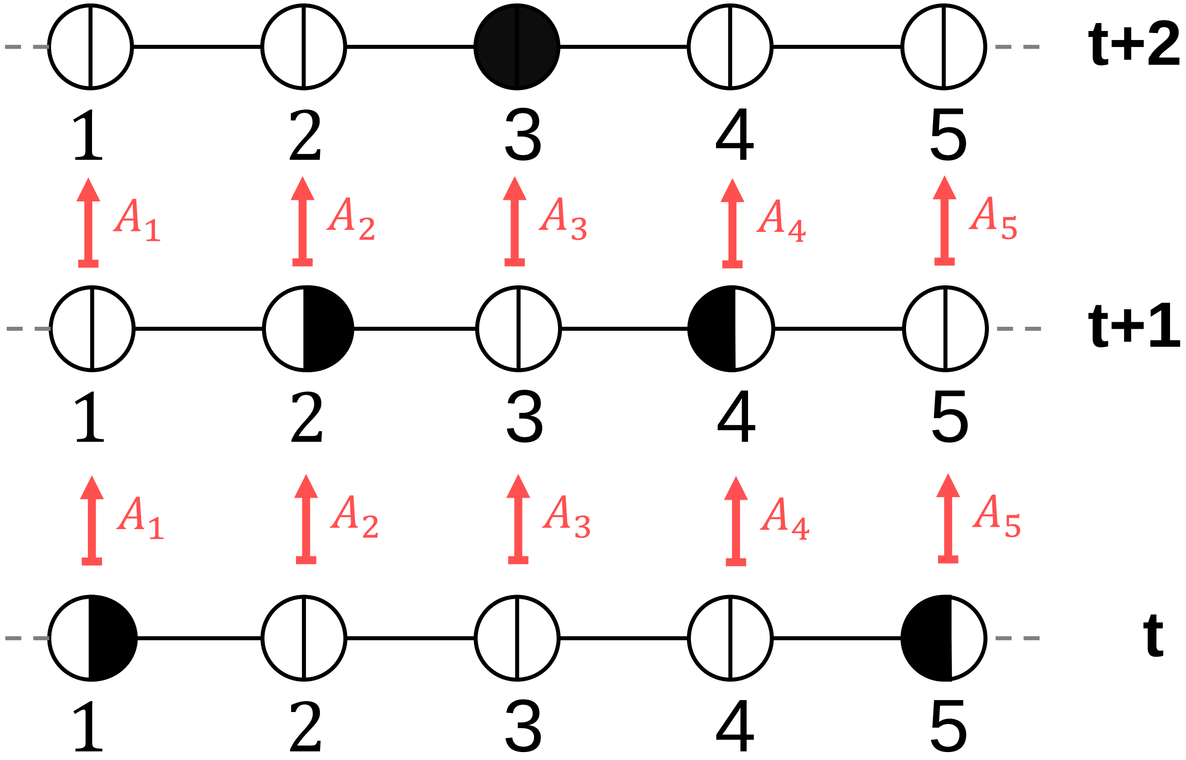
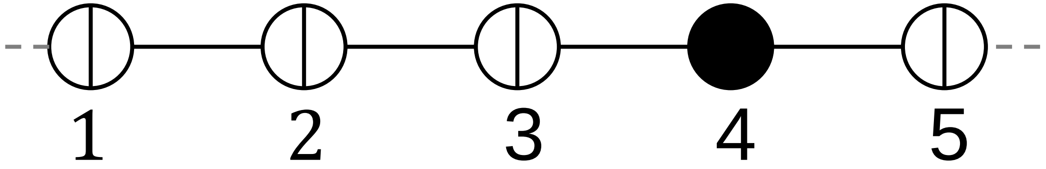
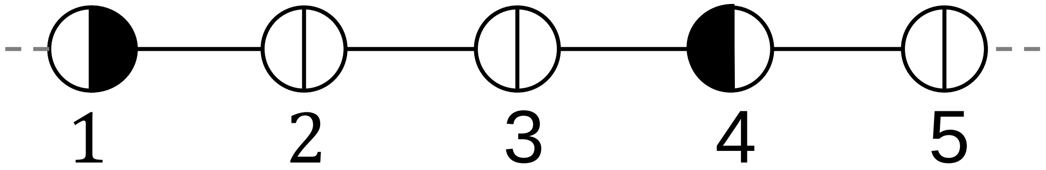


Naive asynchronous counterpart

$A_4 A_3 A_2 G$



$A_4 G$



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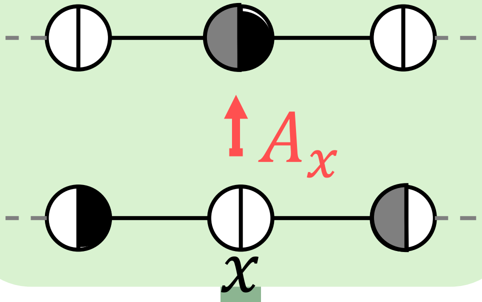

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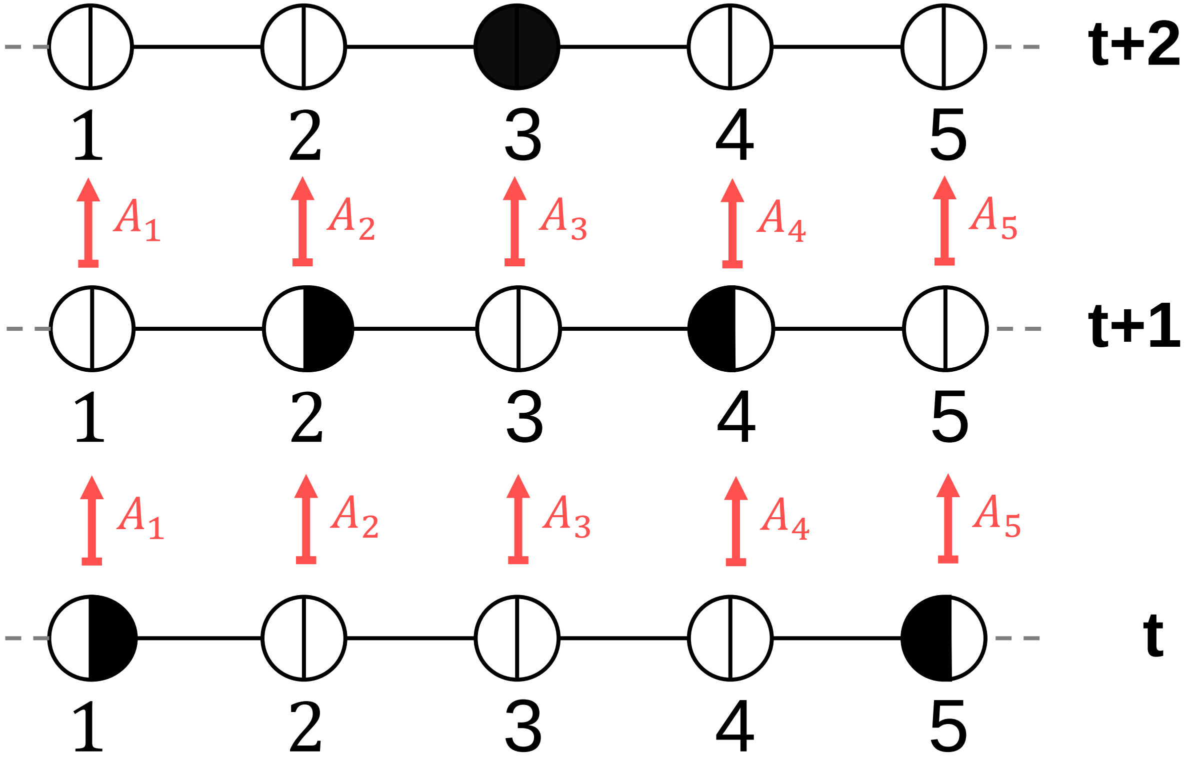

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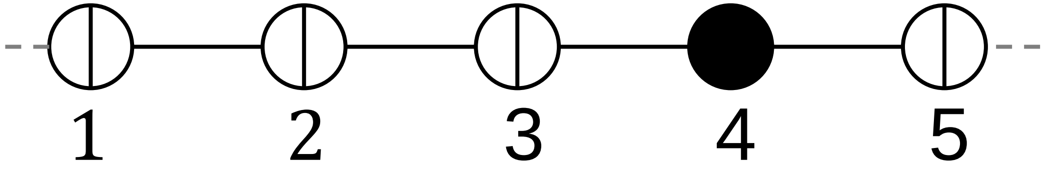

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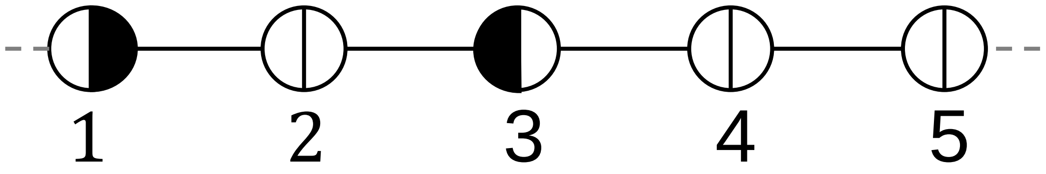
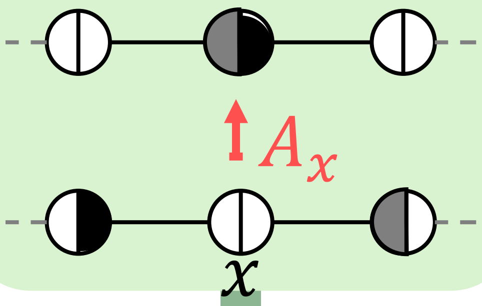


Dynamical systems and rewriting

Local rule



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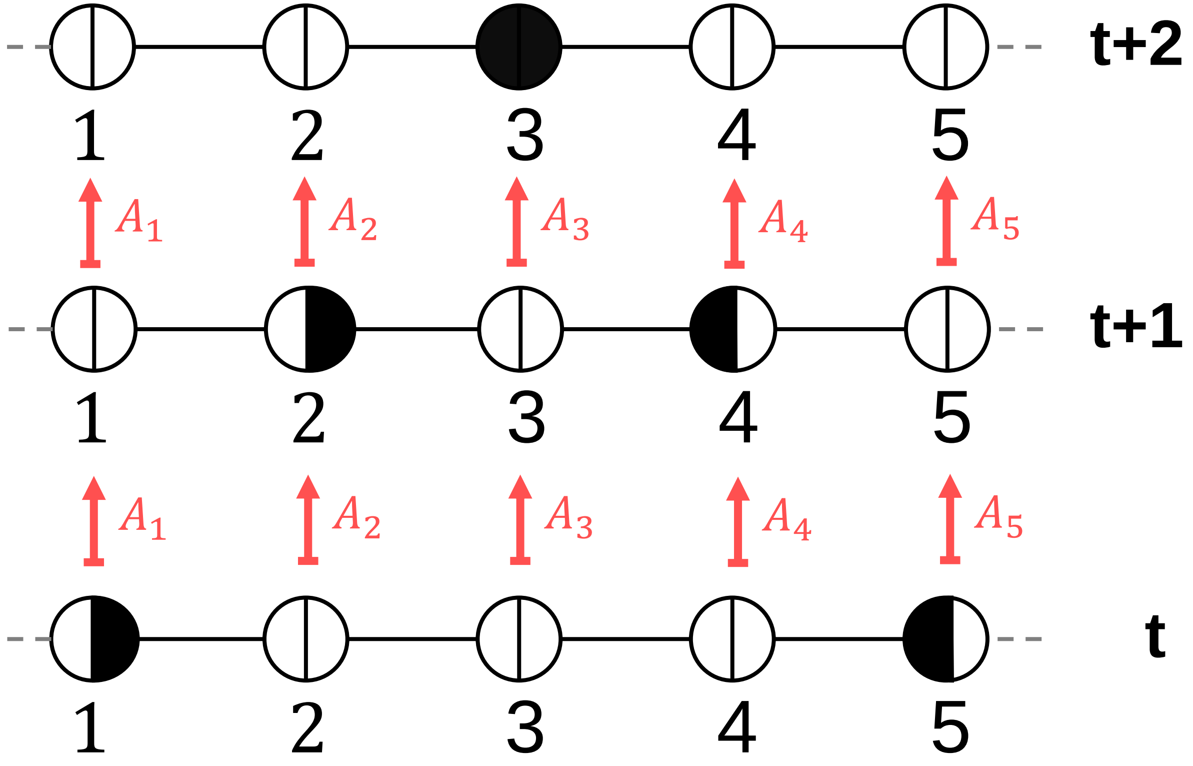

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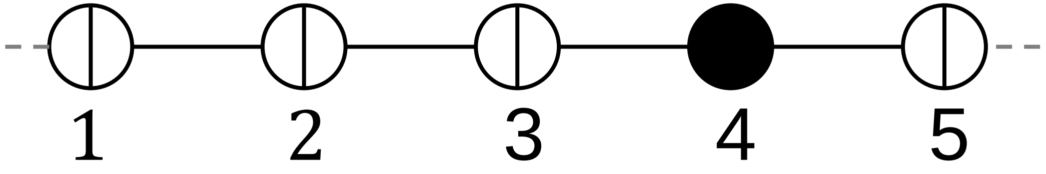
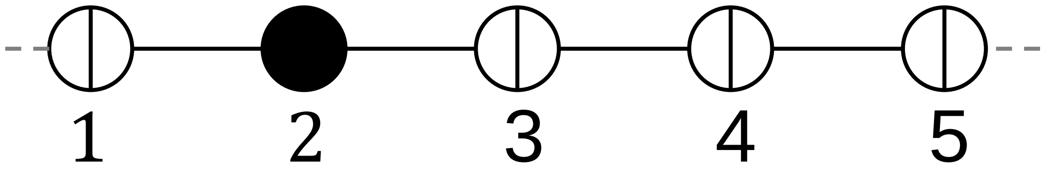


$A_2 A_3 A_4 G$



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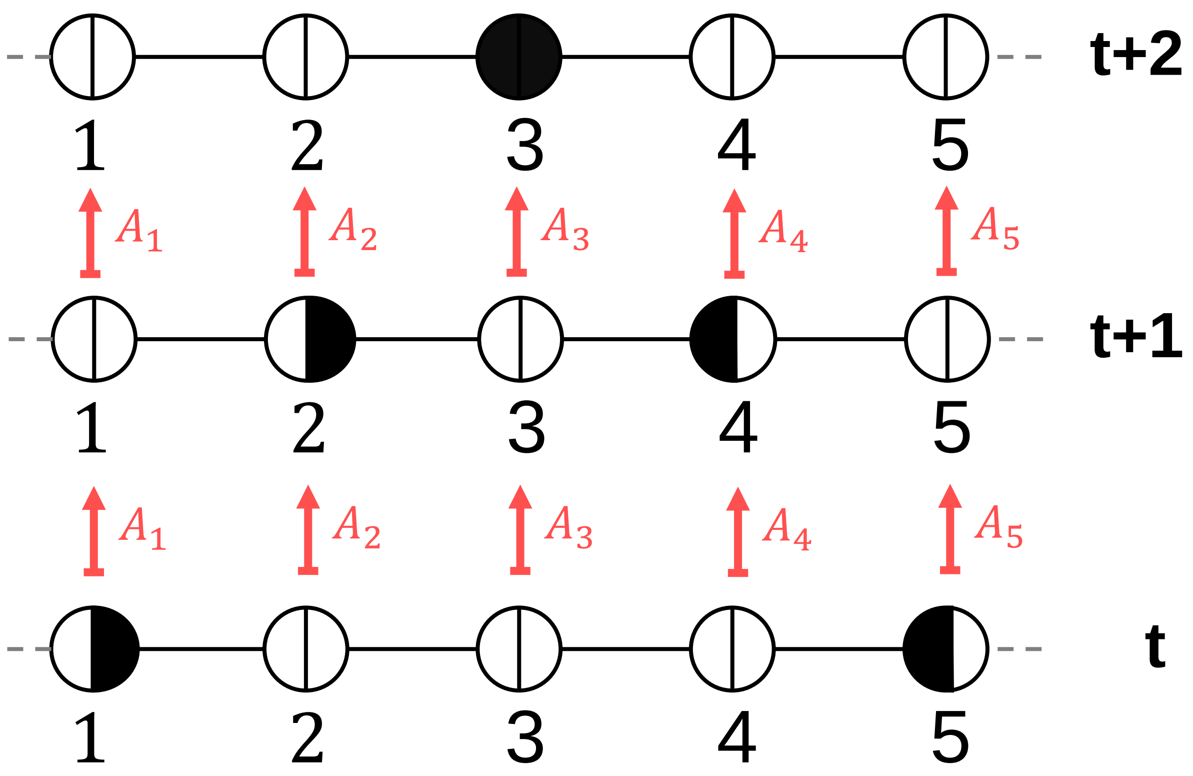
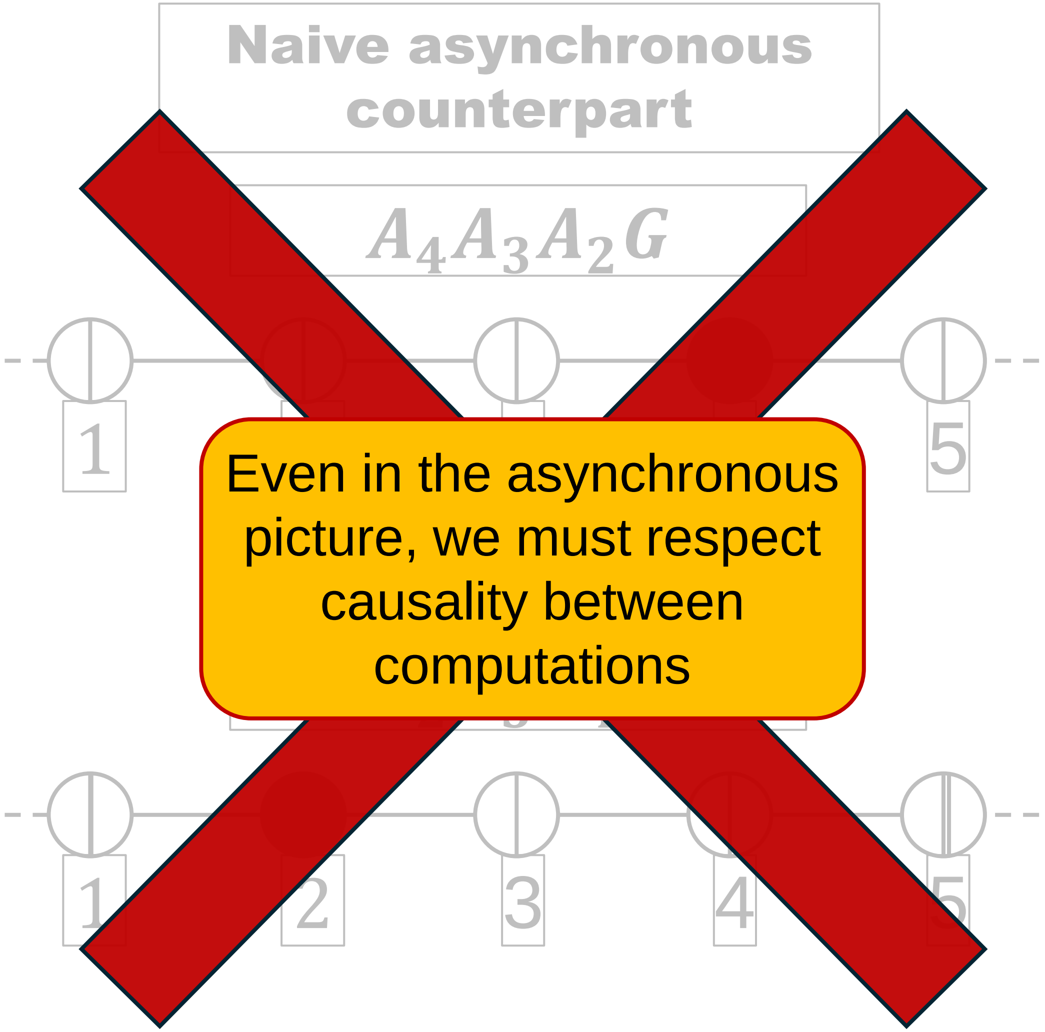
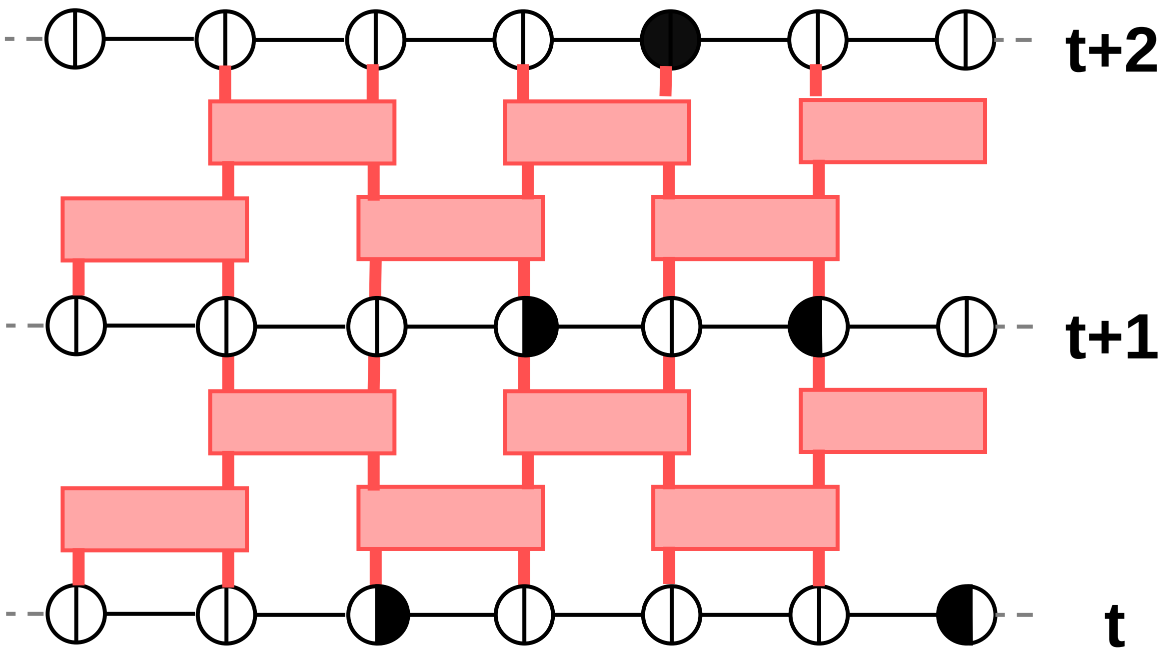


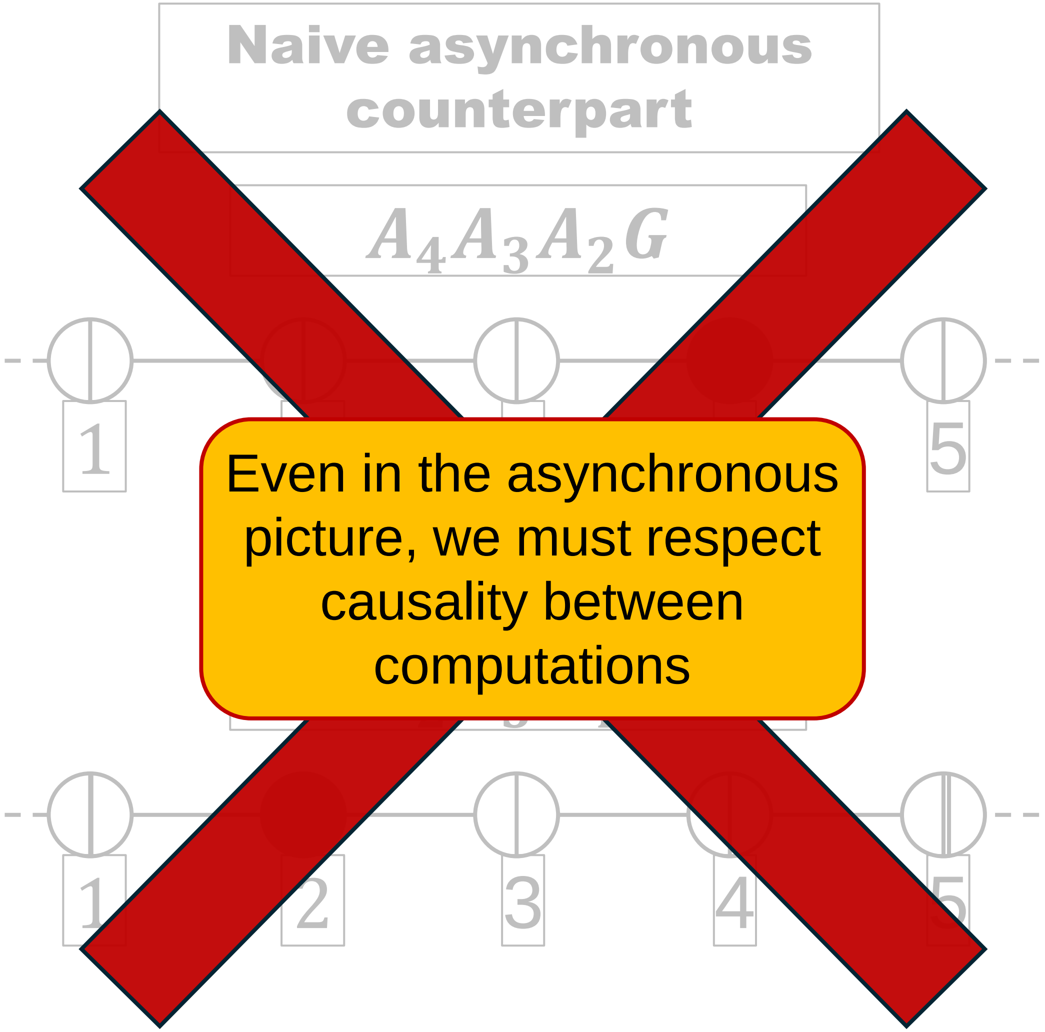


Even in the asynchronous picture, we must respect causality between computations

Dynamical systems and rewriting

Causal structure





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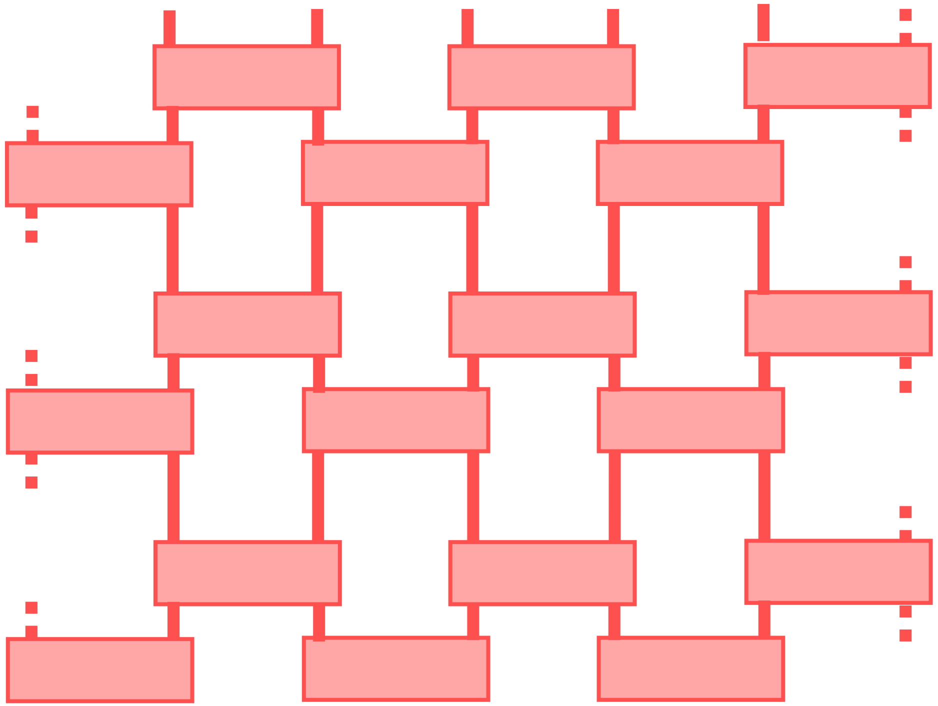
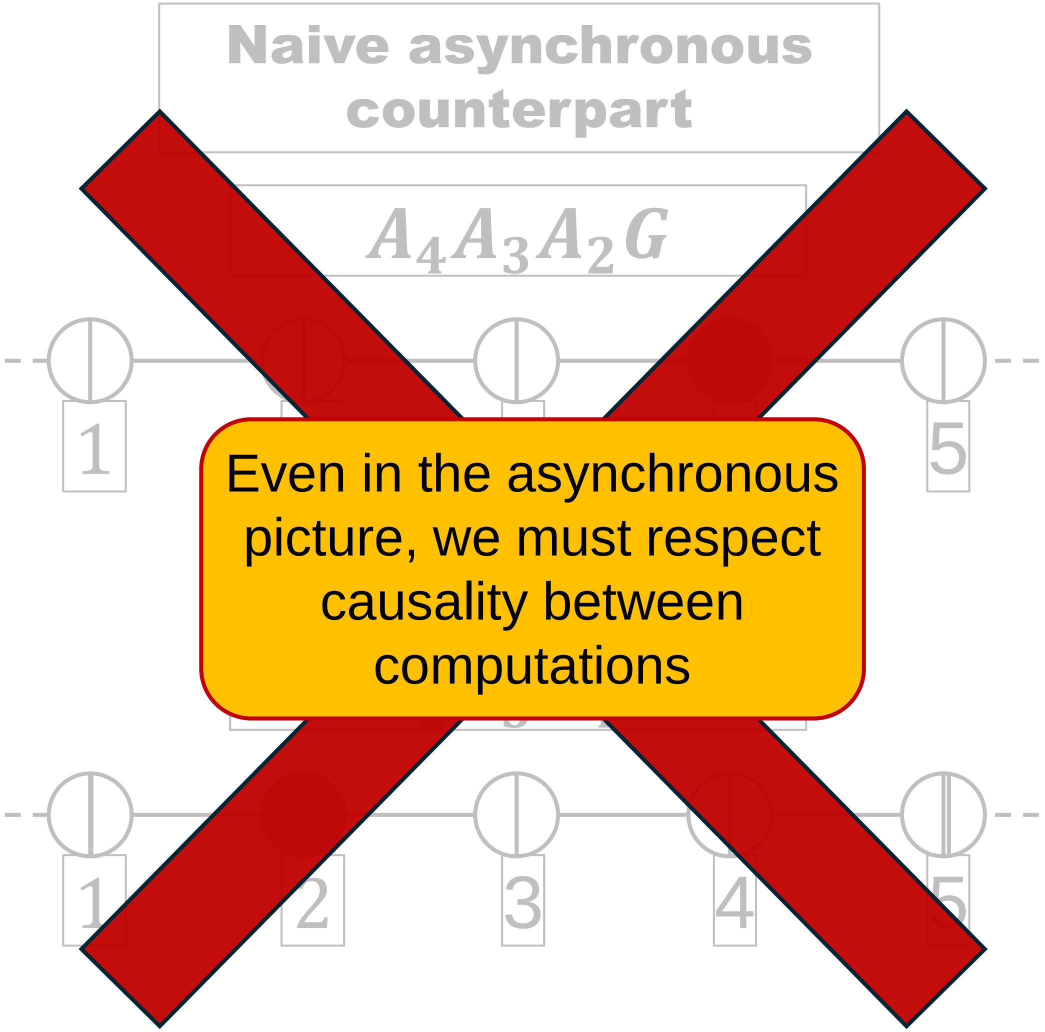
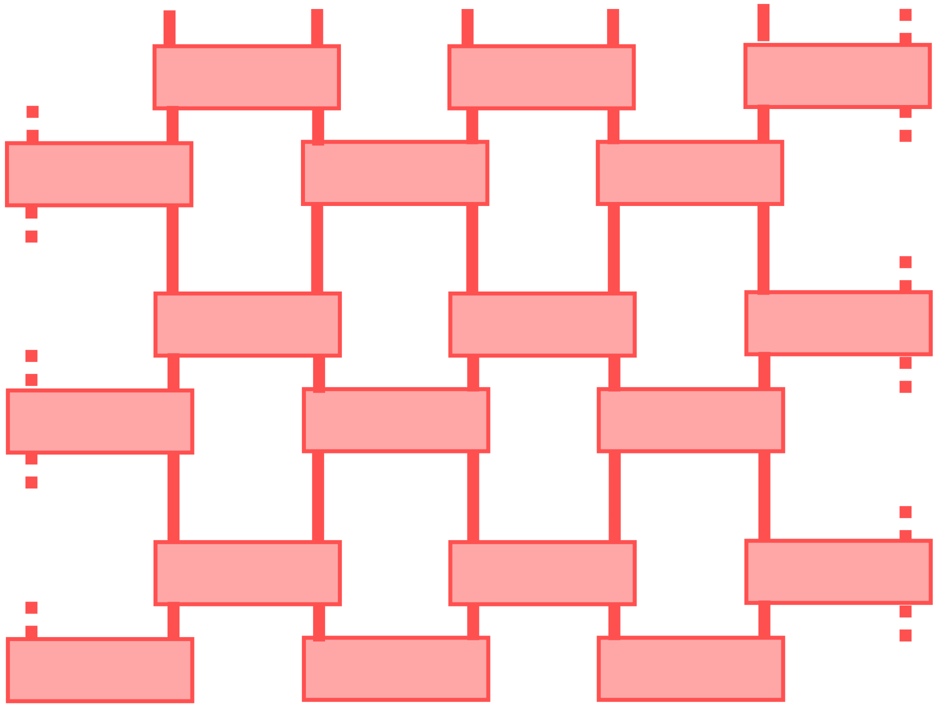
Causal structure



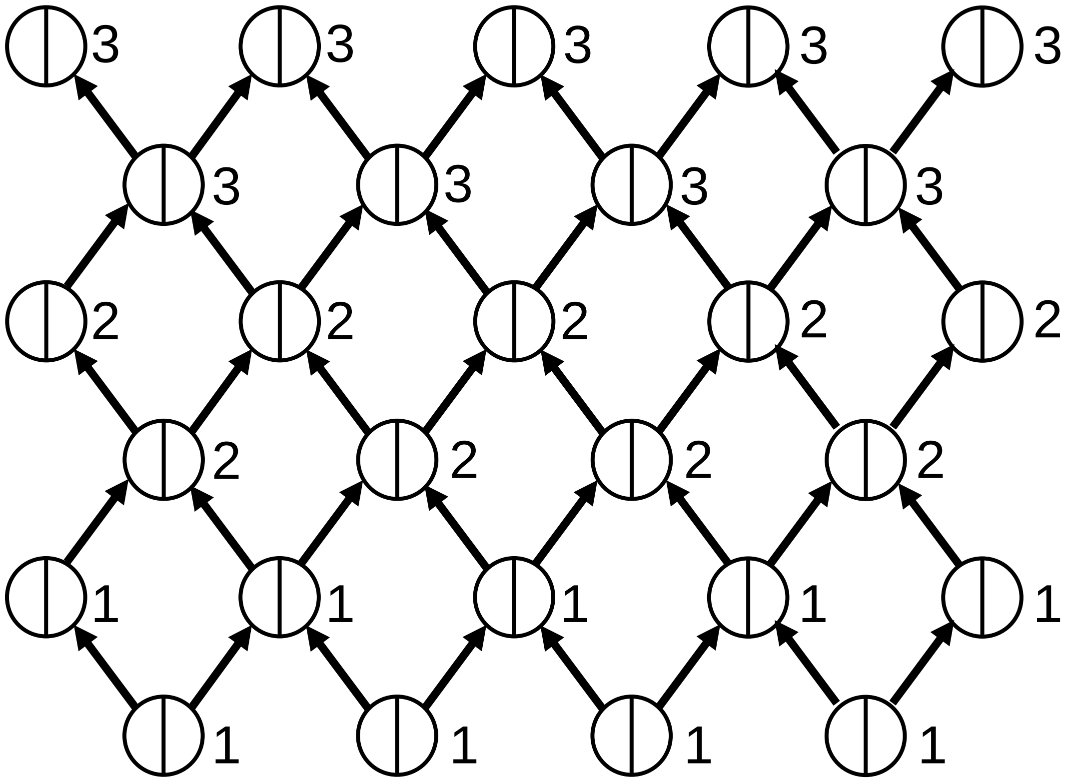


Dynamical systems and rewriting

Causal structure

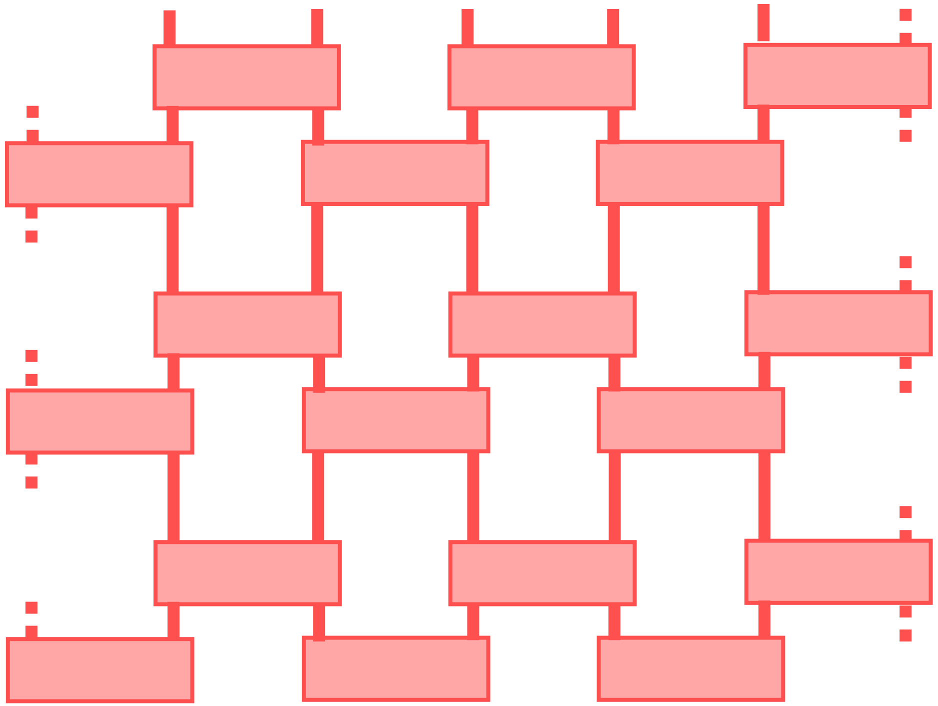


Causal graph rewriting

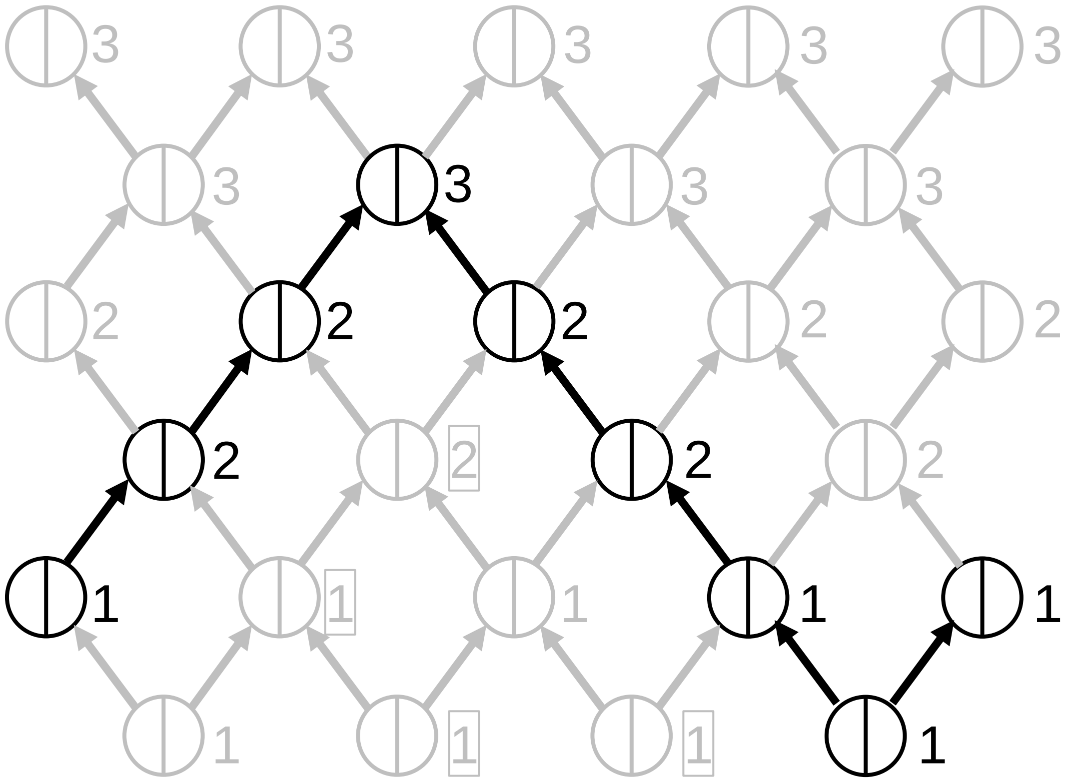


Dynamical systems and rewriting

Causal structure



Causal graph rewriting



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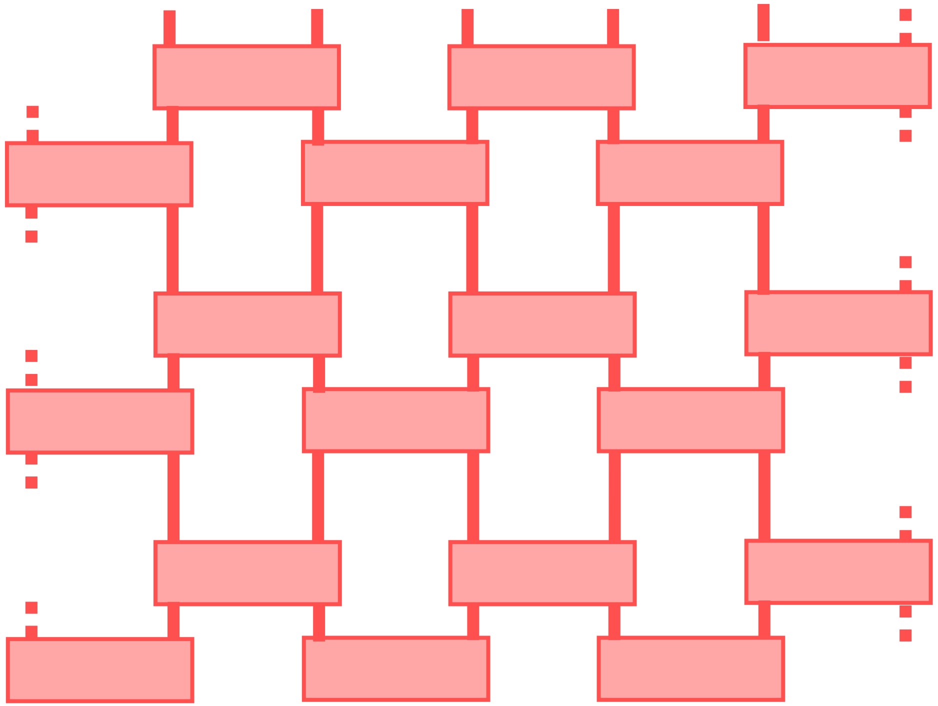

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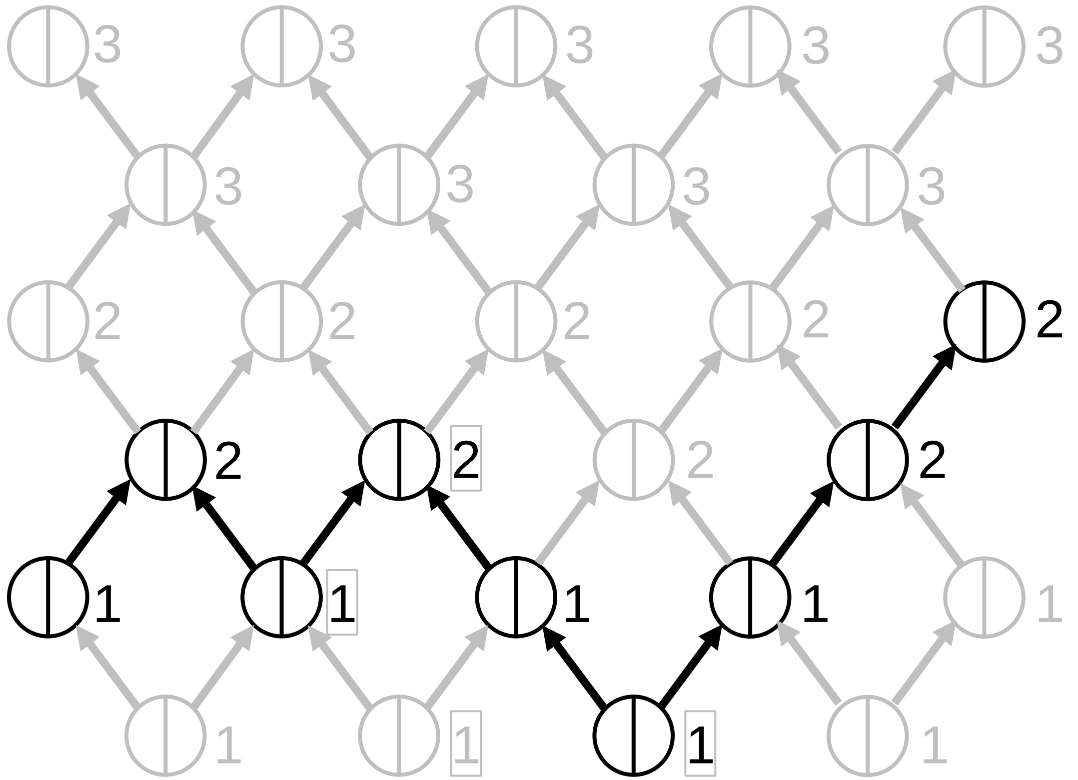

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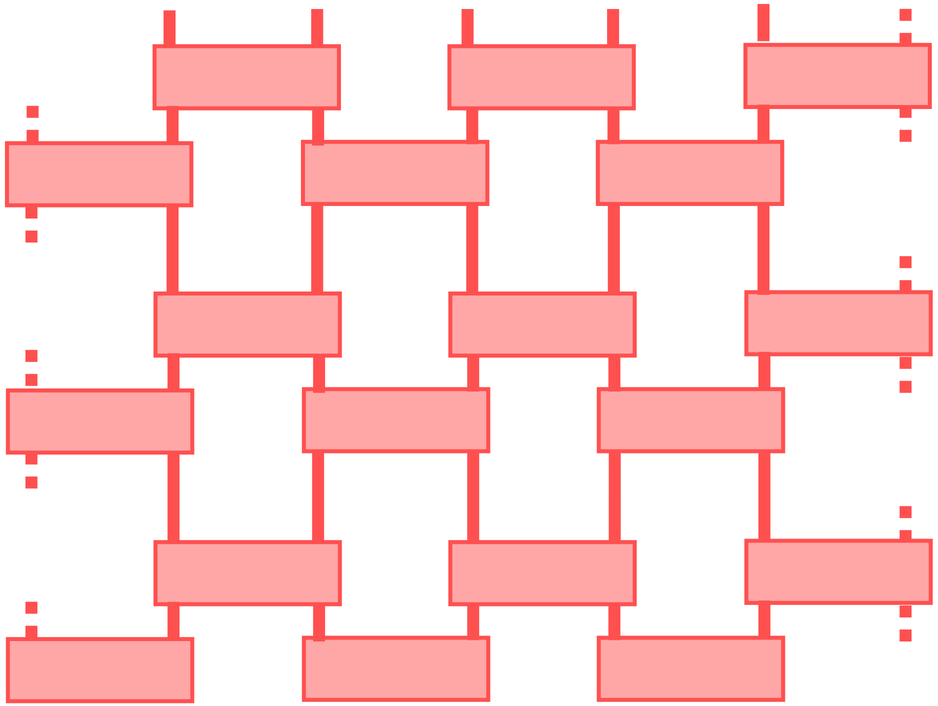

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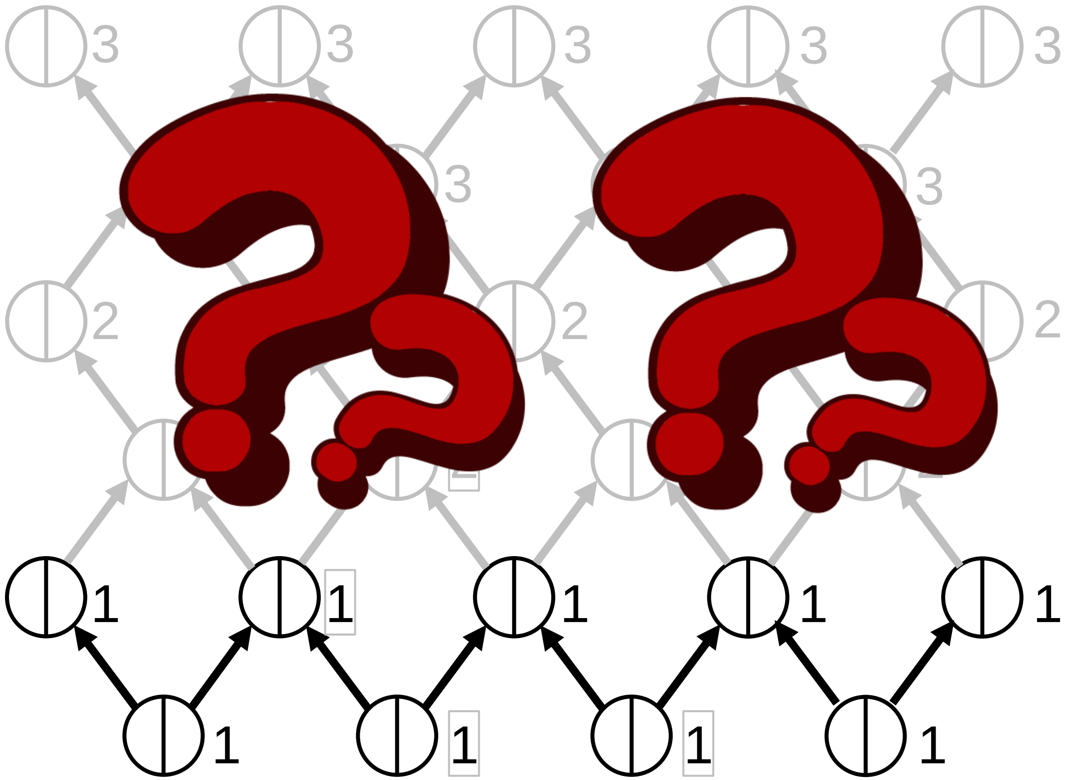
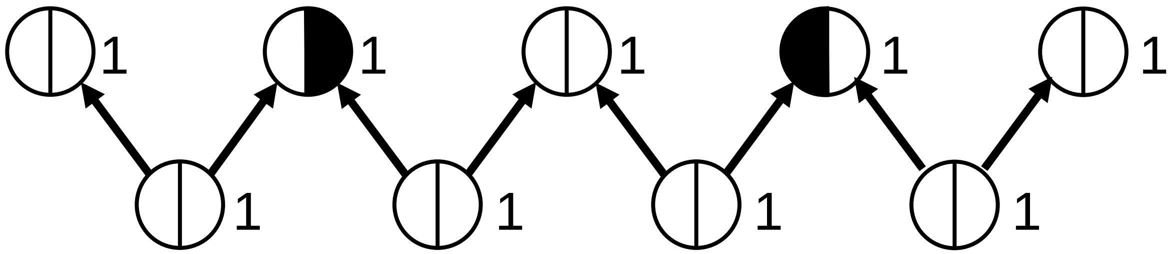
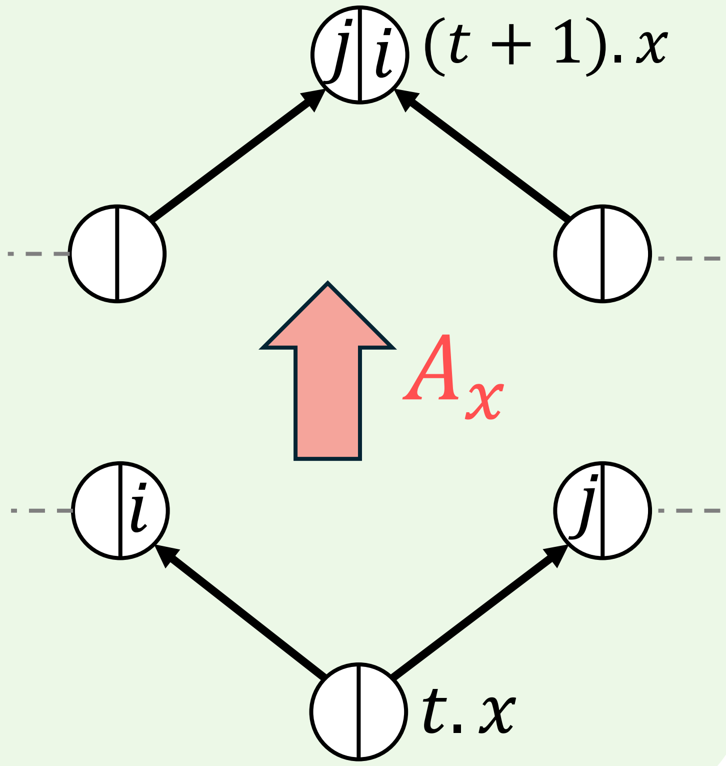


Particle system example



Local rule



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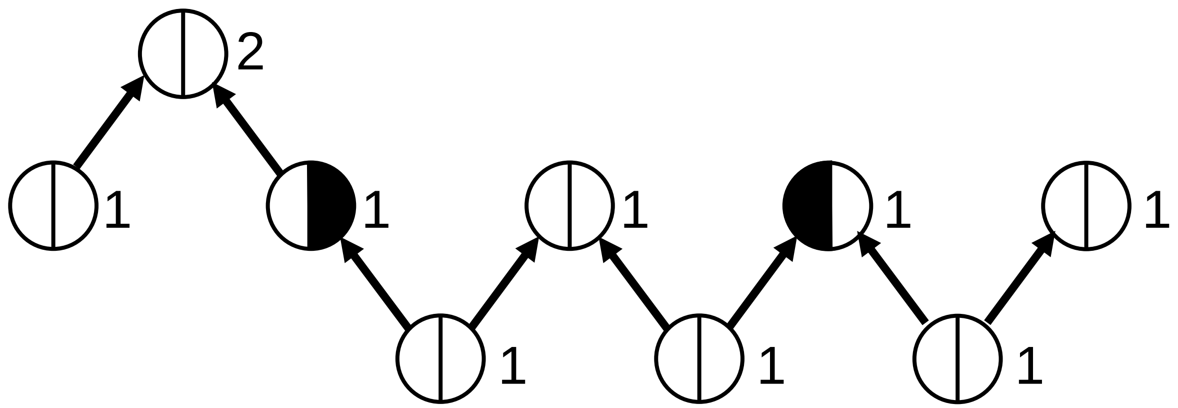

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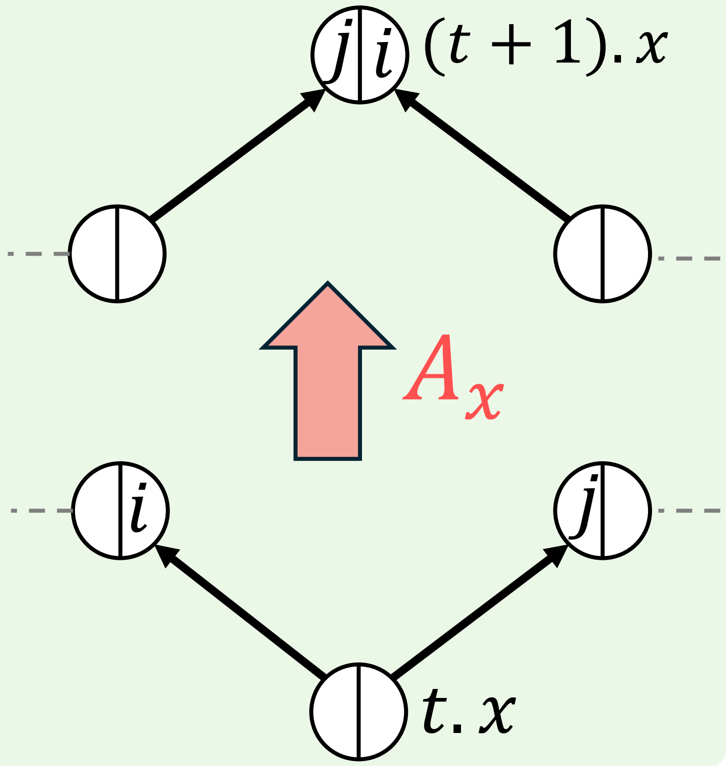

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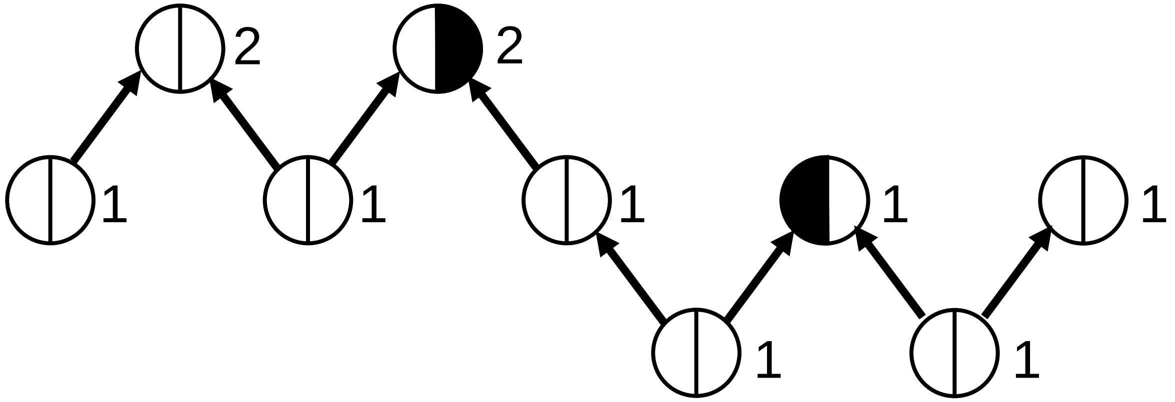

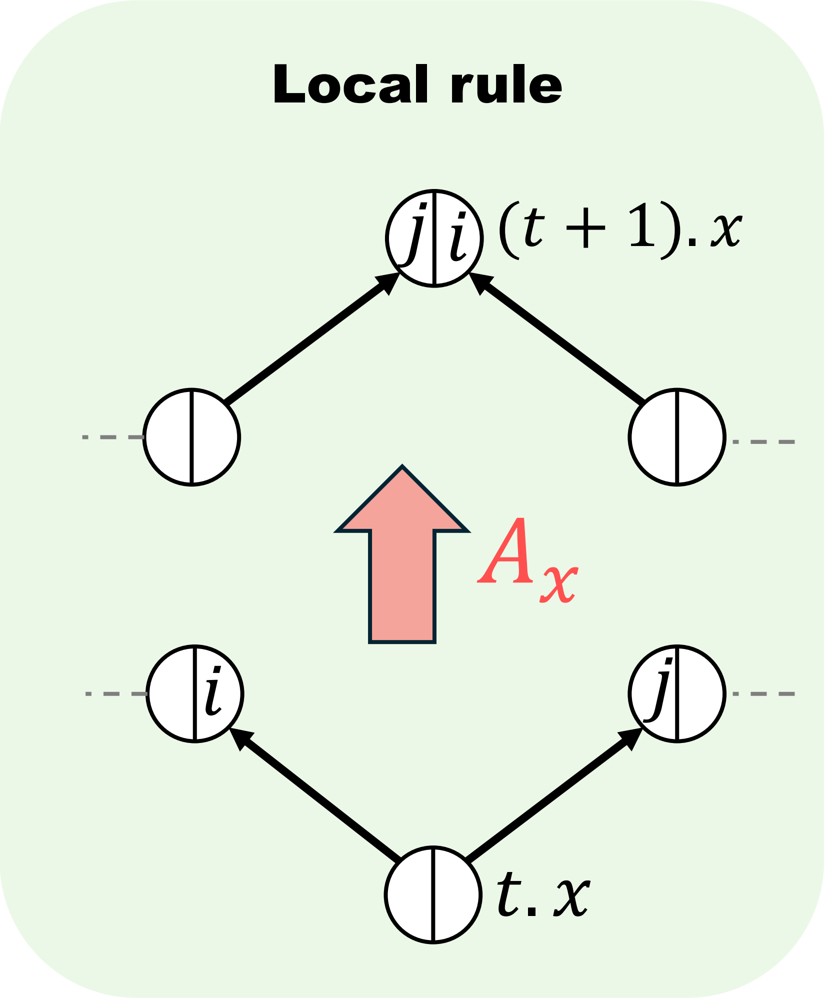

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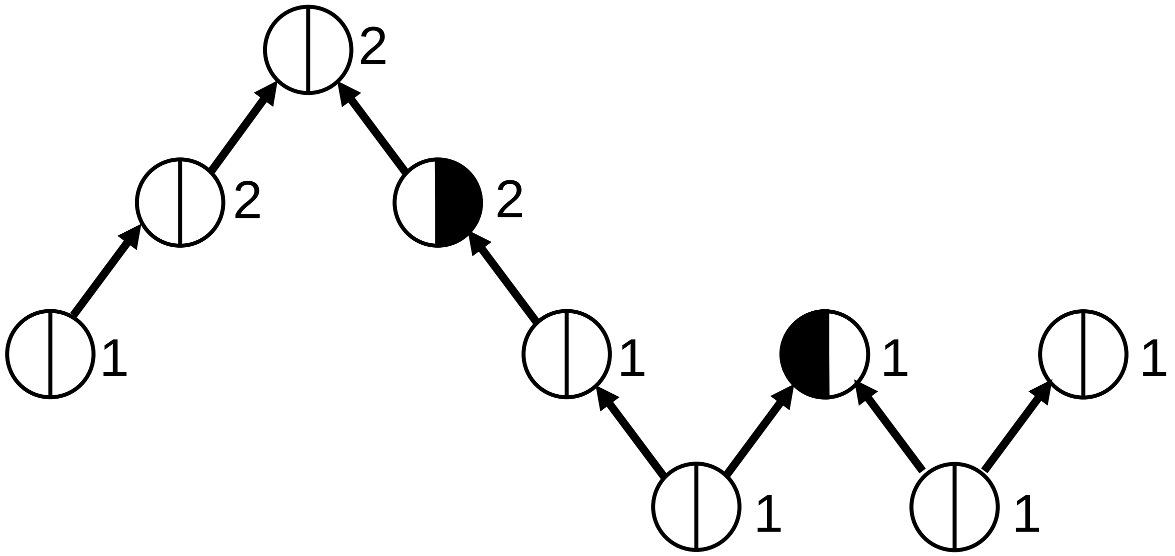

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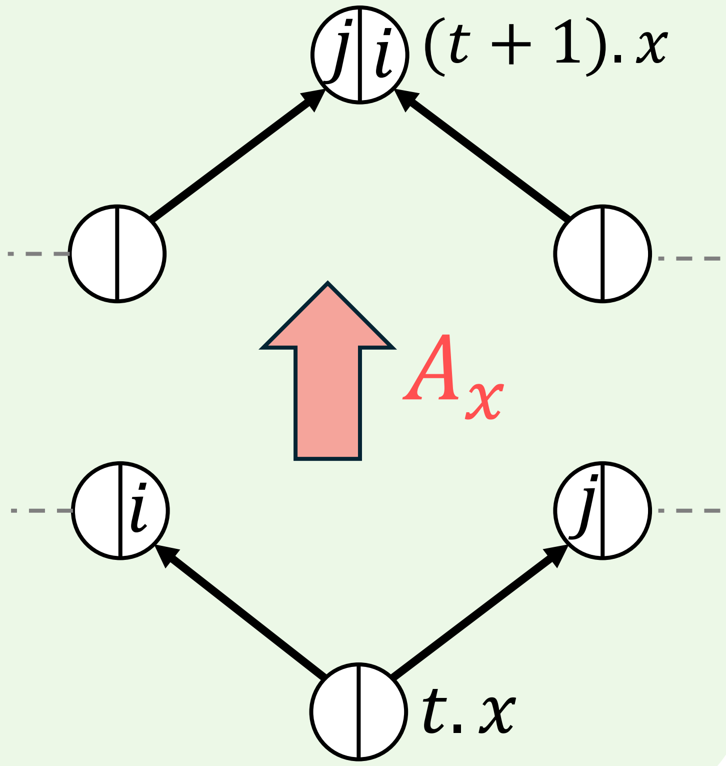

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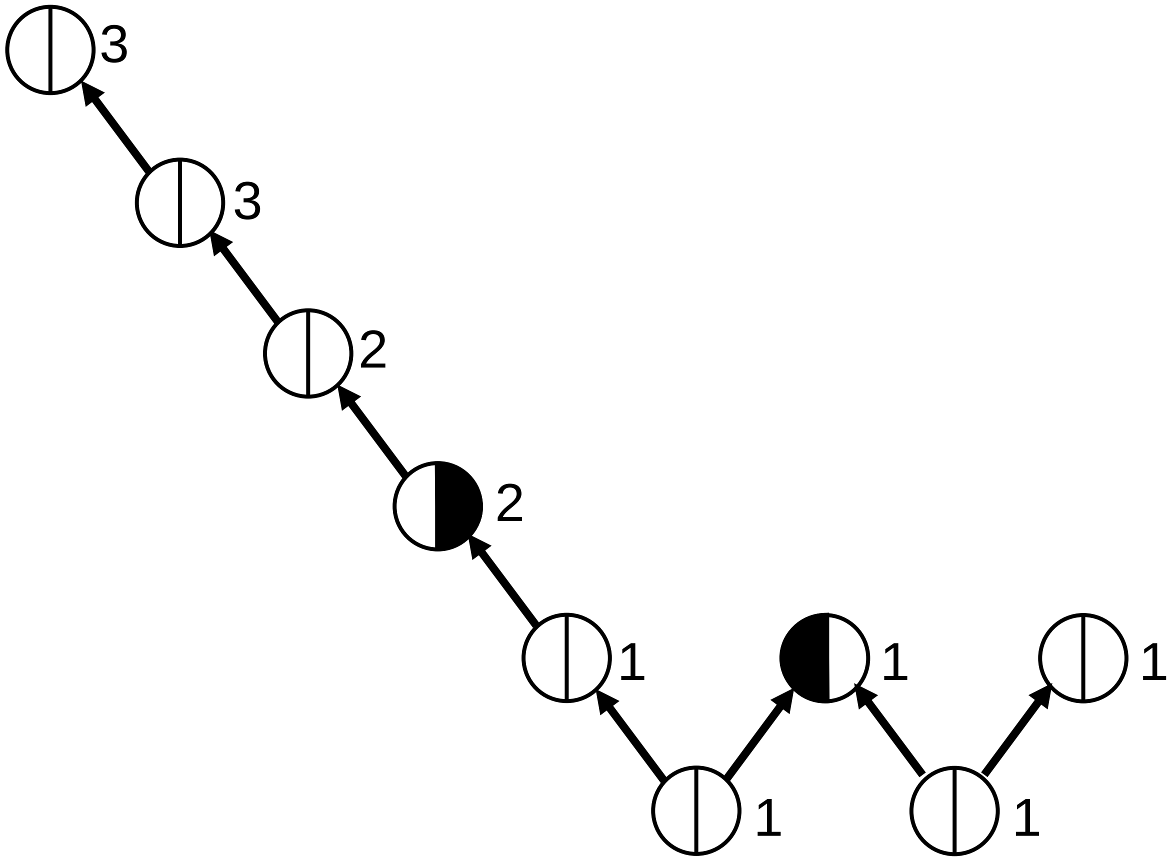

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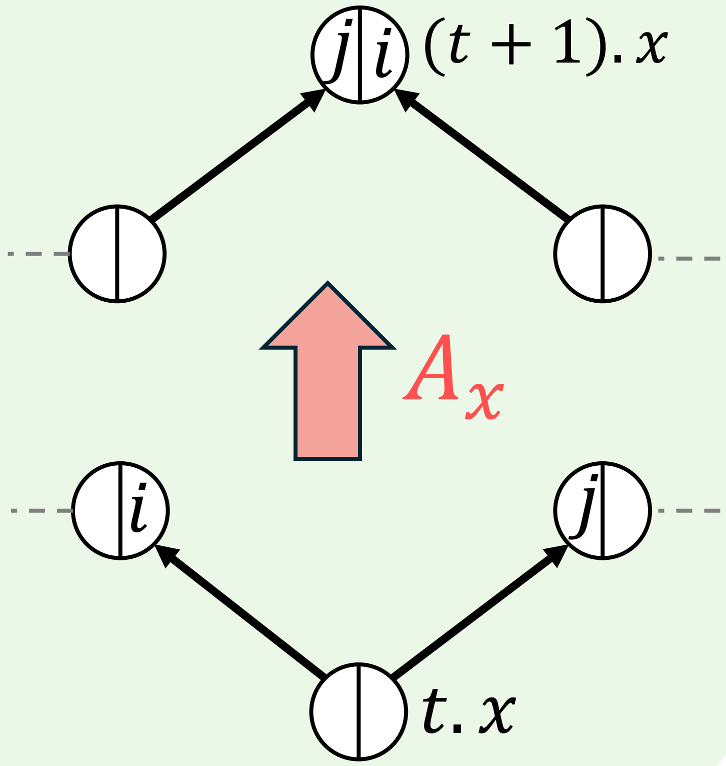

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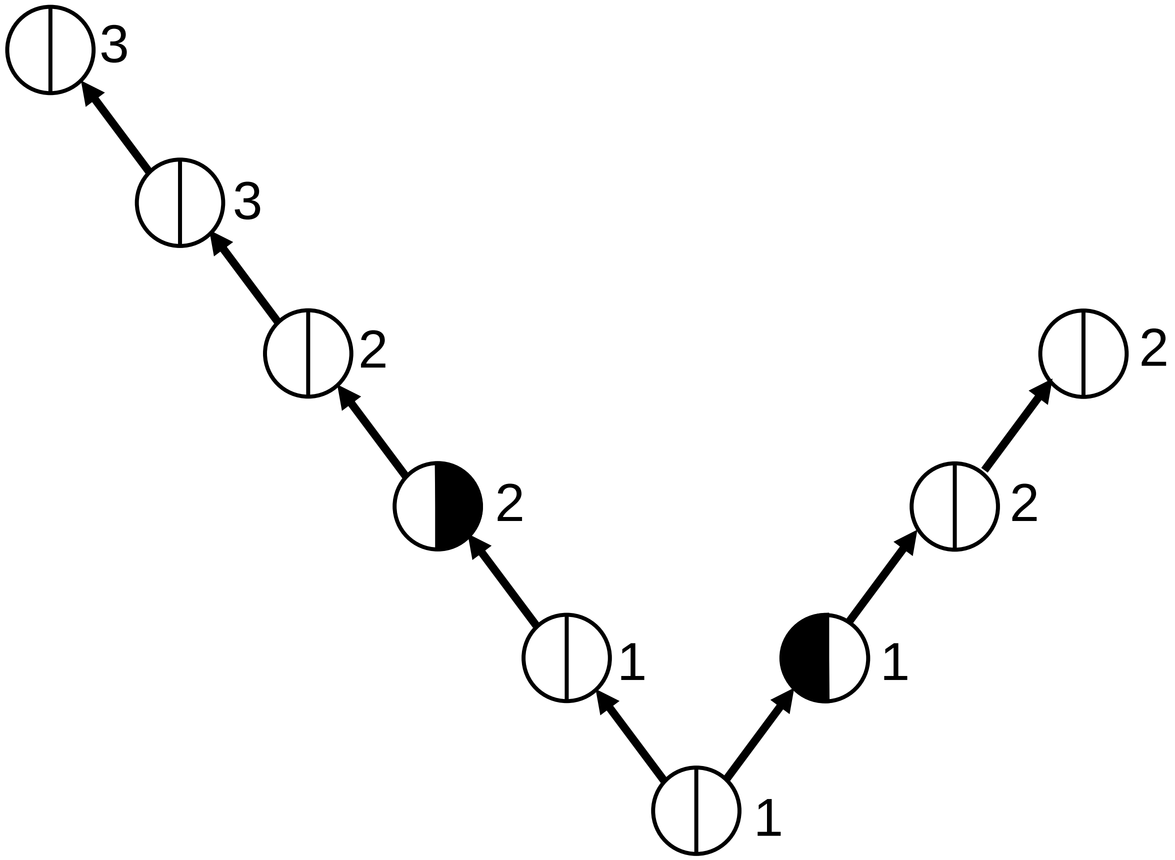

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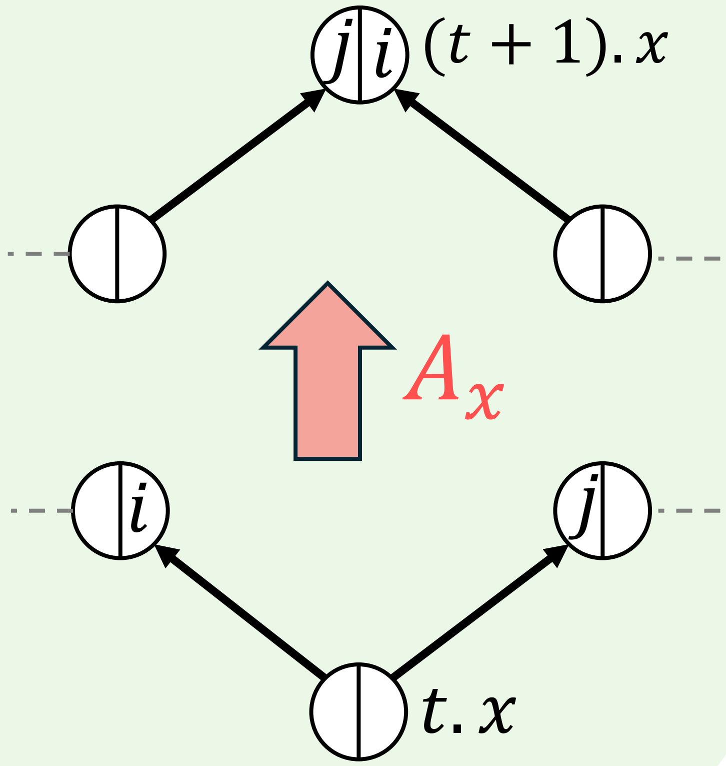

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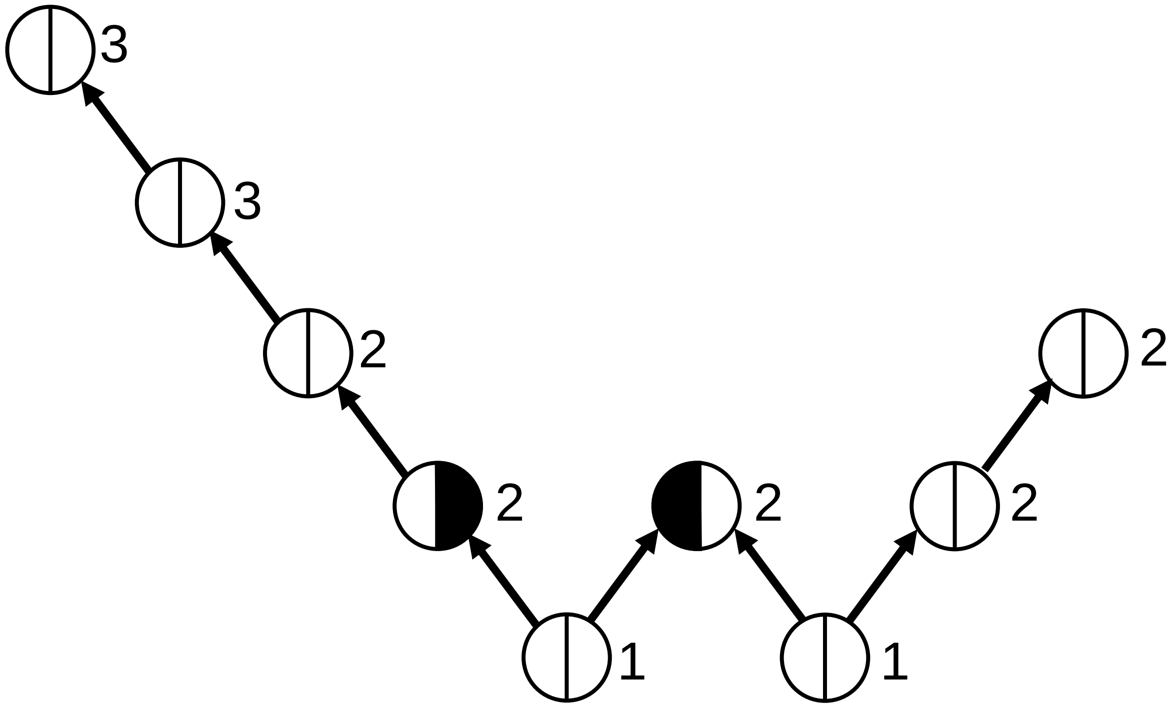

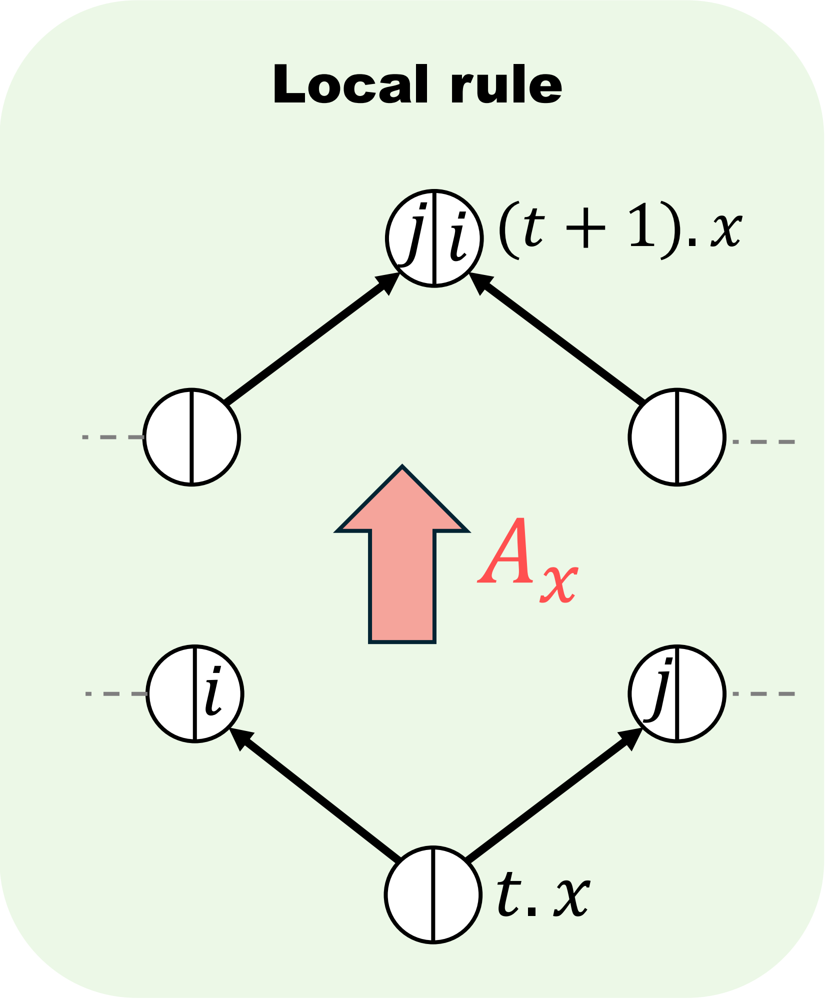

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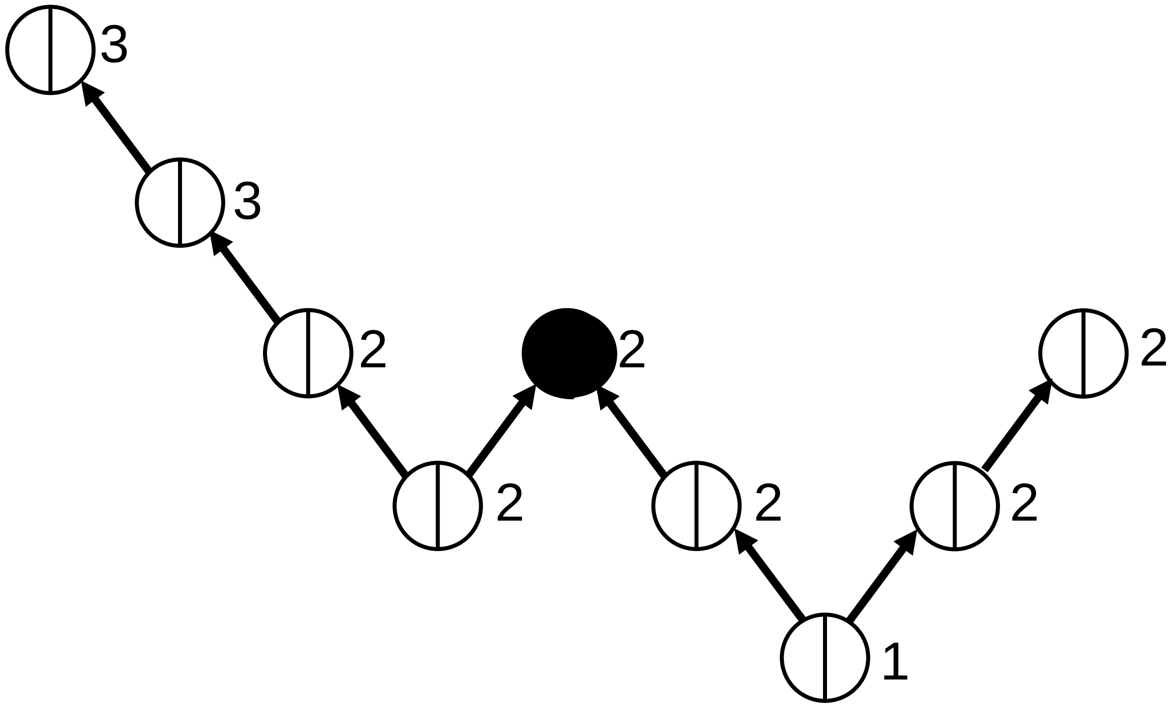

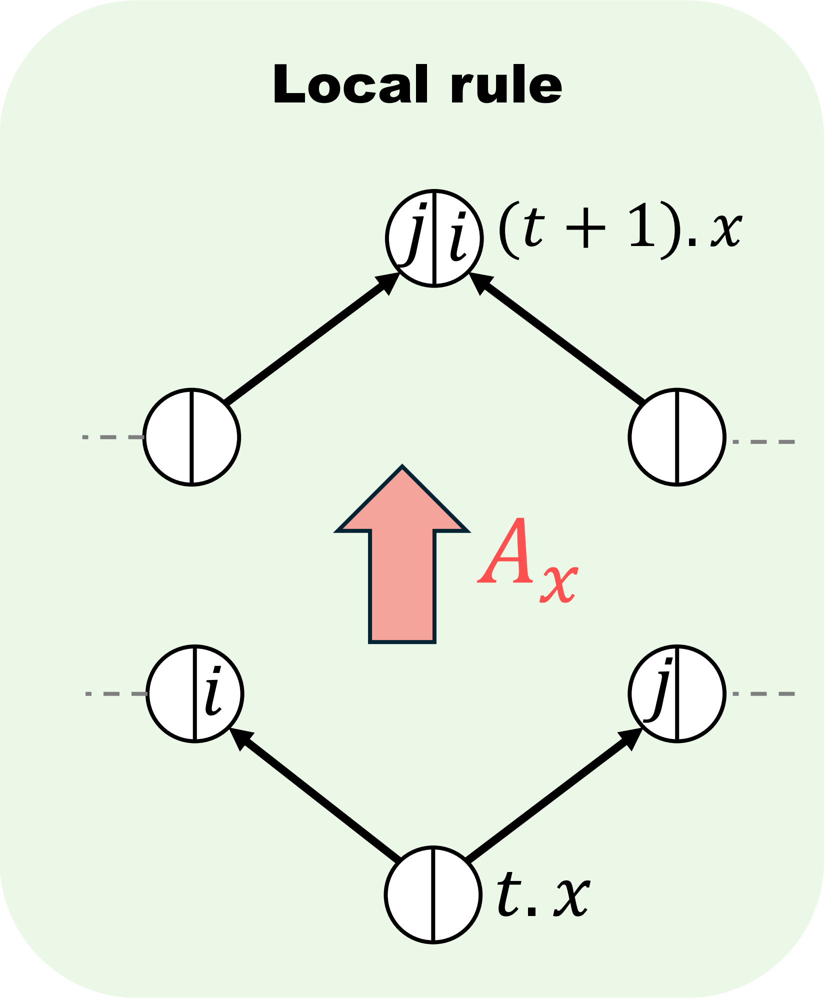

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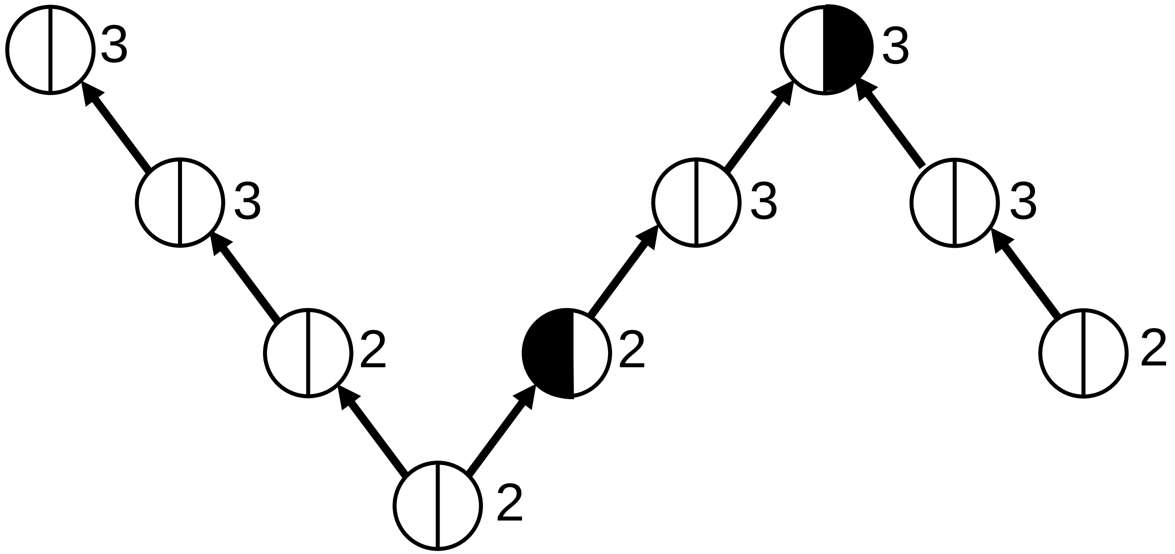

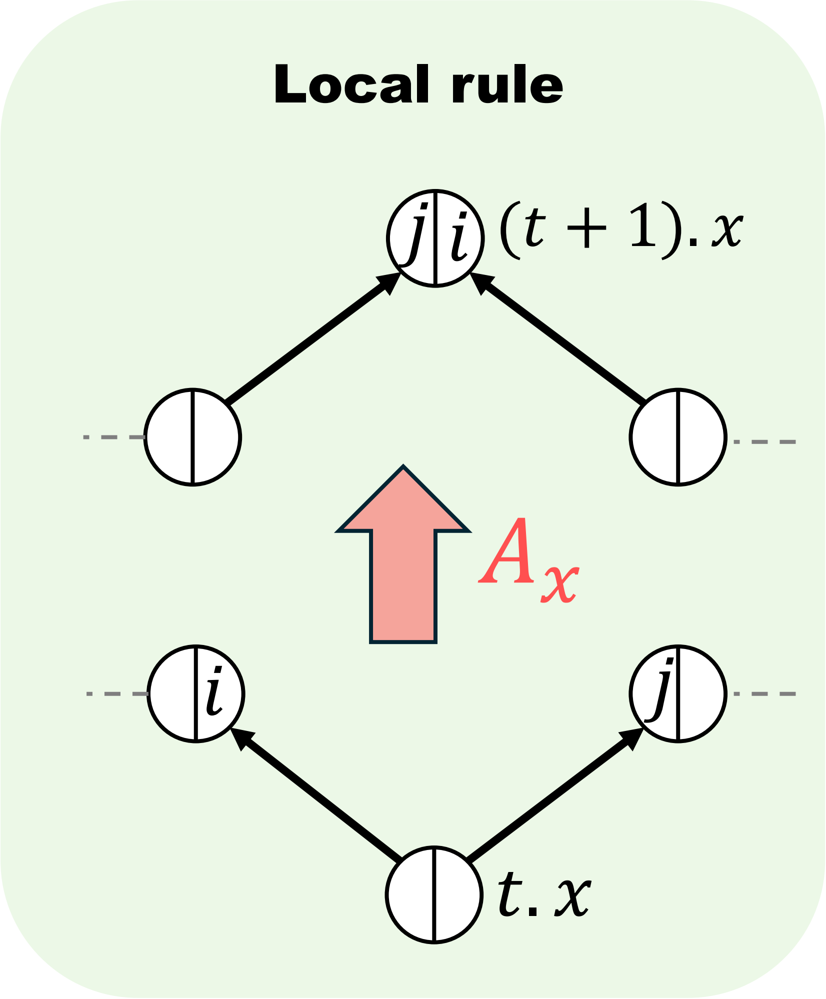

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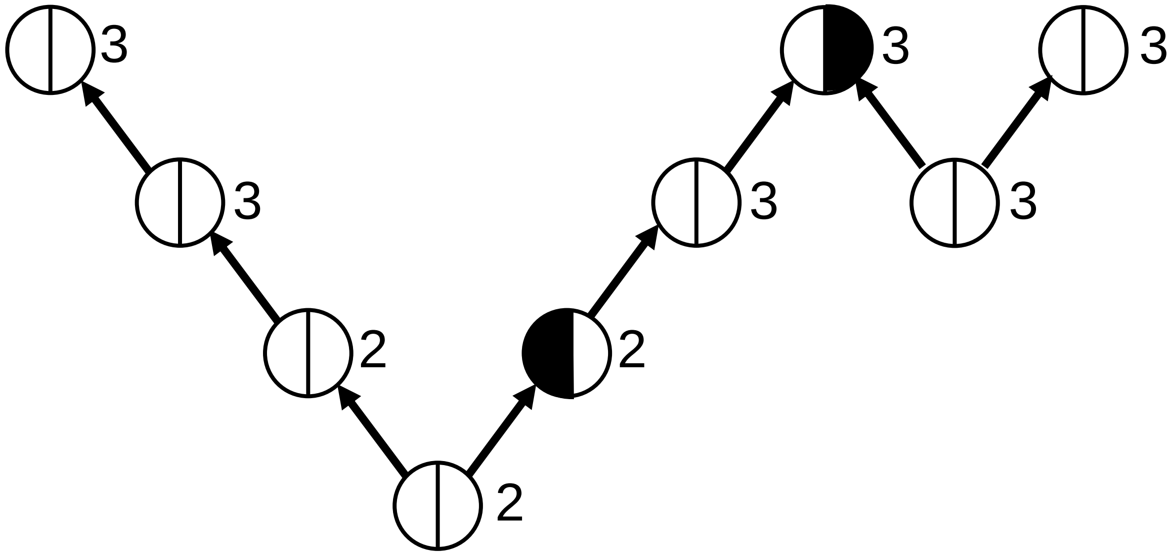

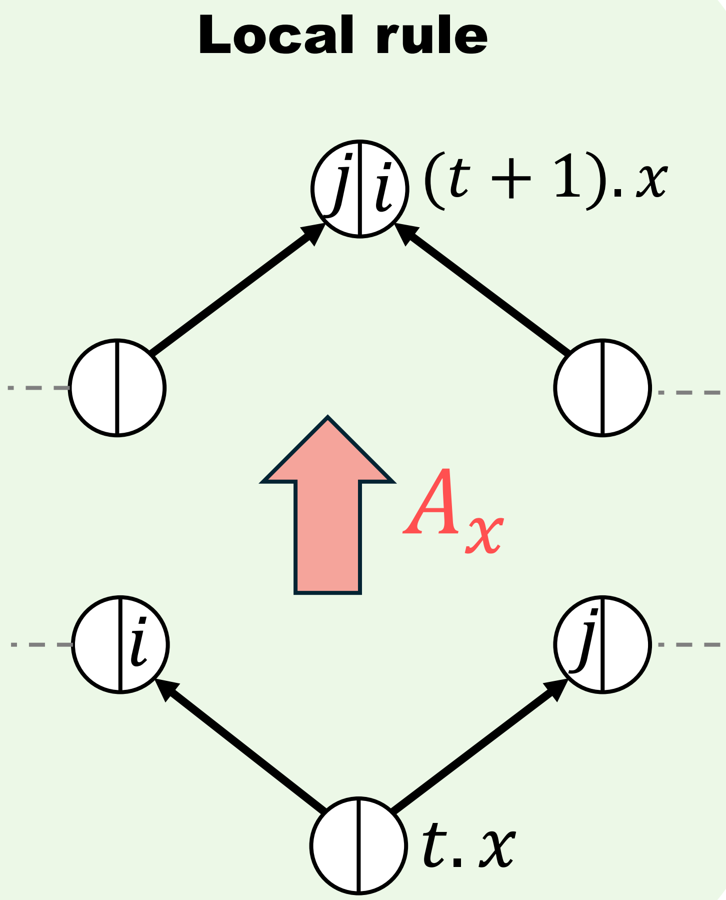

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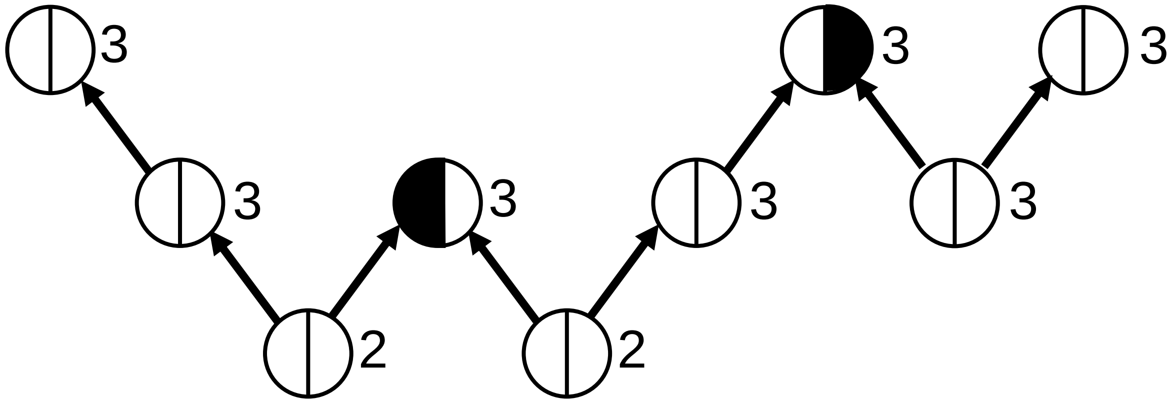

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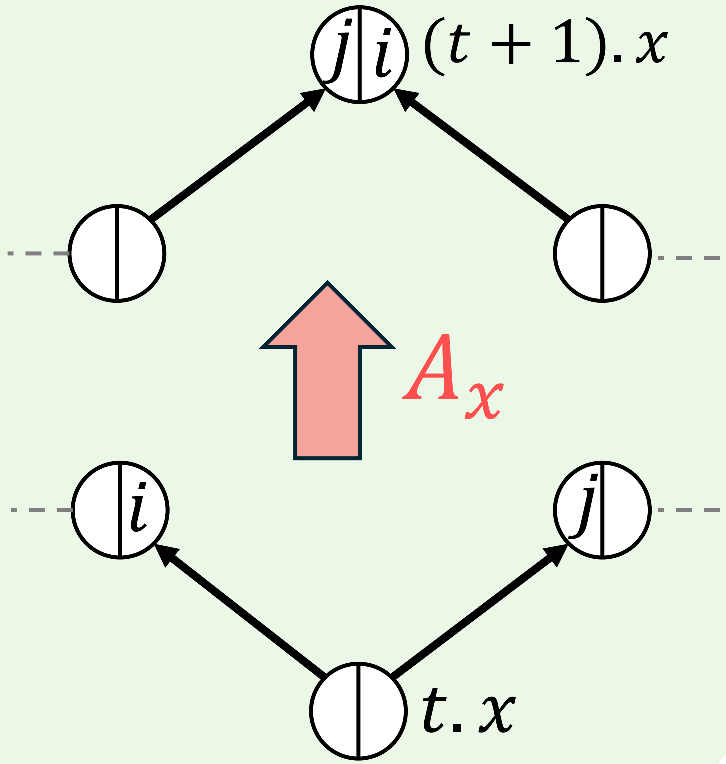

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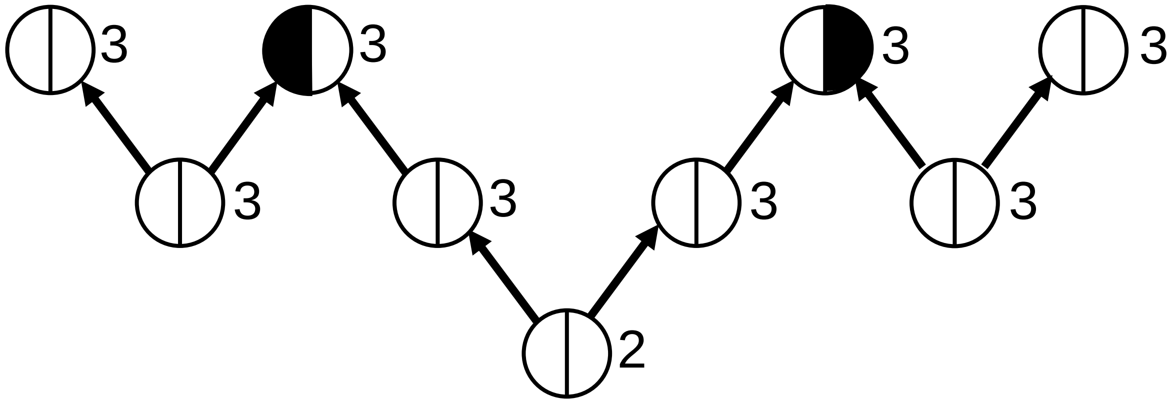

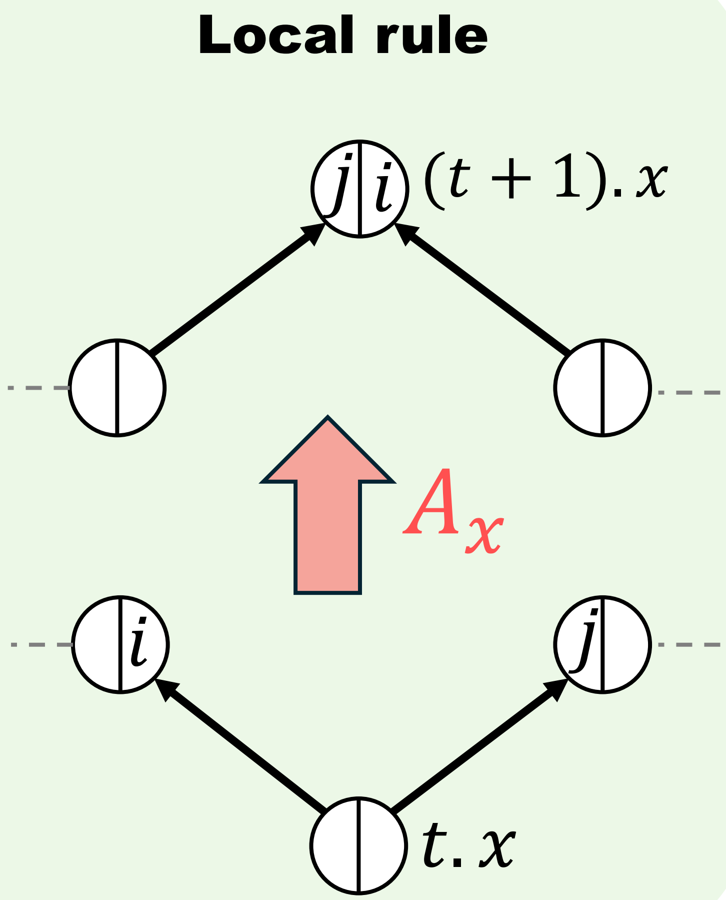

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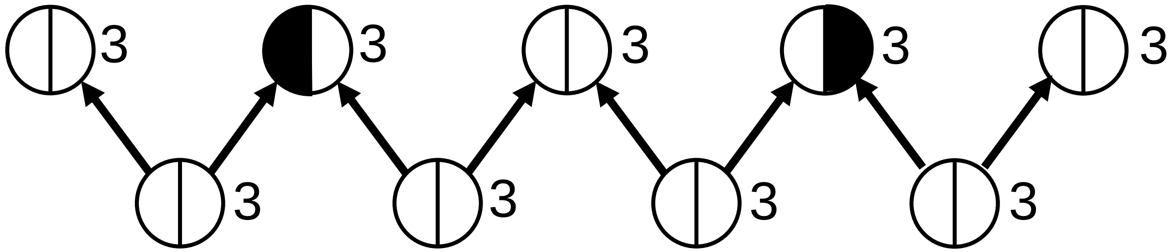

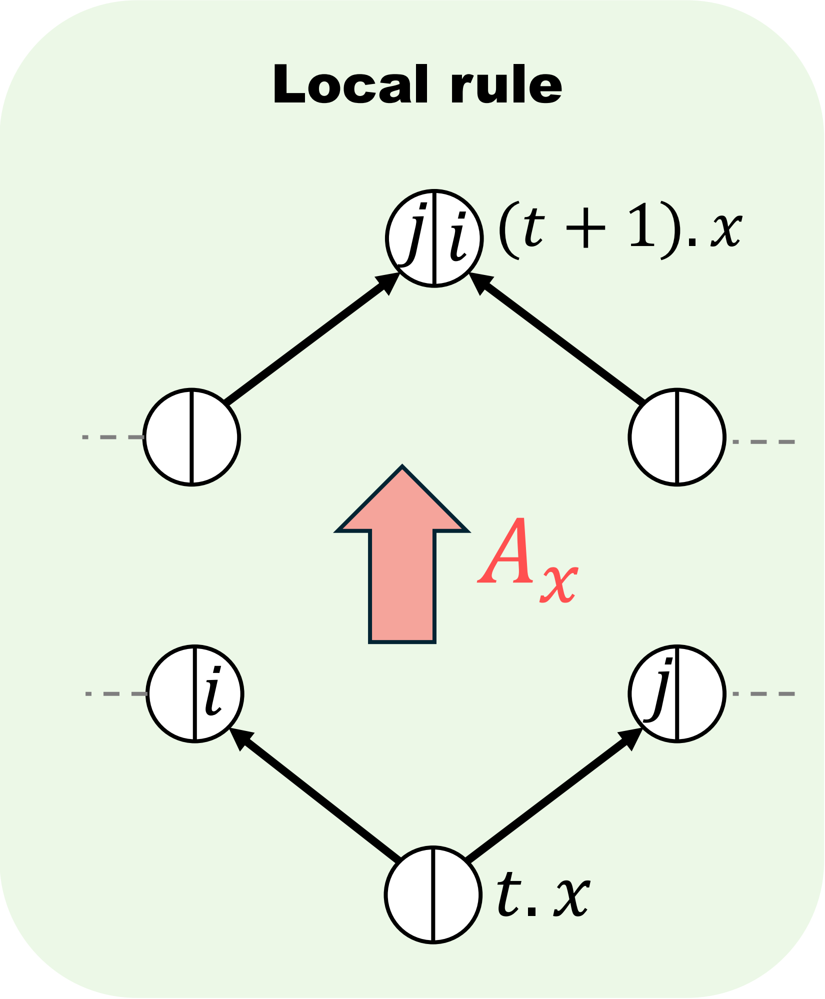

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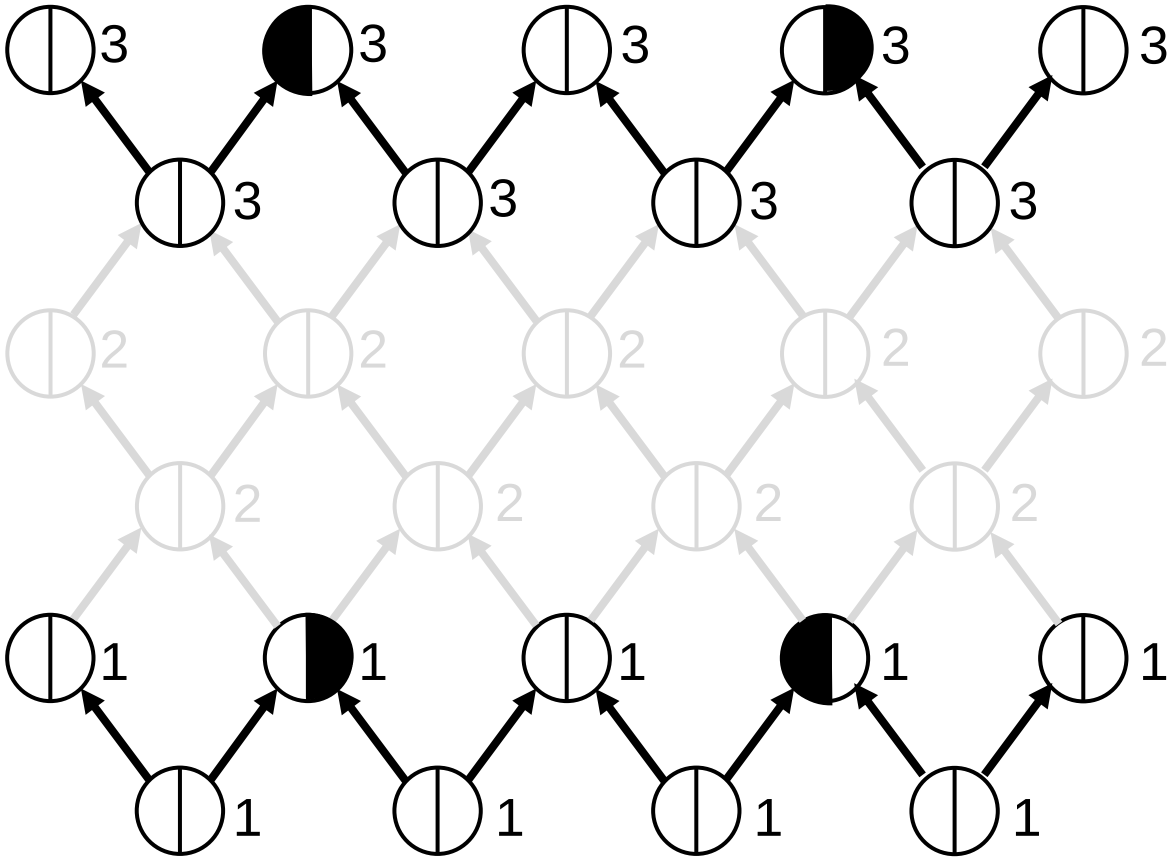
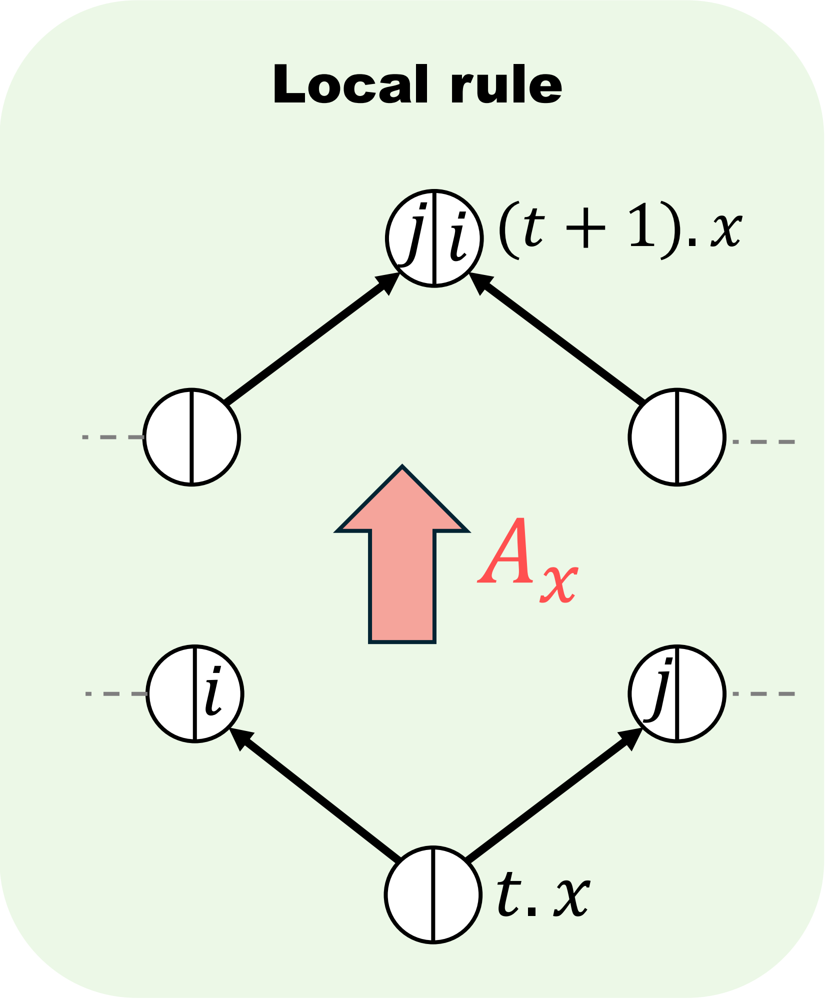
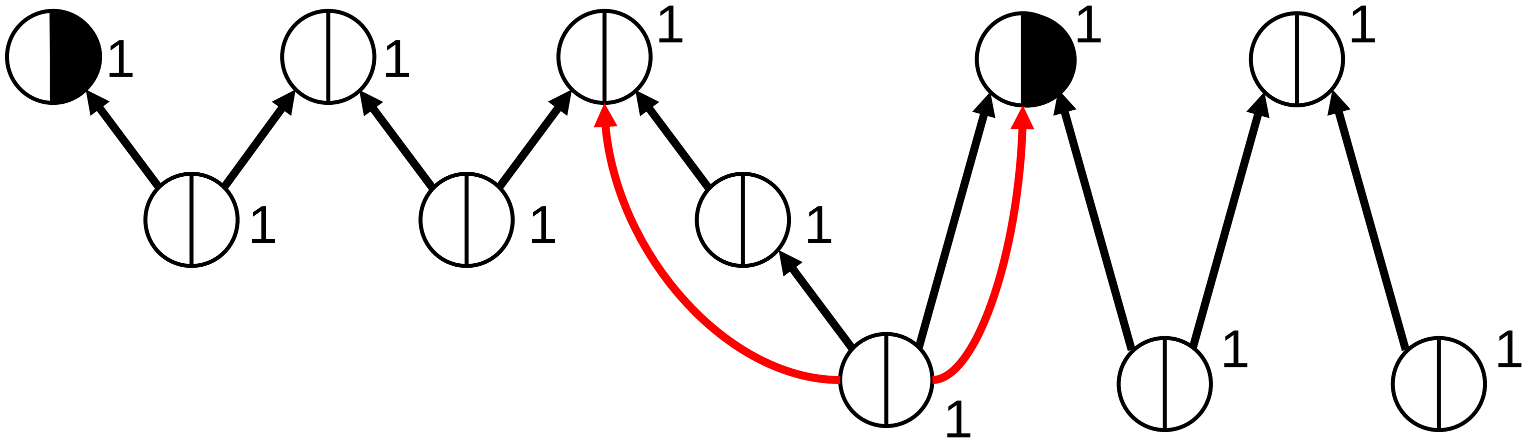
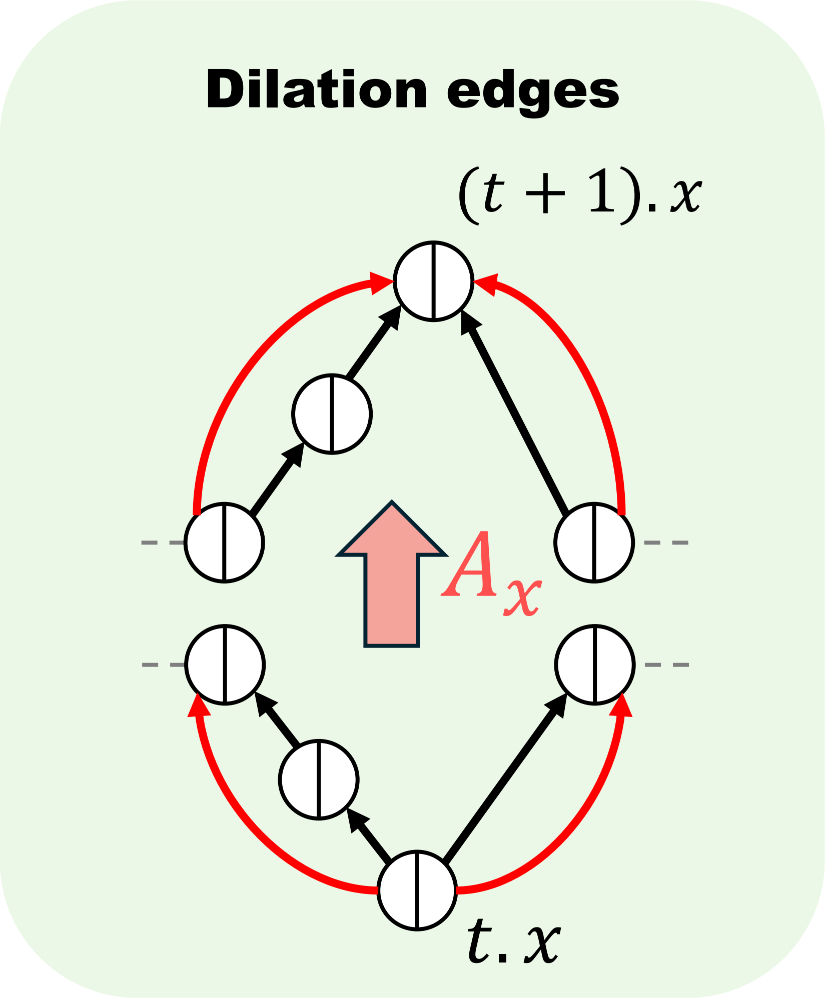




Time dilation example





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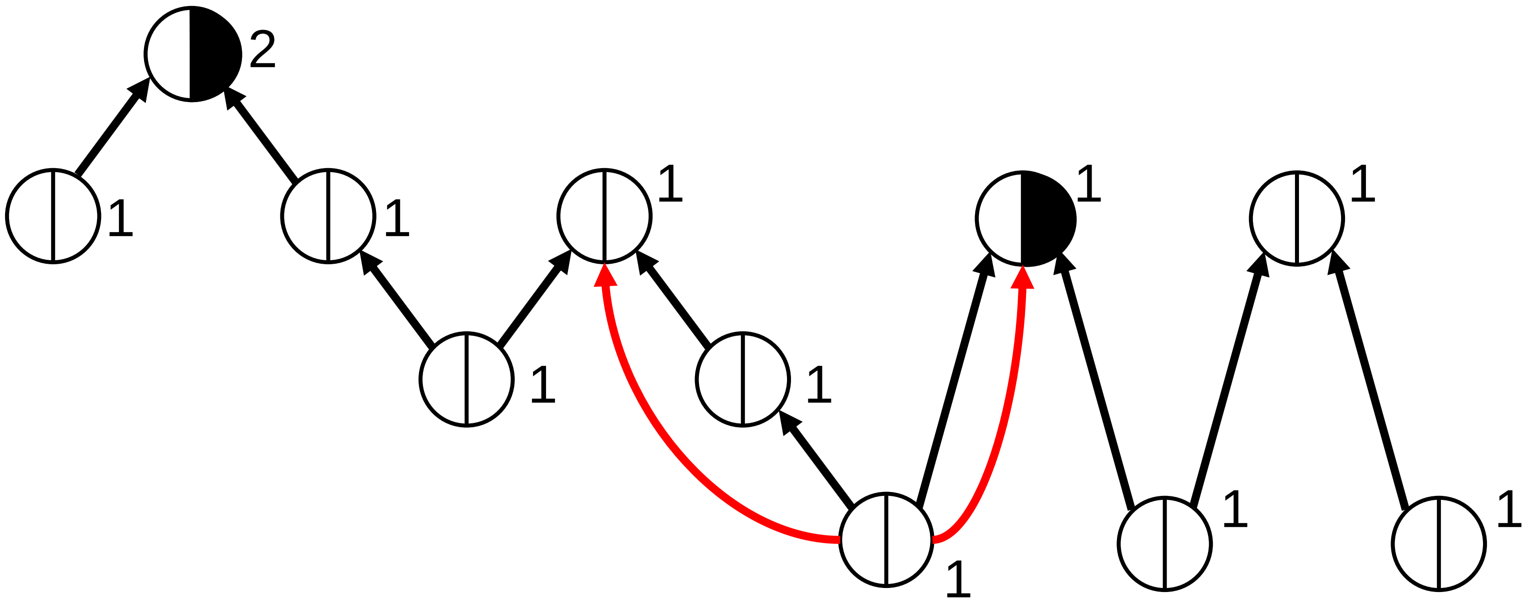

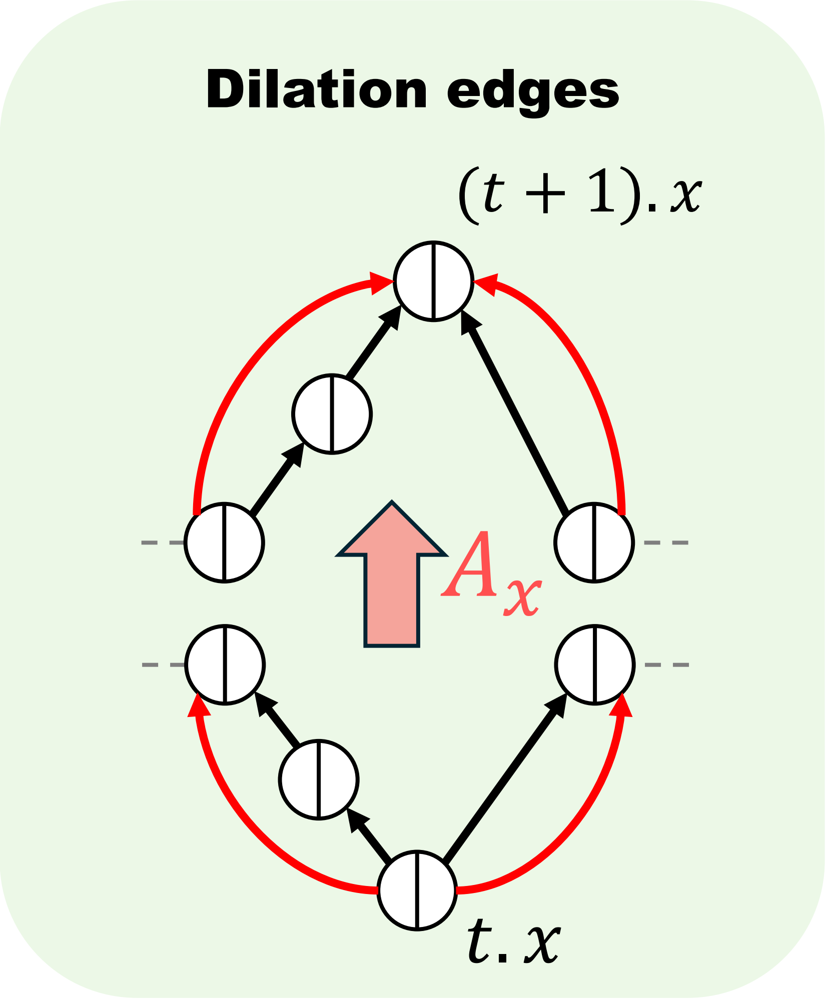

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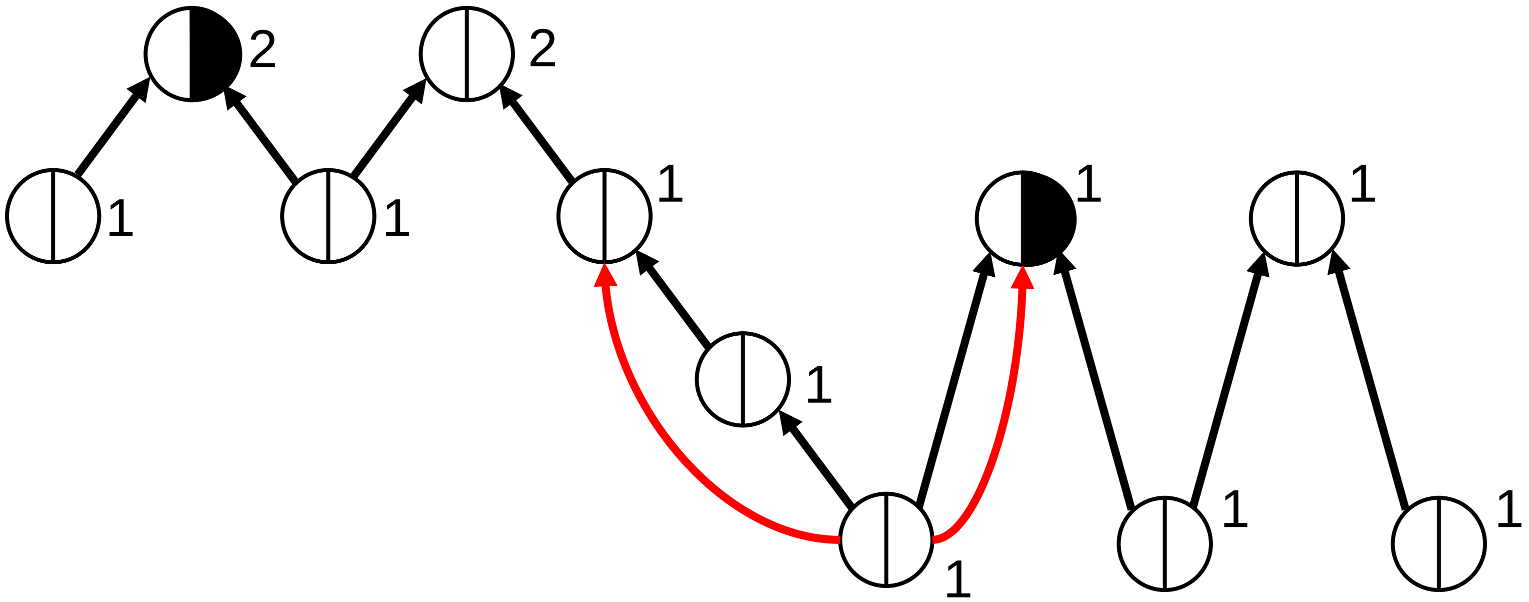

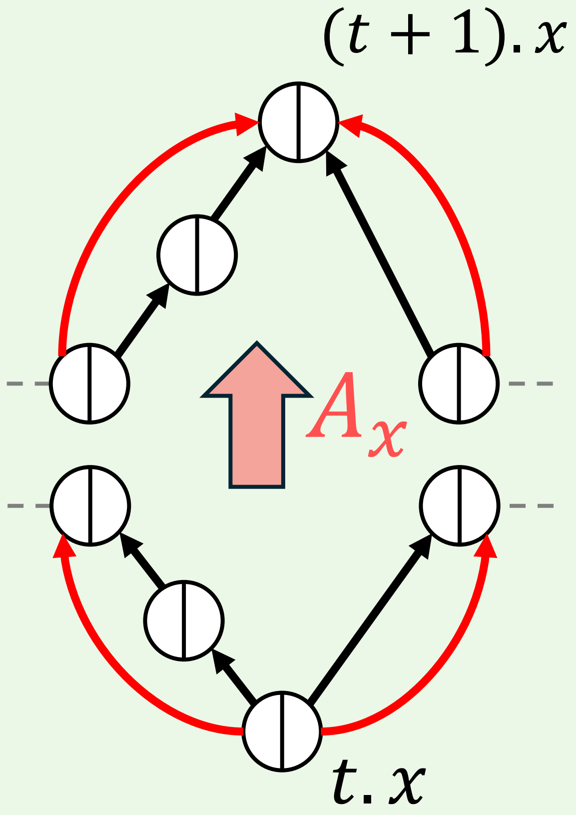

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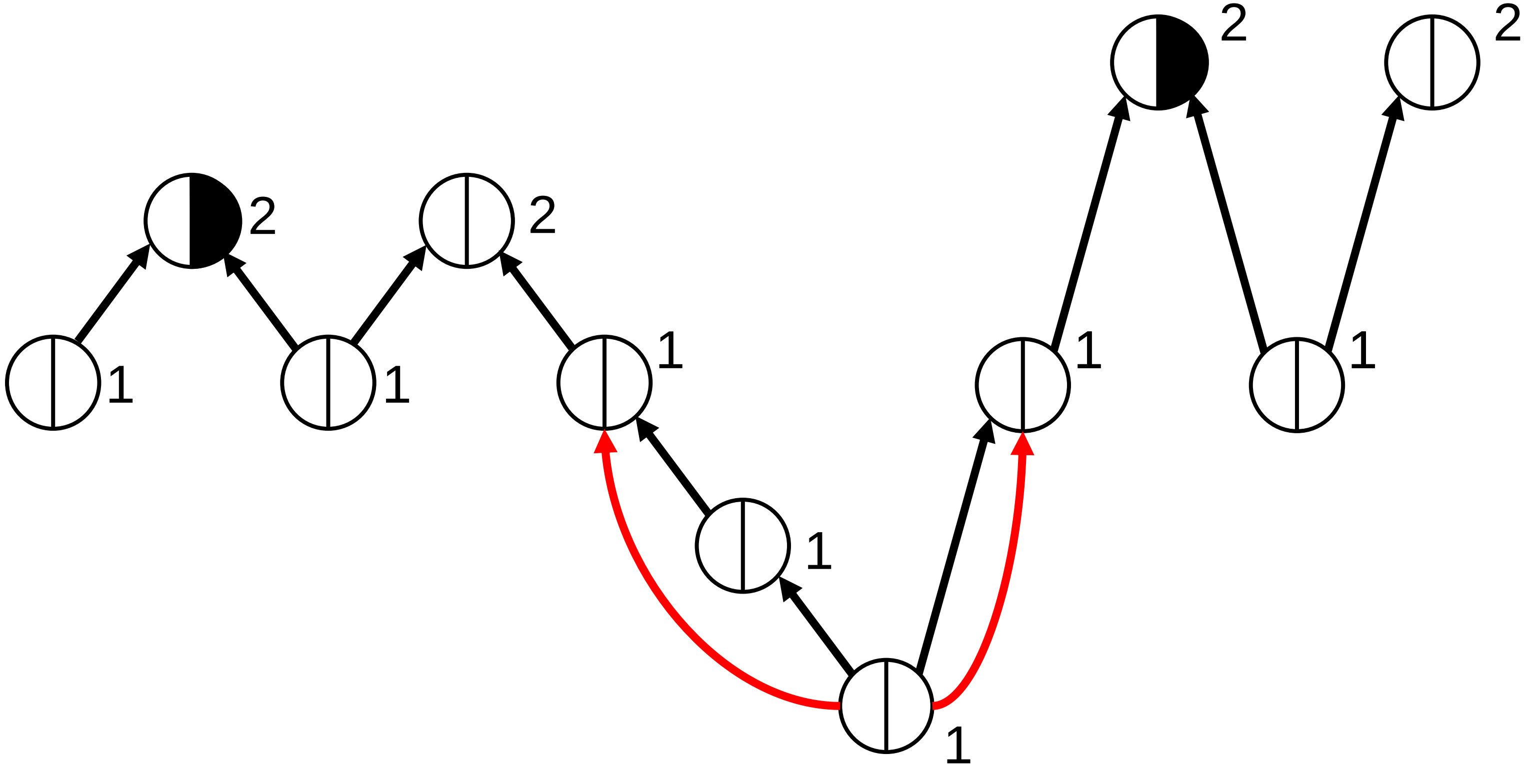

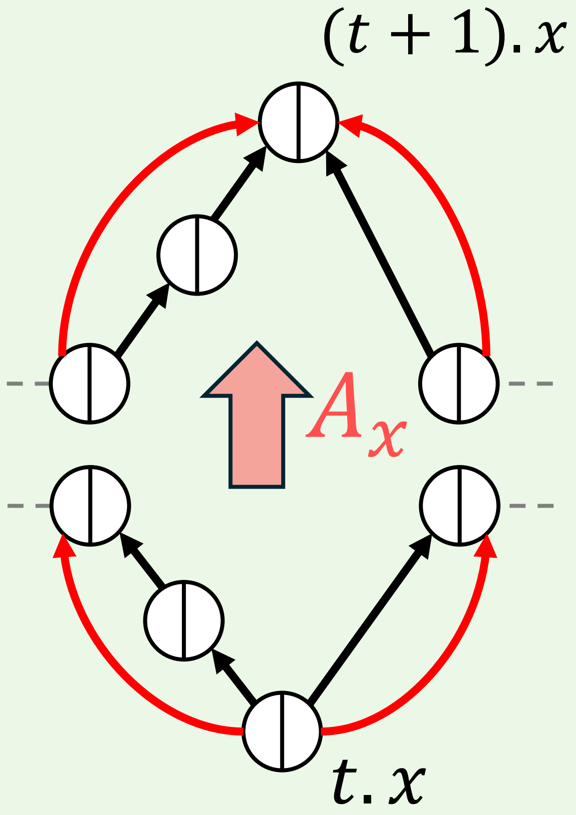

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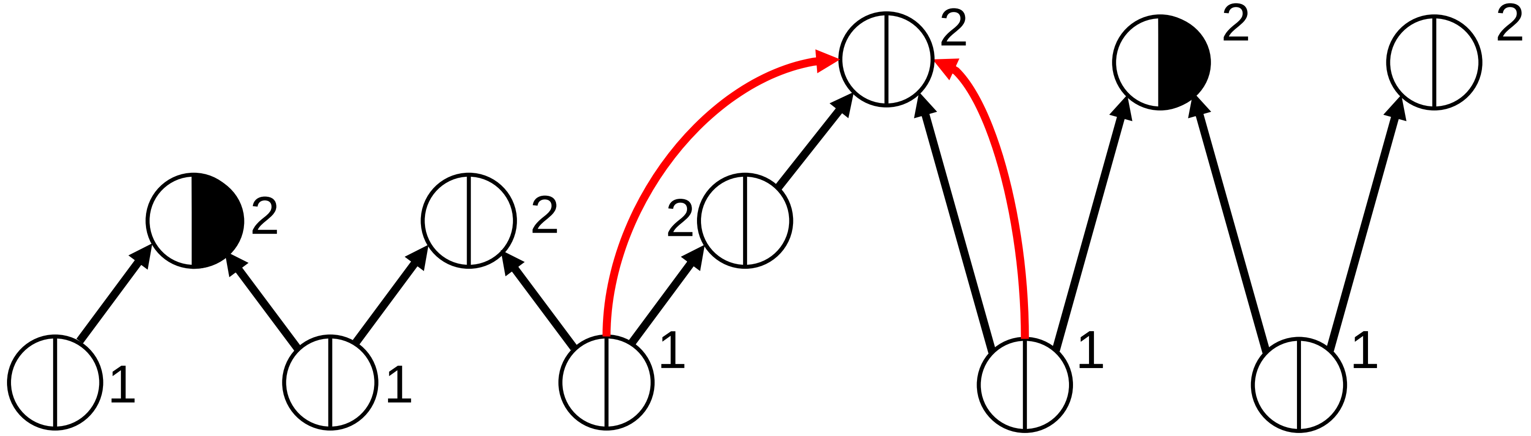

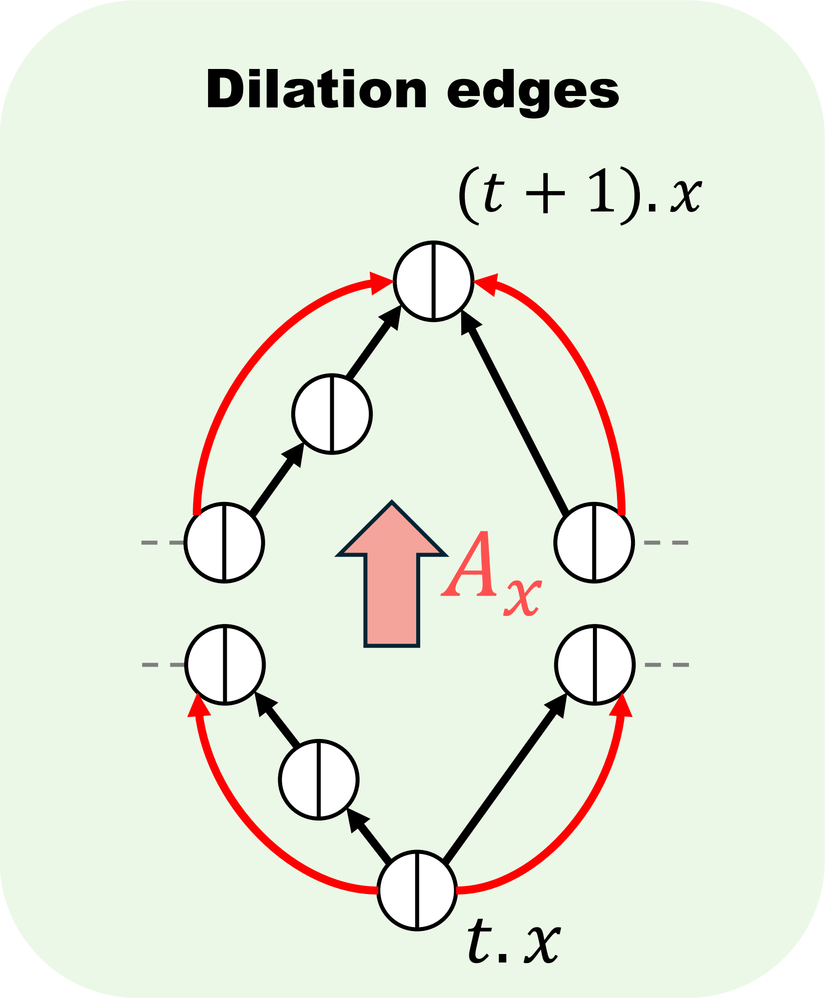

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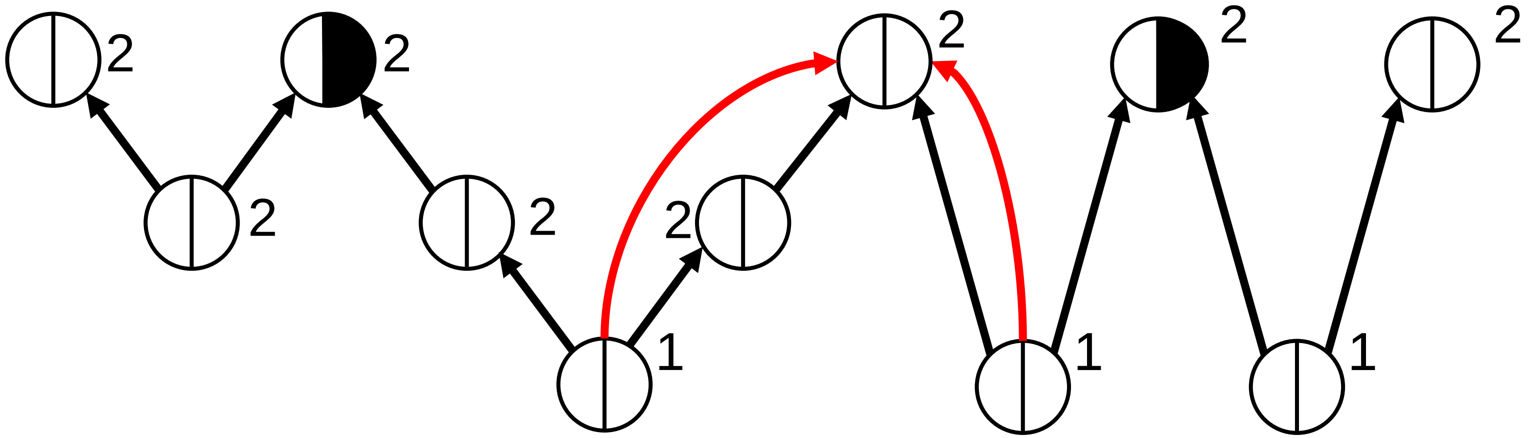
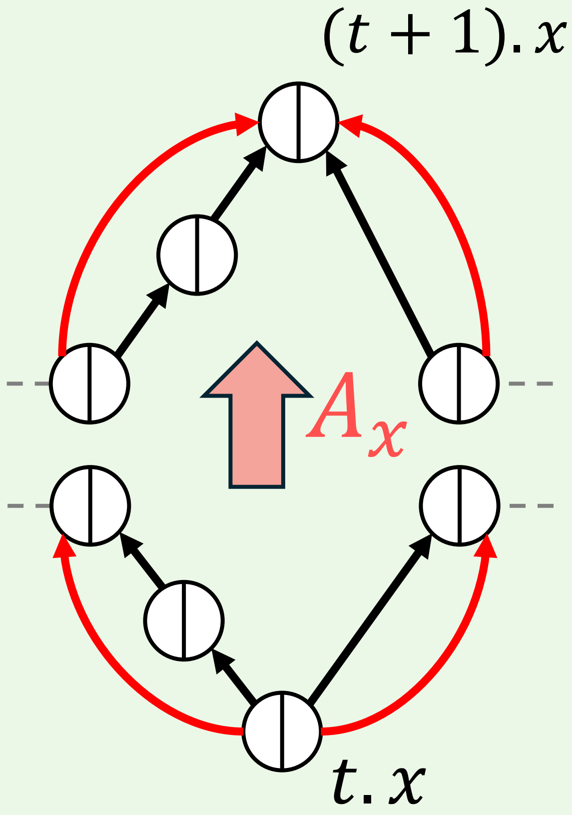


Dilation edges



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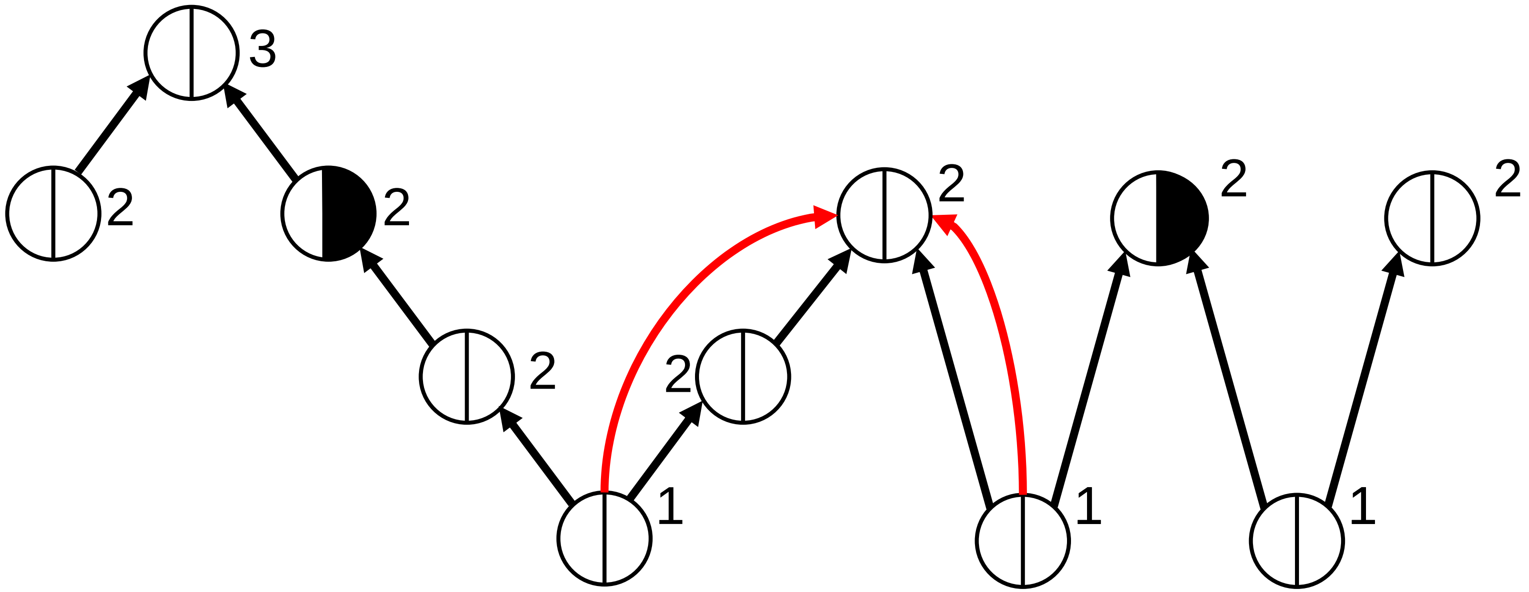

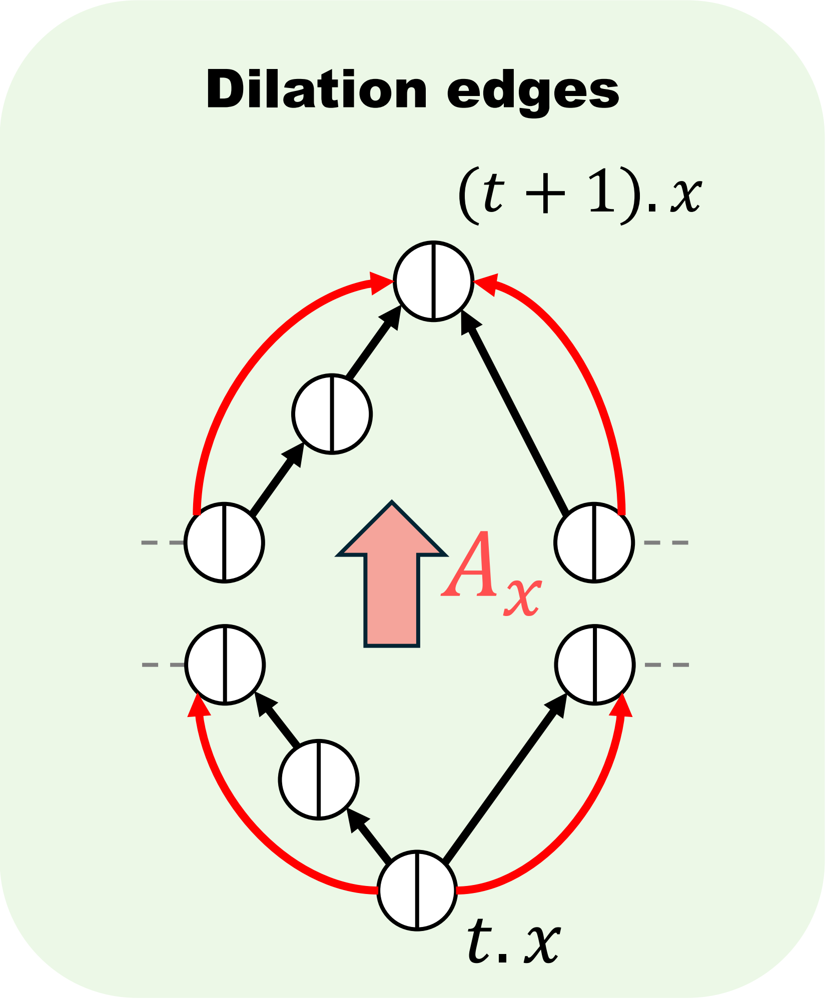

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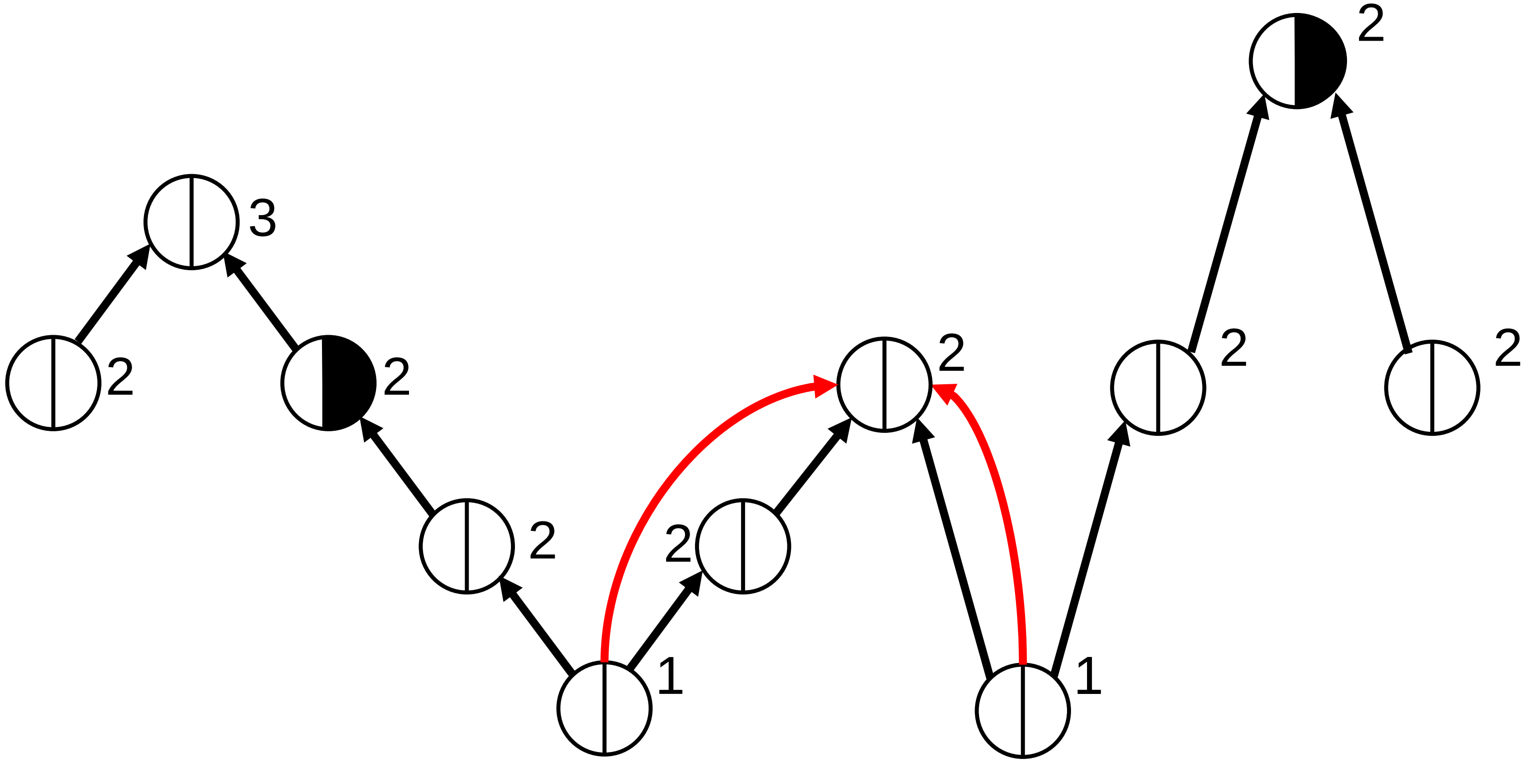

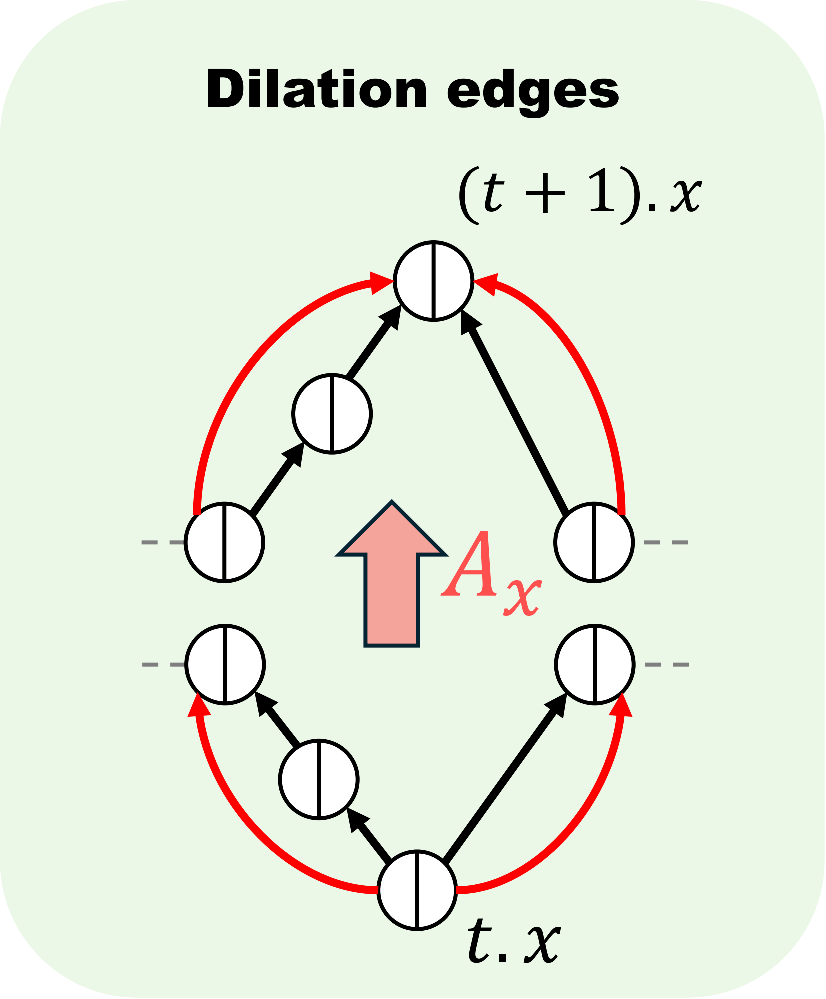

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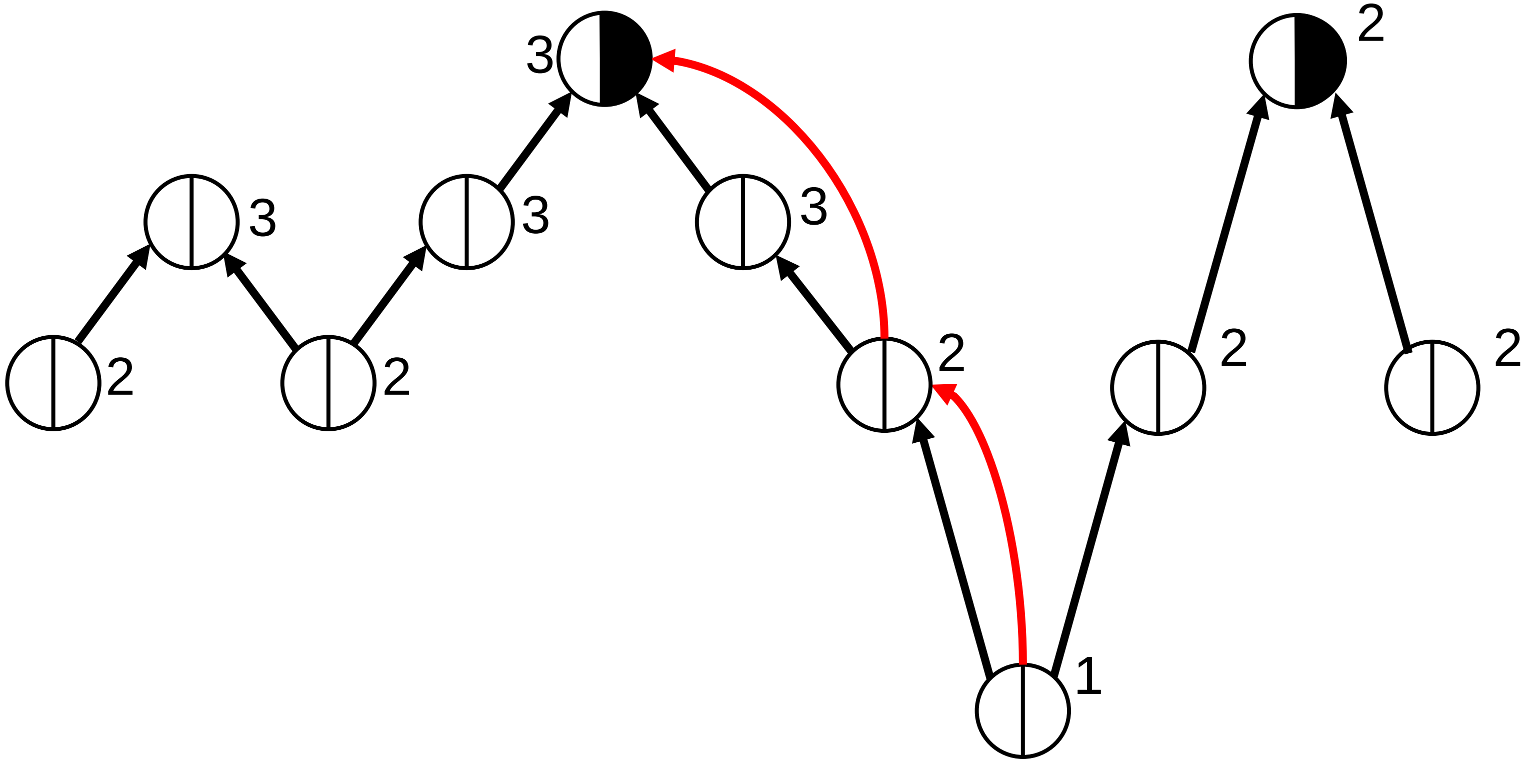

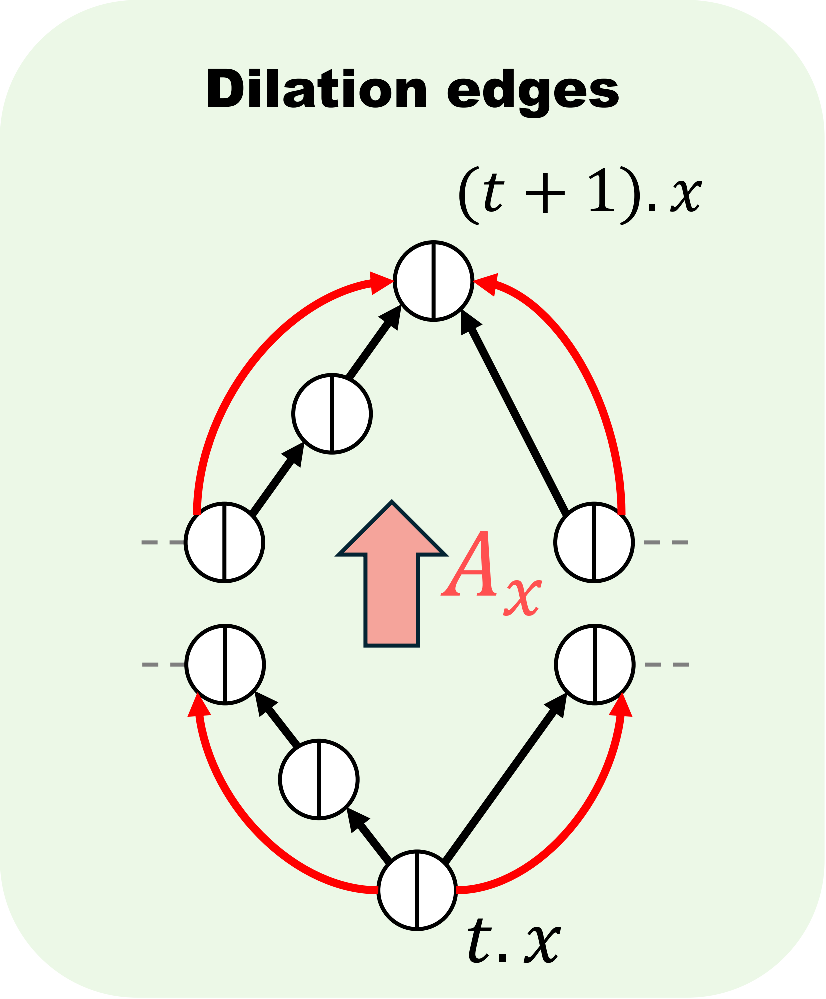

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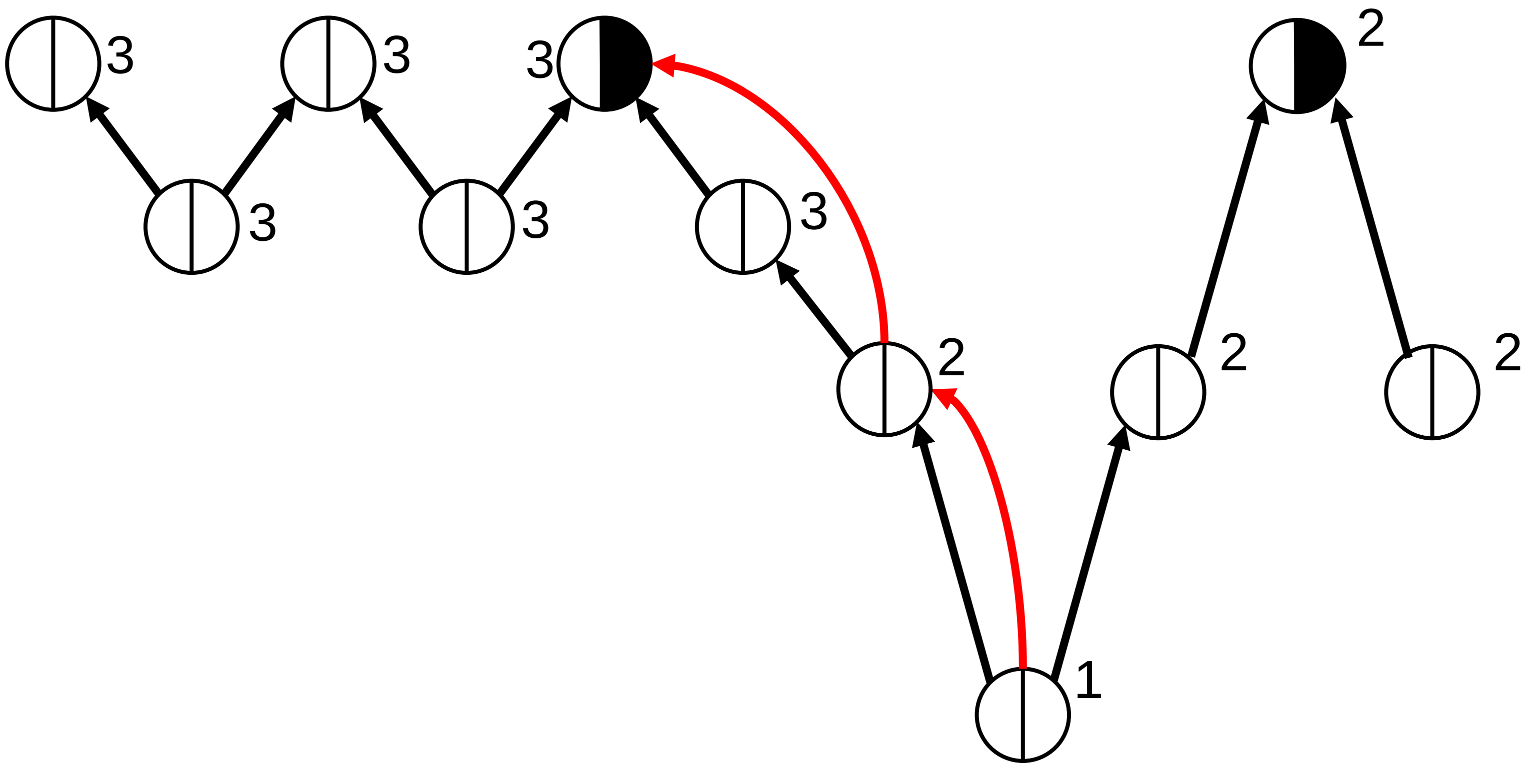

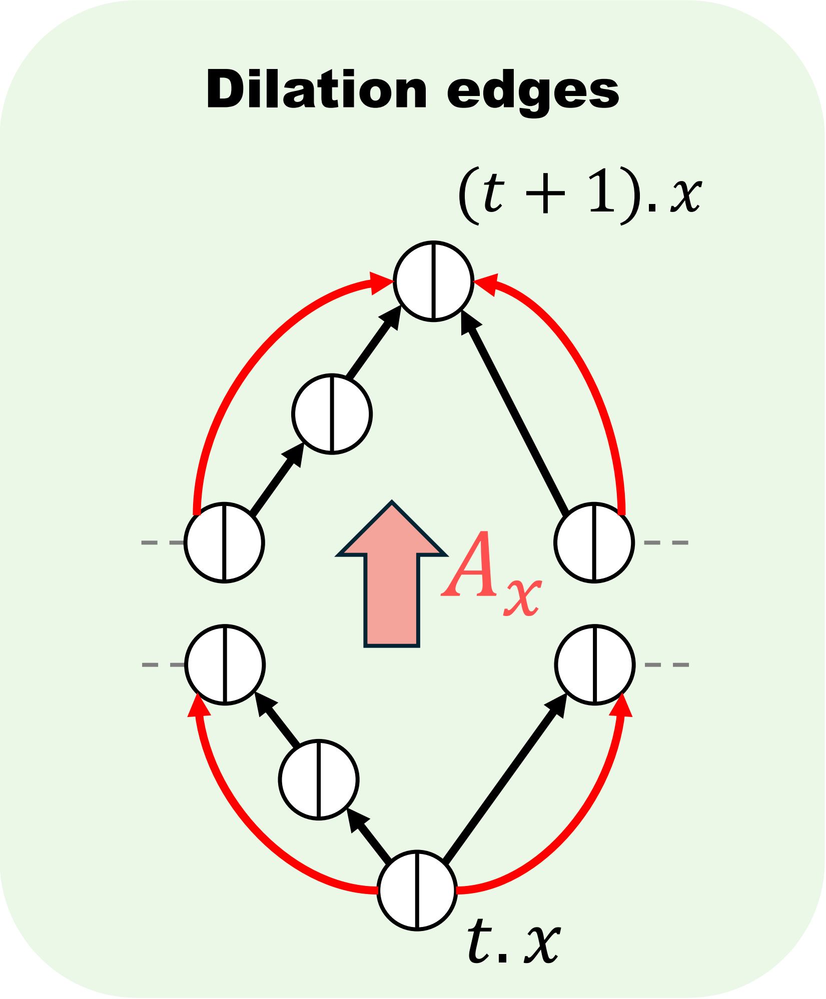

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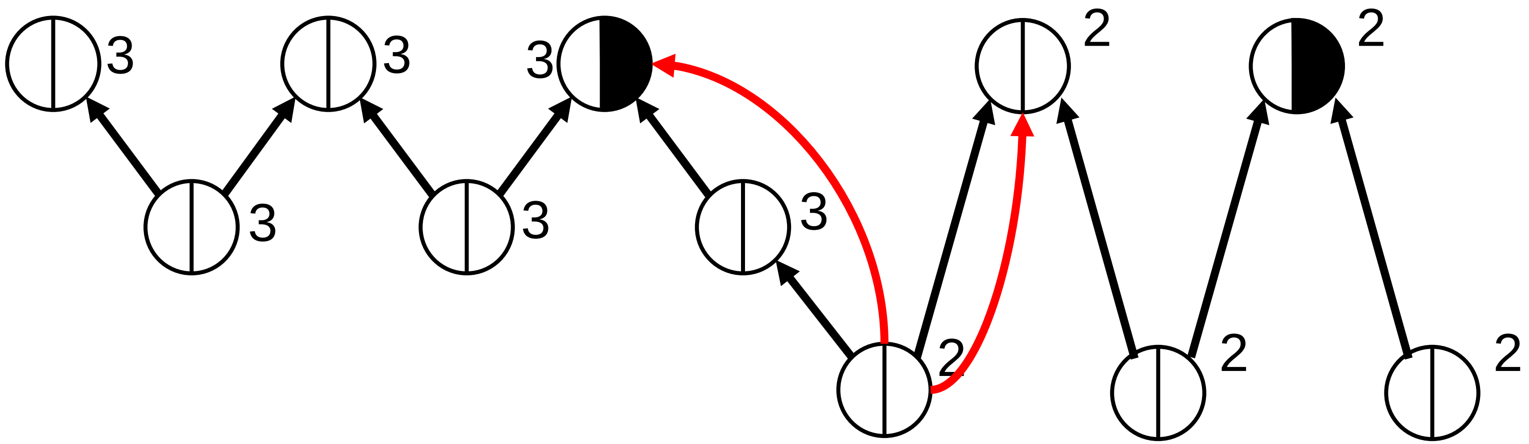

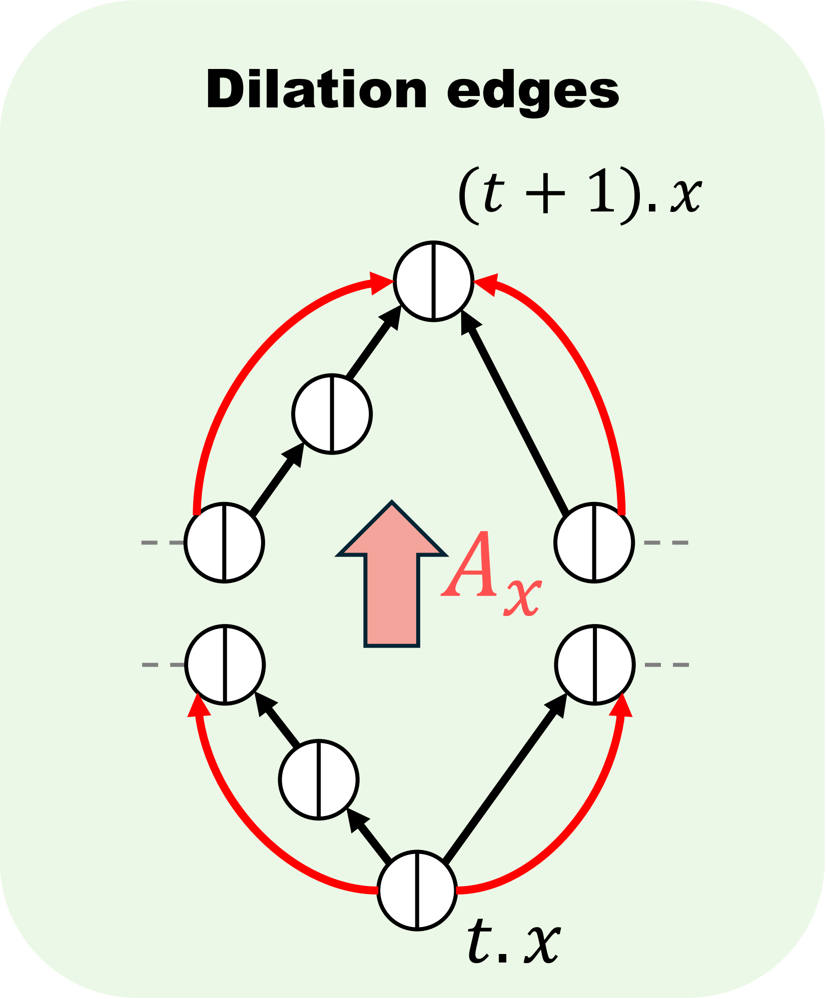

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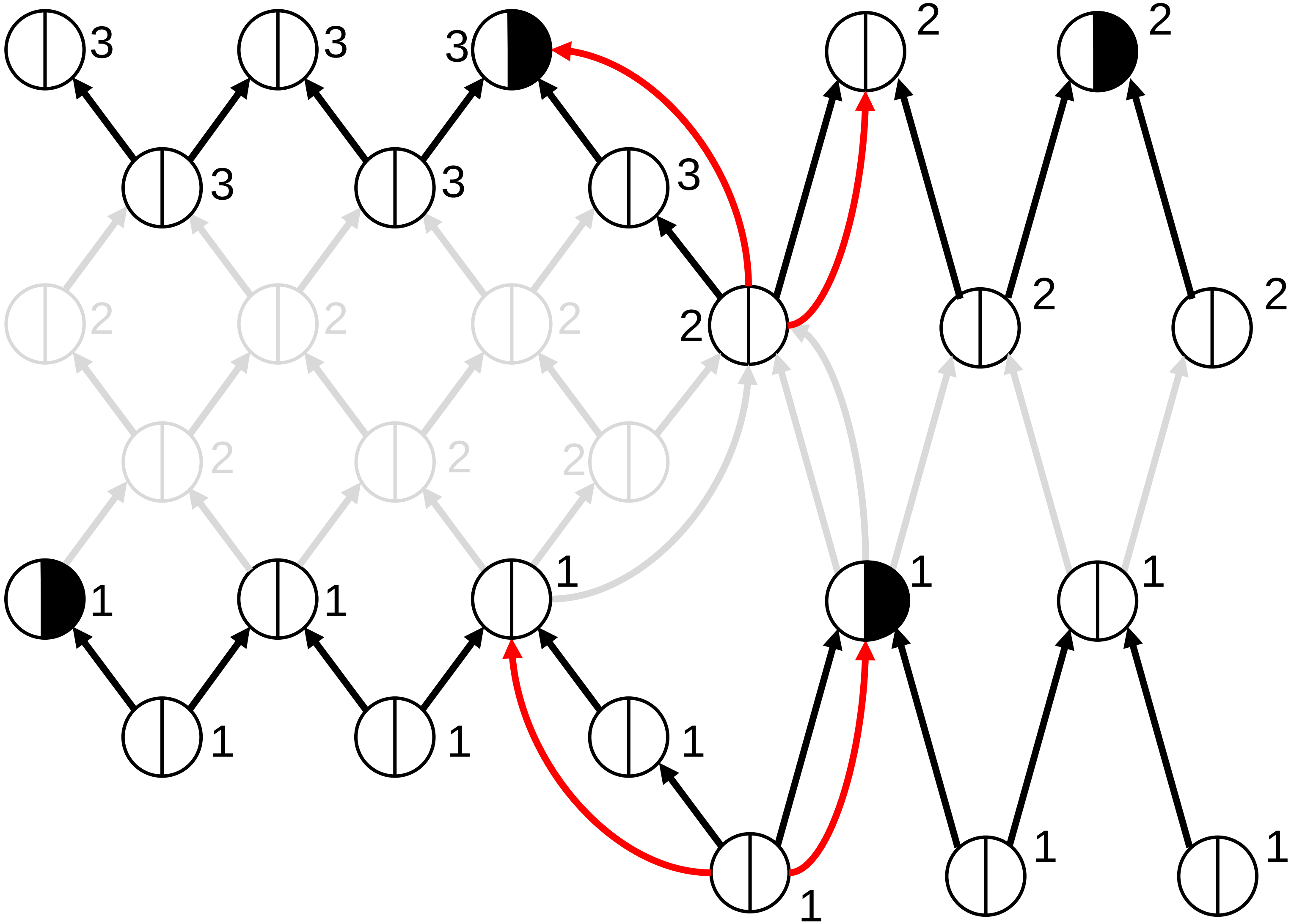
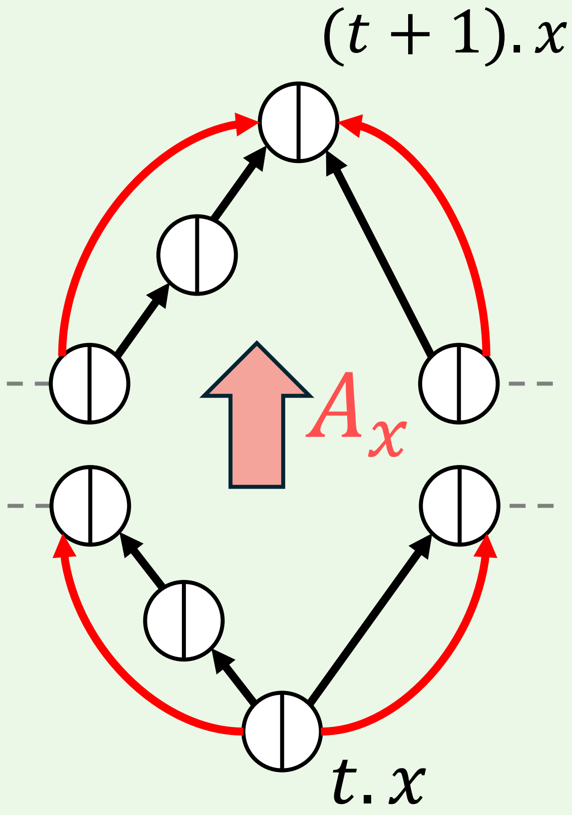
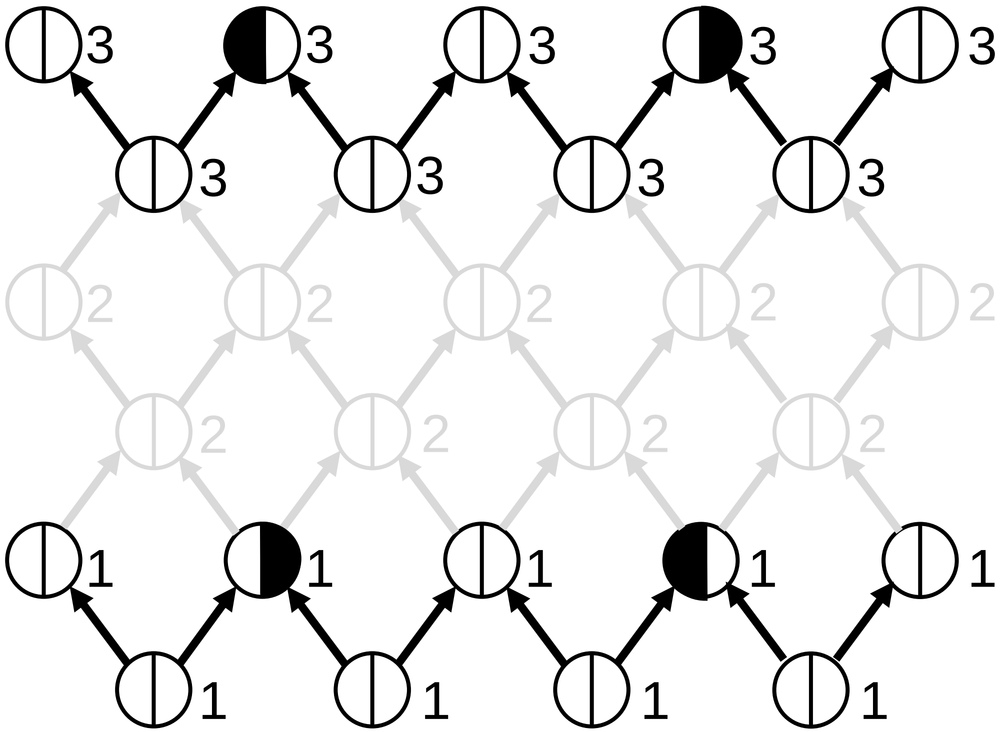


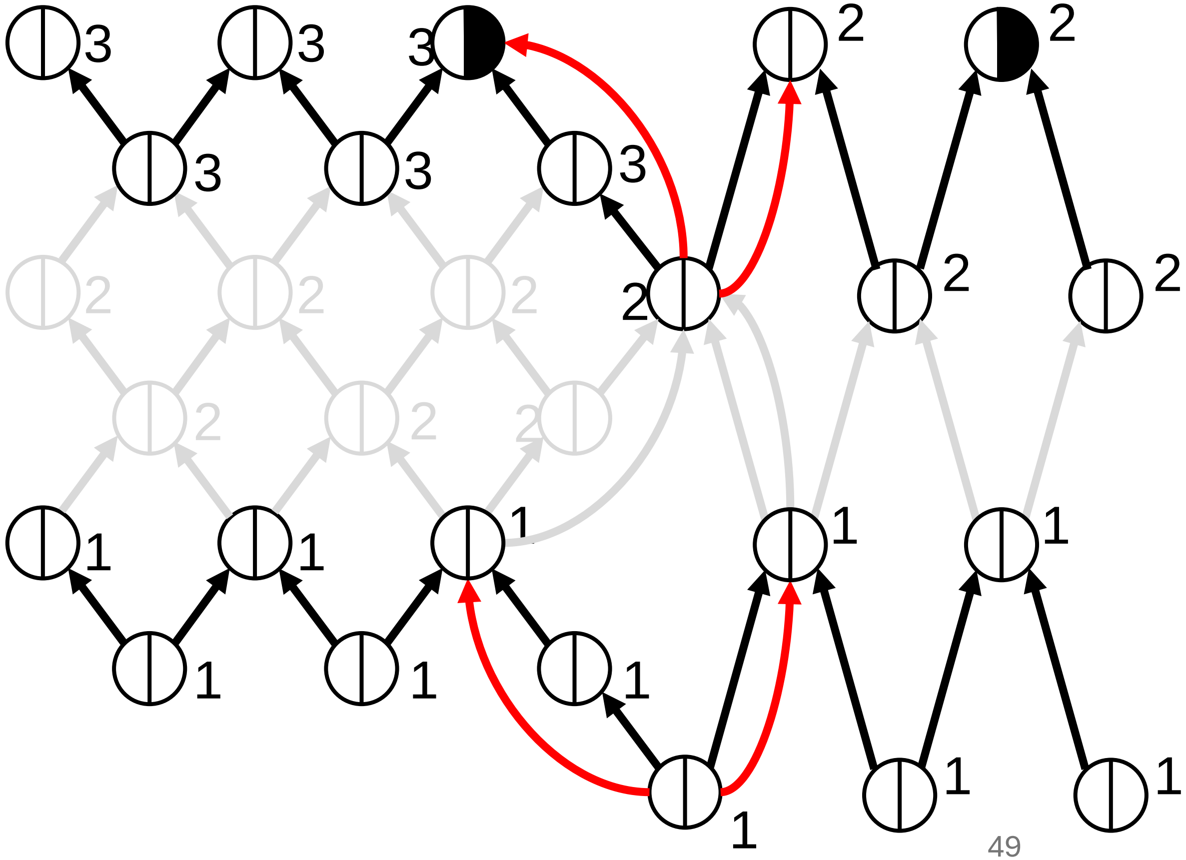


Summary

Using graph rewriting we **can simulate synchronous** dynamical system …



… but also represent some **intrinsically asynchronous** evolution.



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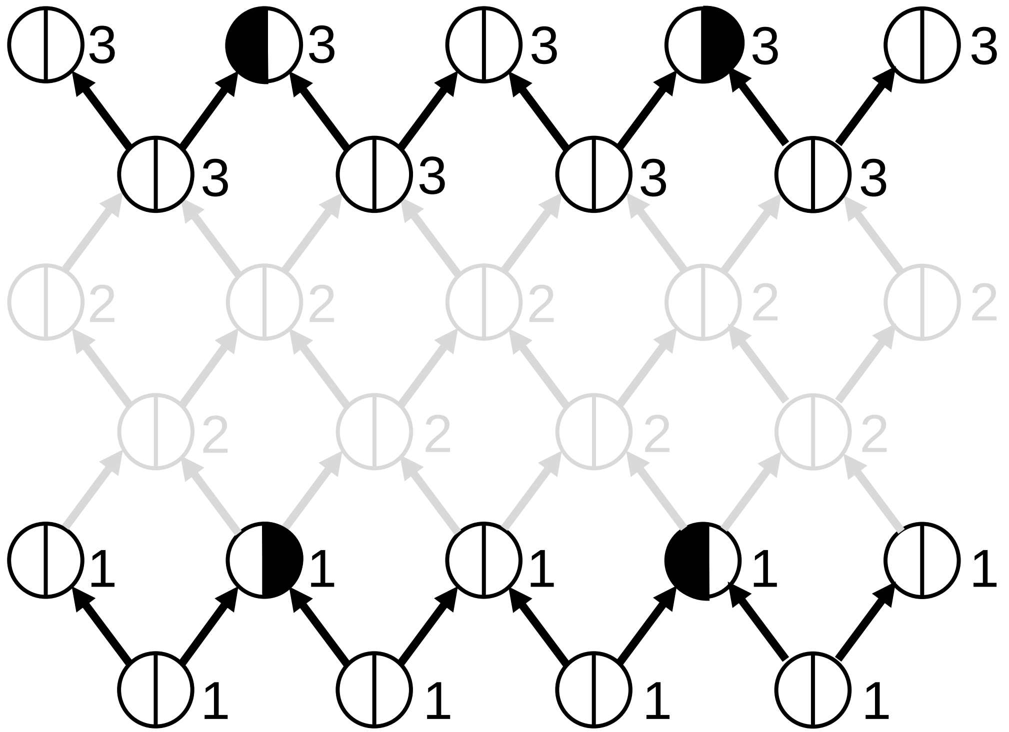

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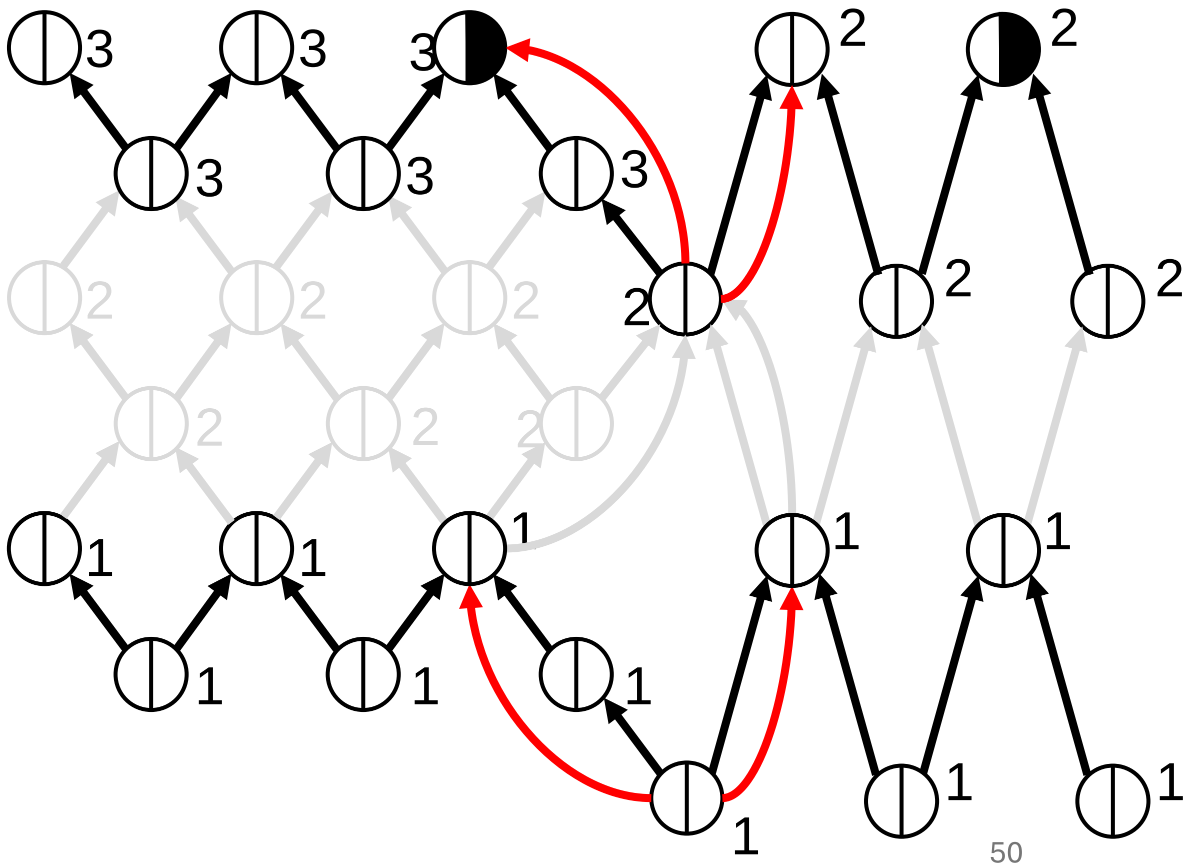
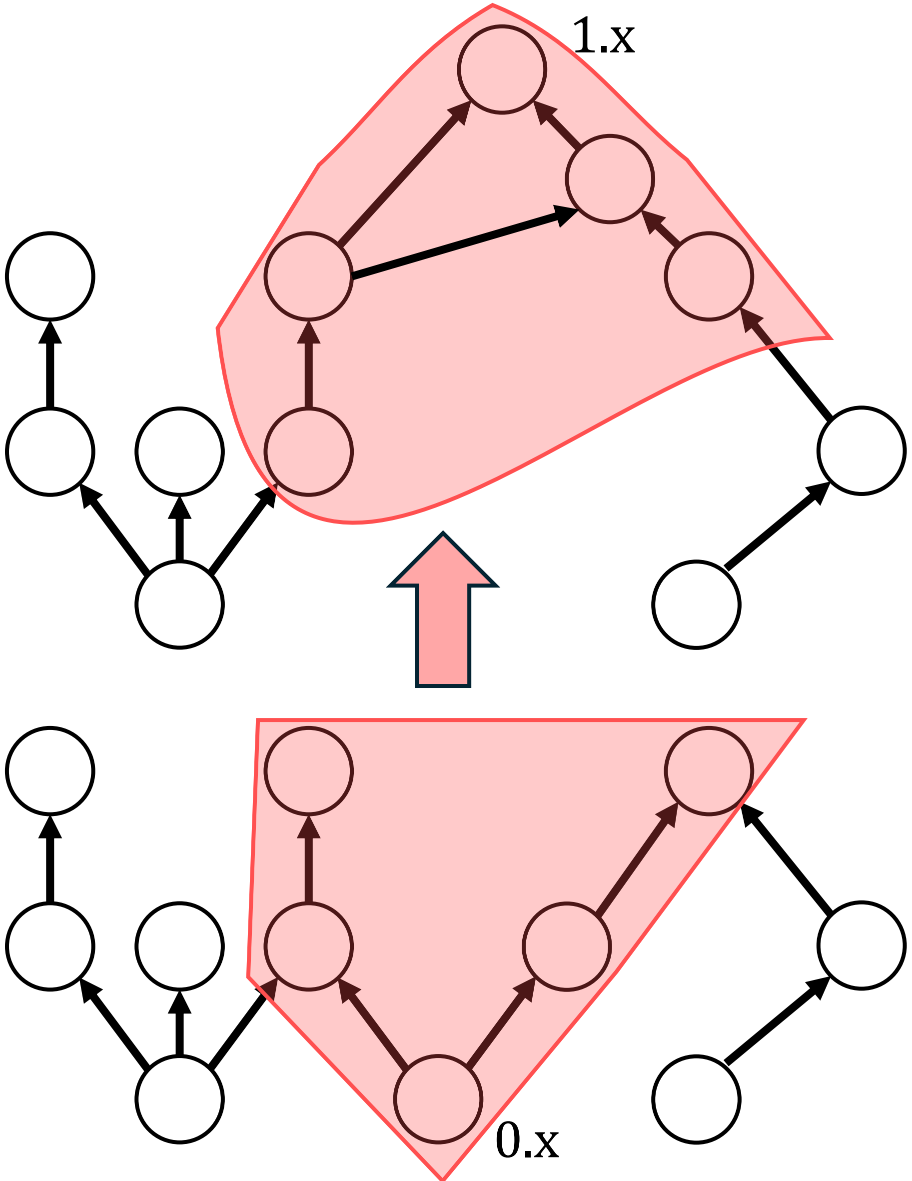


General framework



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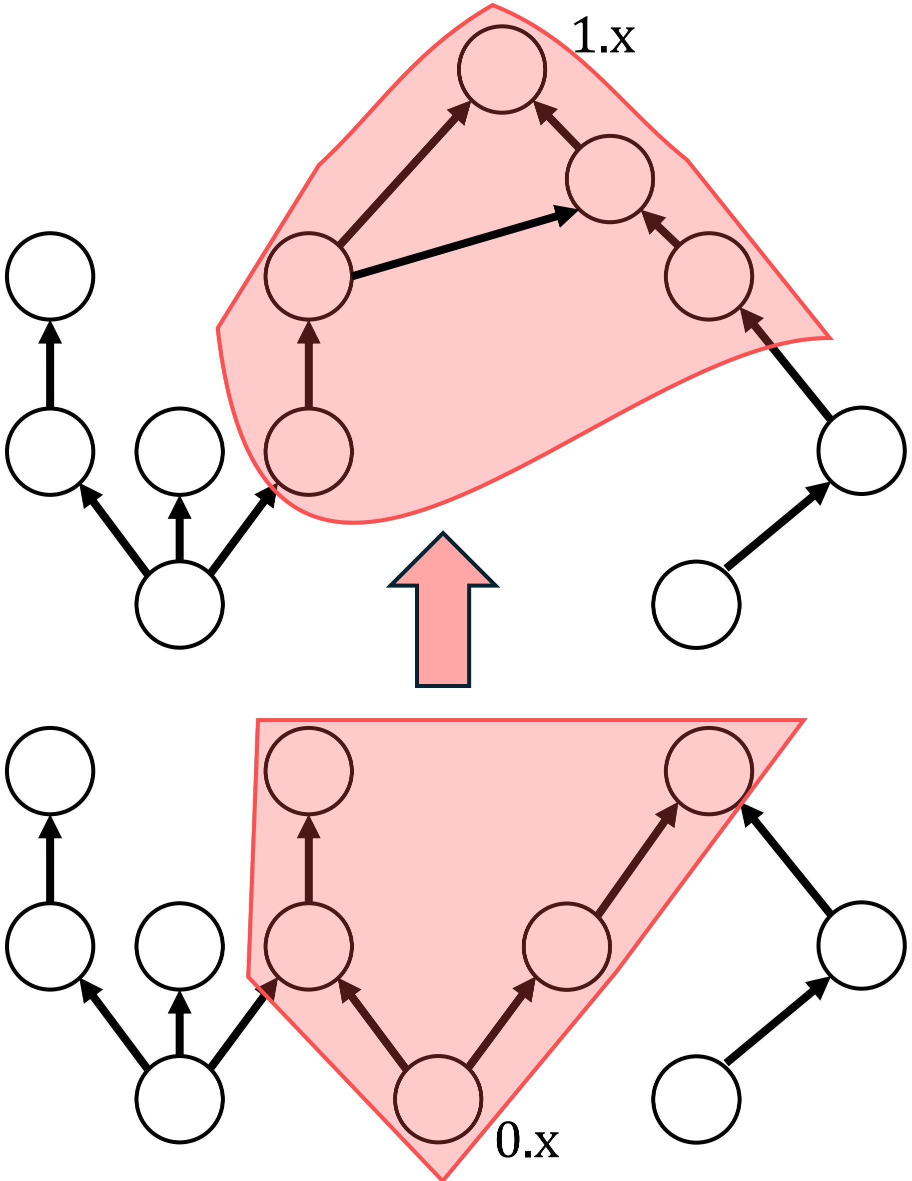
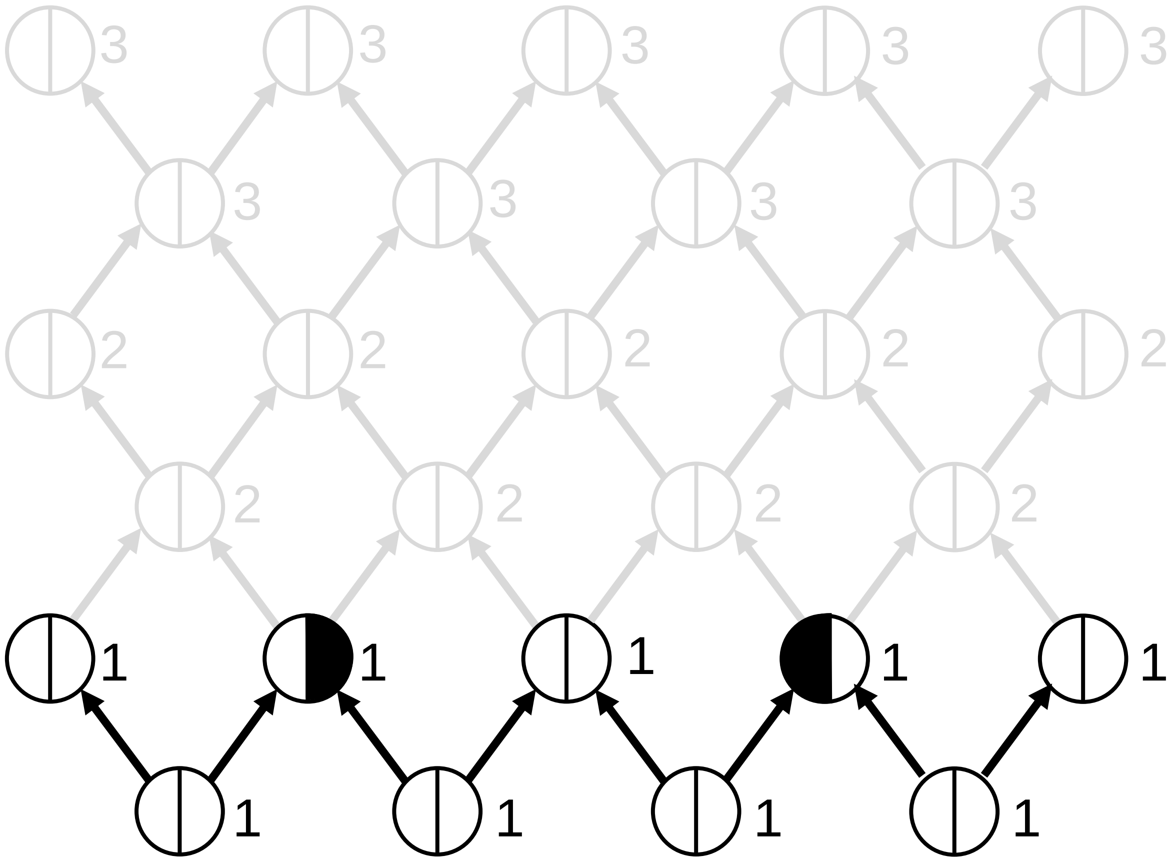


In general which local rewriting rules are physical ?

- **Determinism**
- Reversibility

A first attempt at defining determinism



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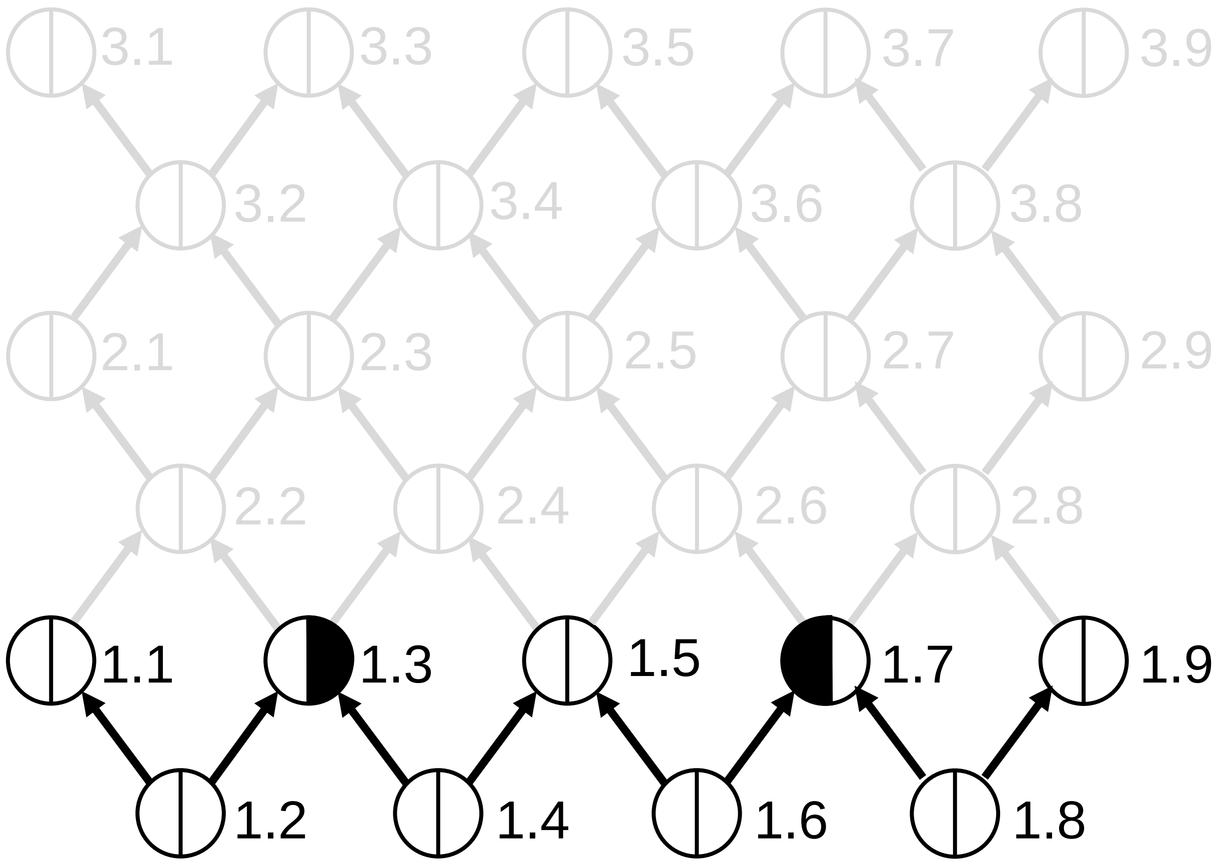

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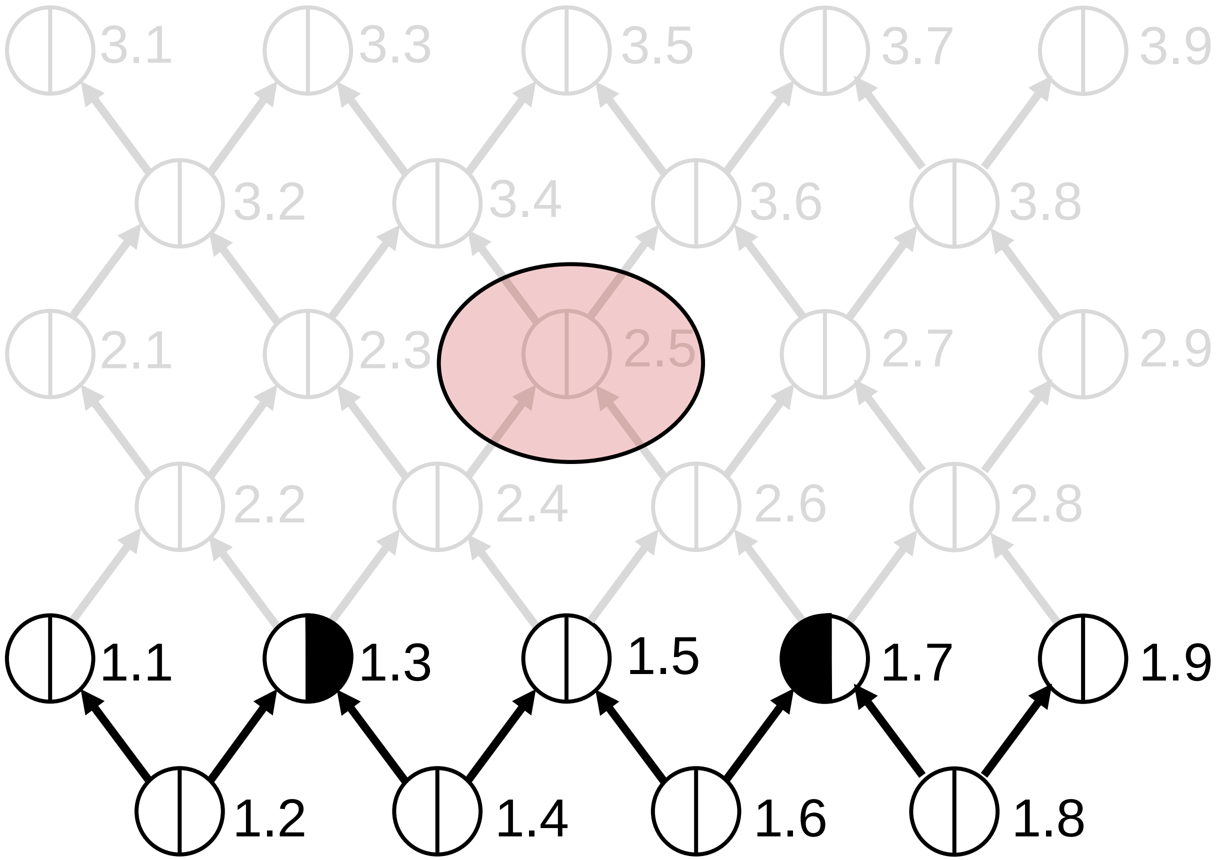

Determinism

The **state** of 2.5 is **always the same**. It does not depend on the rewriting strategy.

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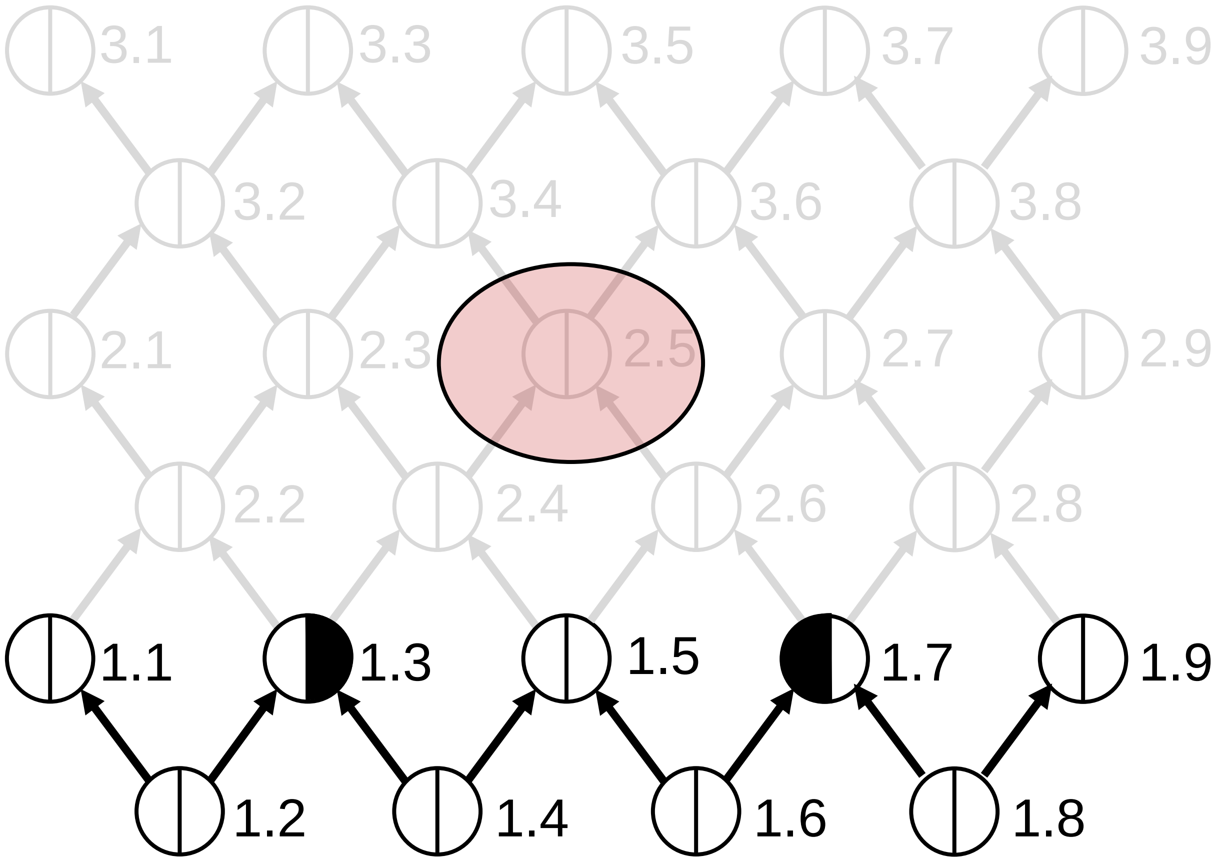


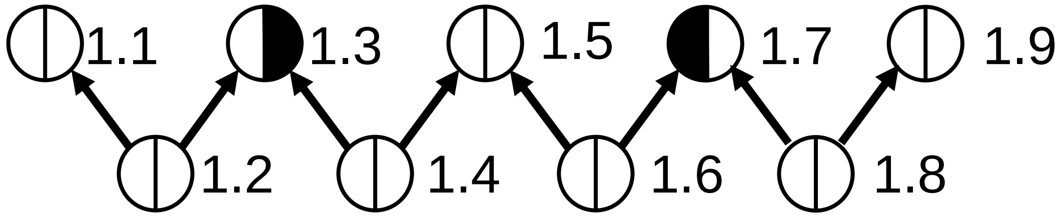
Determinism

The **state** of 2.5 is **always the same**. It does not depend on the rewriting strategy.

It does depend on the local shape of the cut !

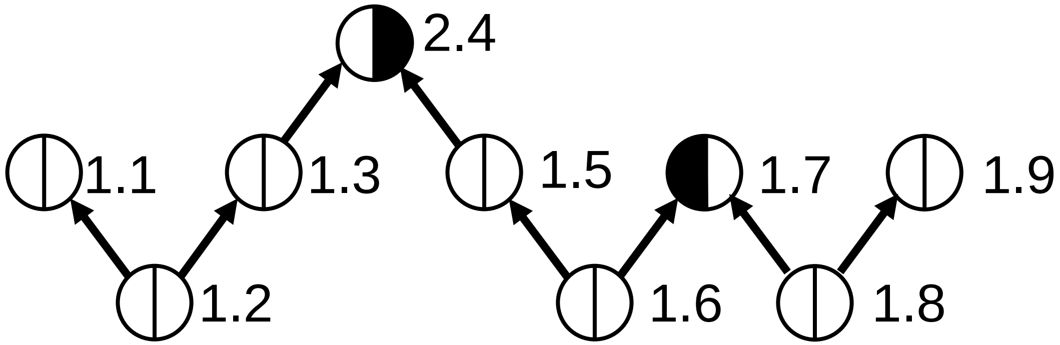
States depend on the cut

G



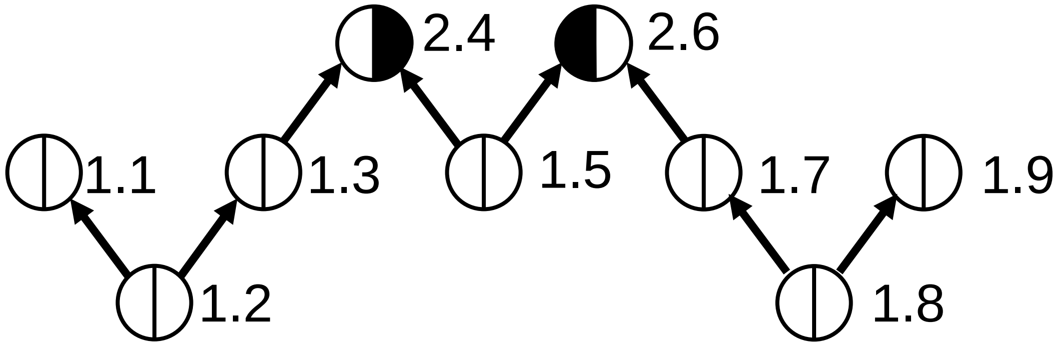
States depend on the cut

$A_4 G$



States depend on the cut

$A_6 A_4 G$



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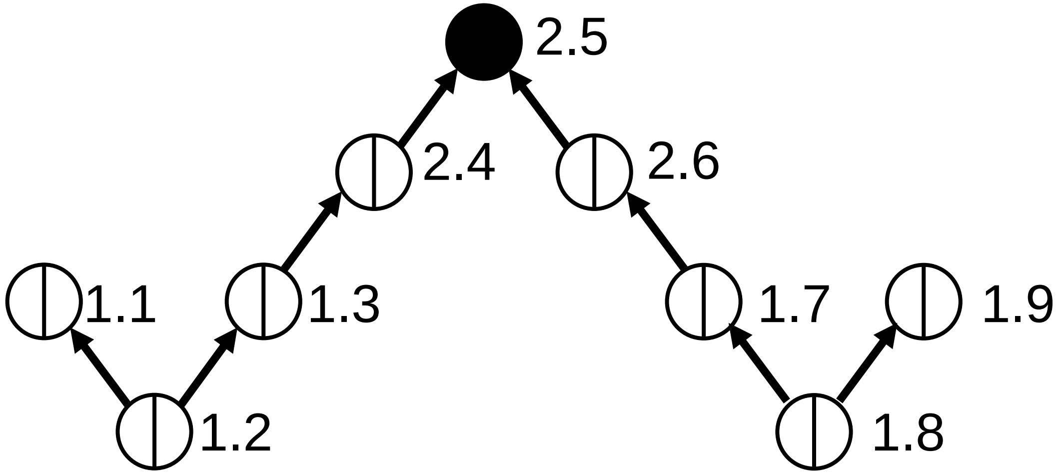

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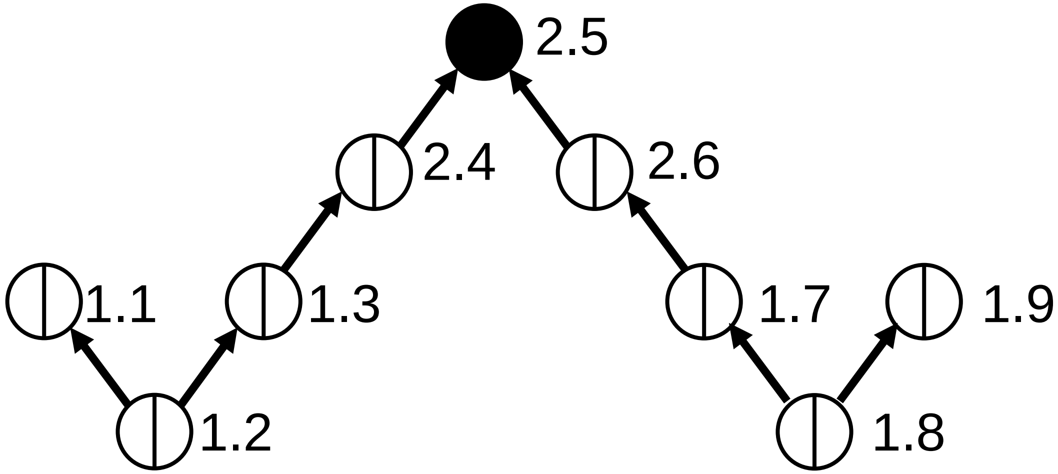

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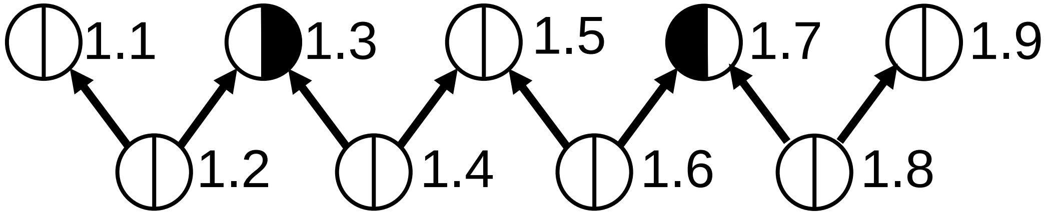

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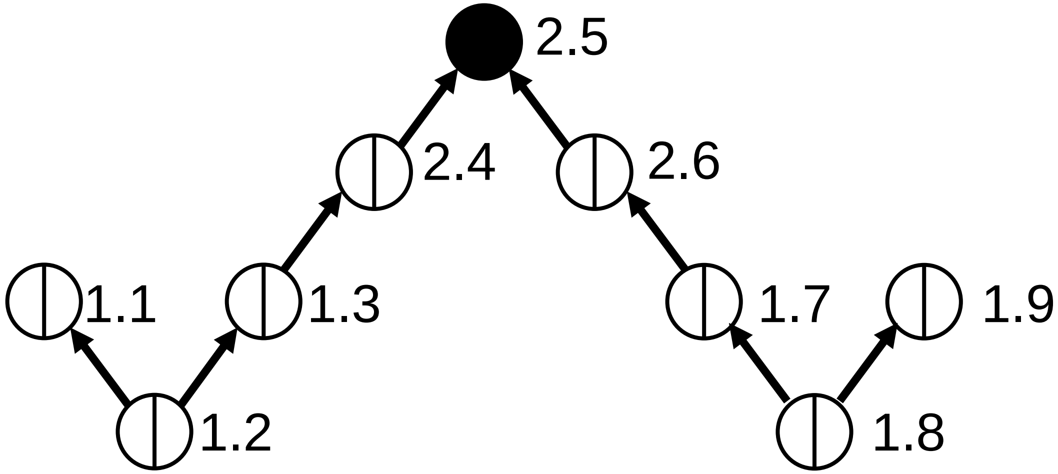

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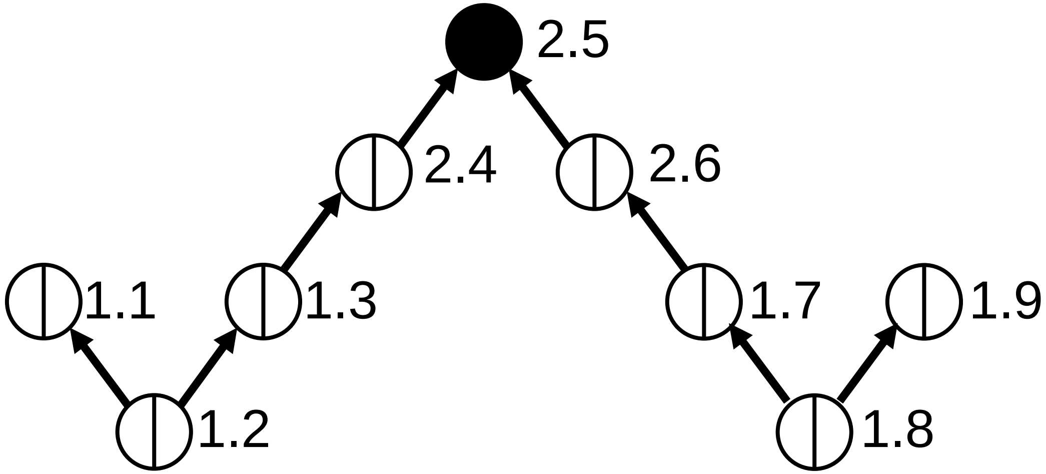

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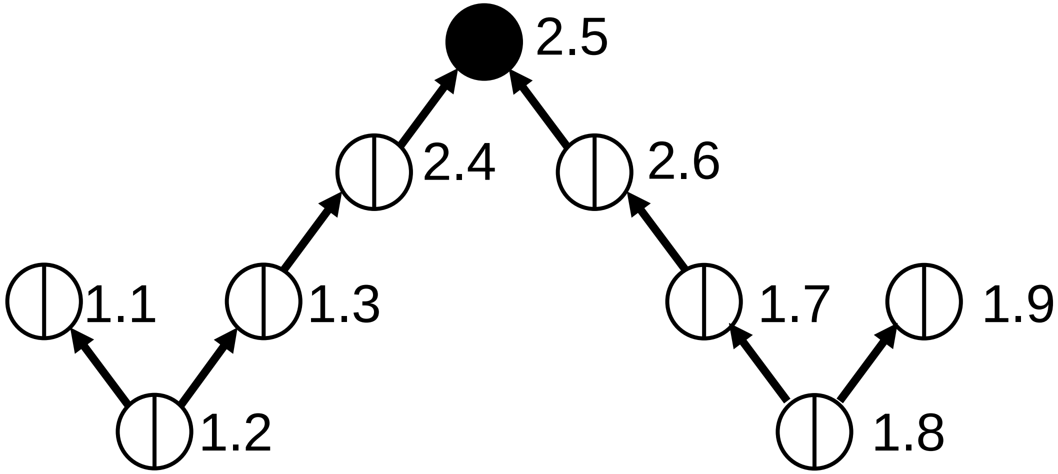
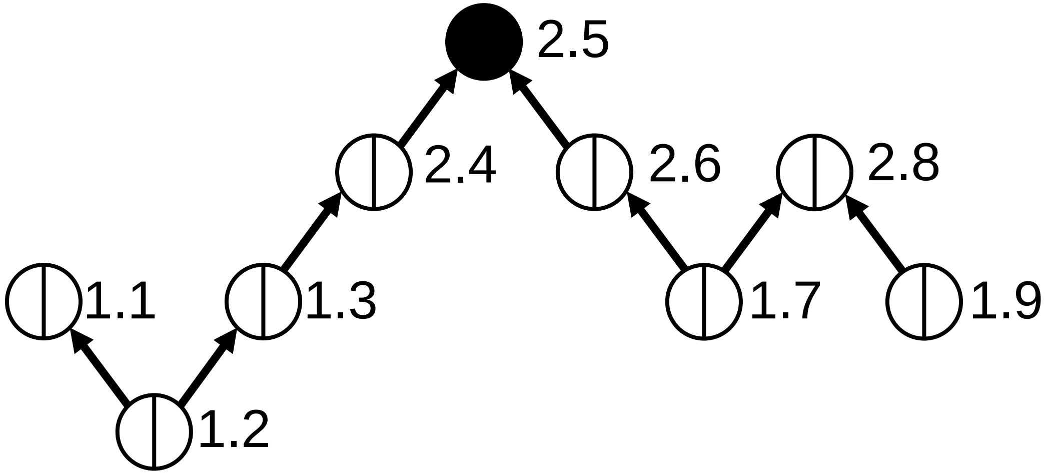


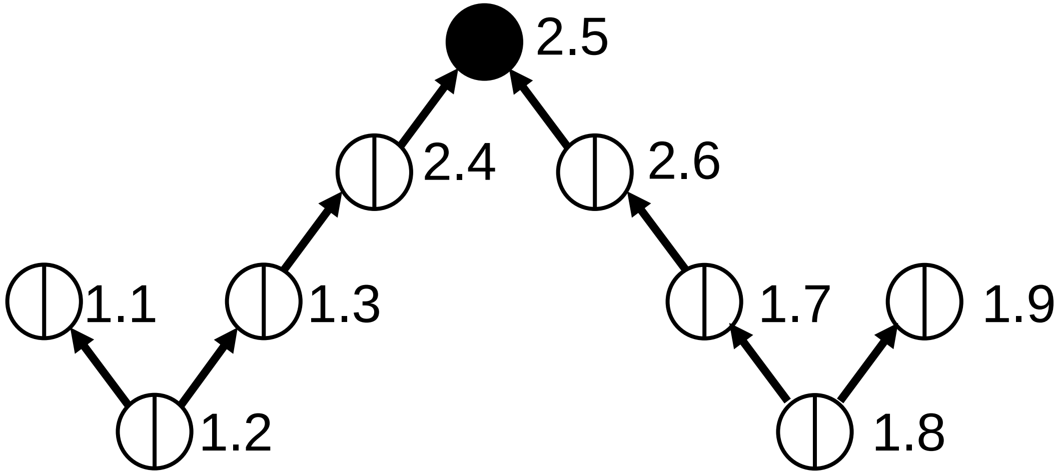
$A_8 A_5 A_6 A_4 G$

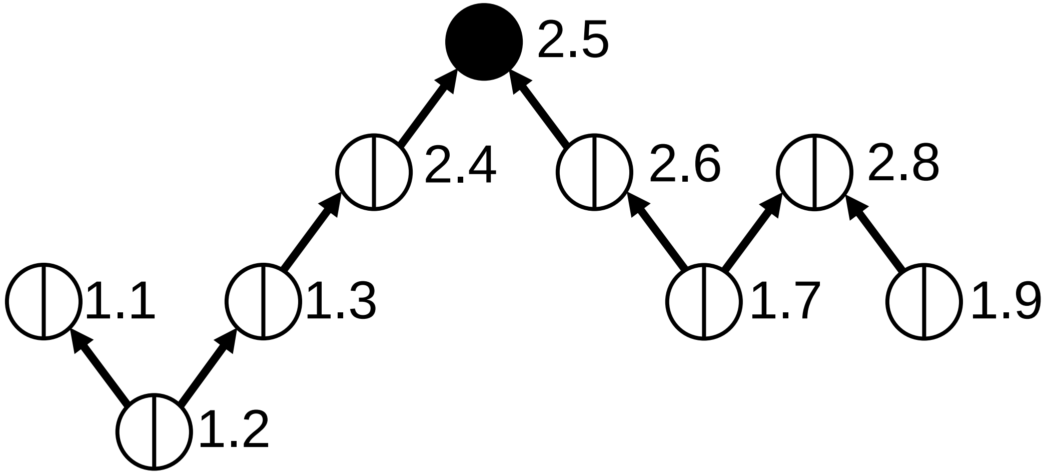


States depend on the cut

$A_5 A_6 A_4 G$

$A_{8564} G$





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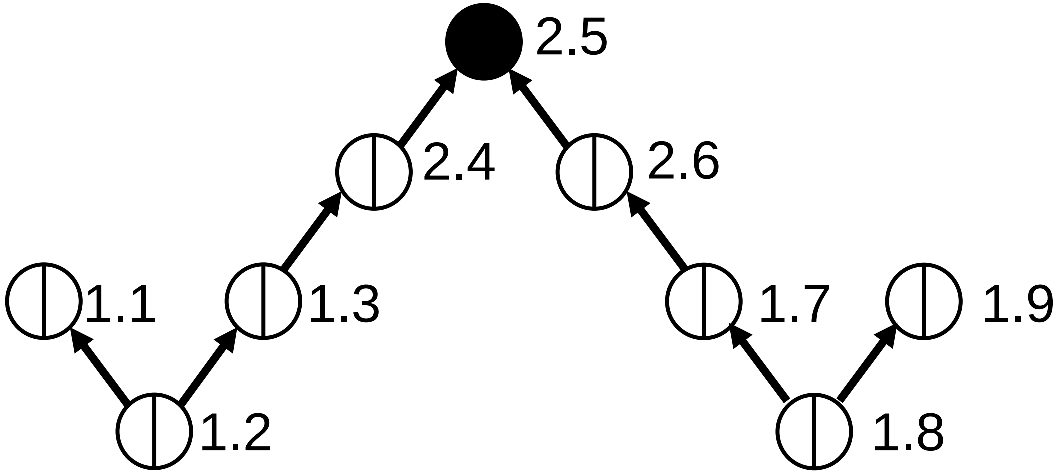

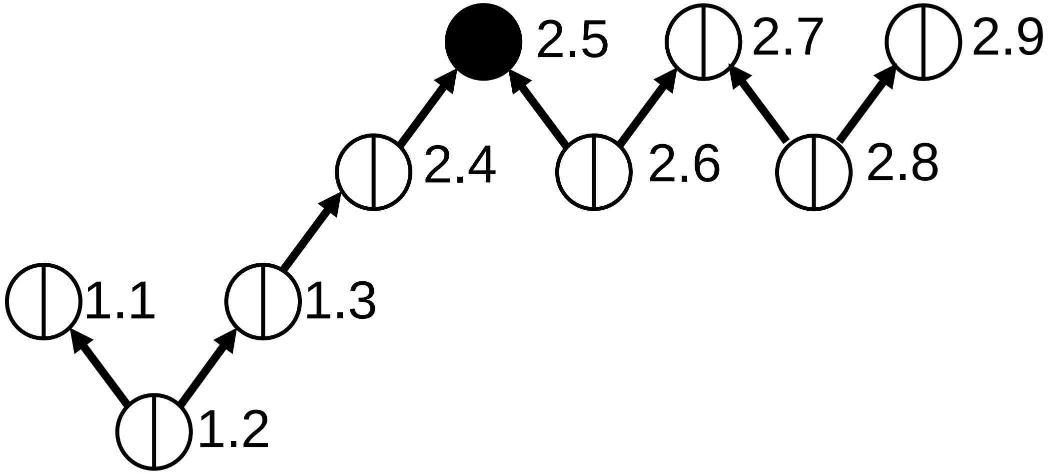

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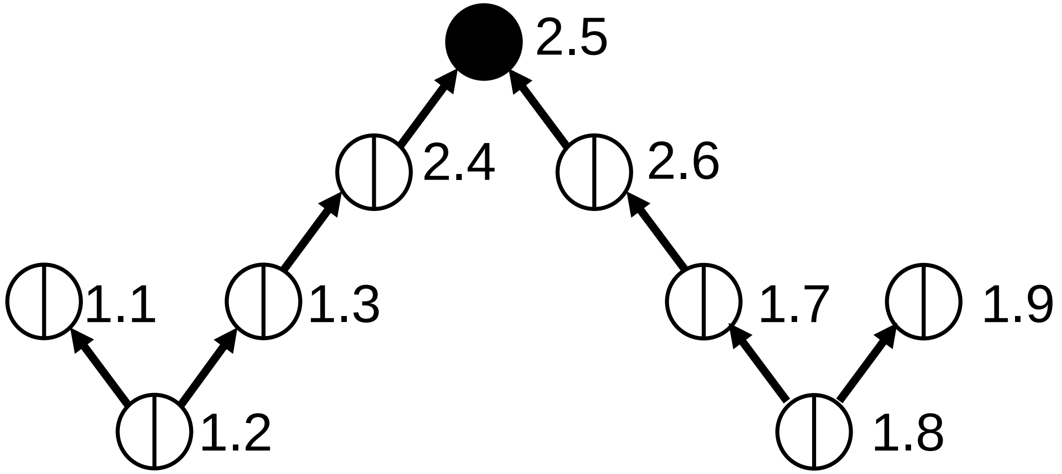

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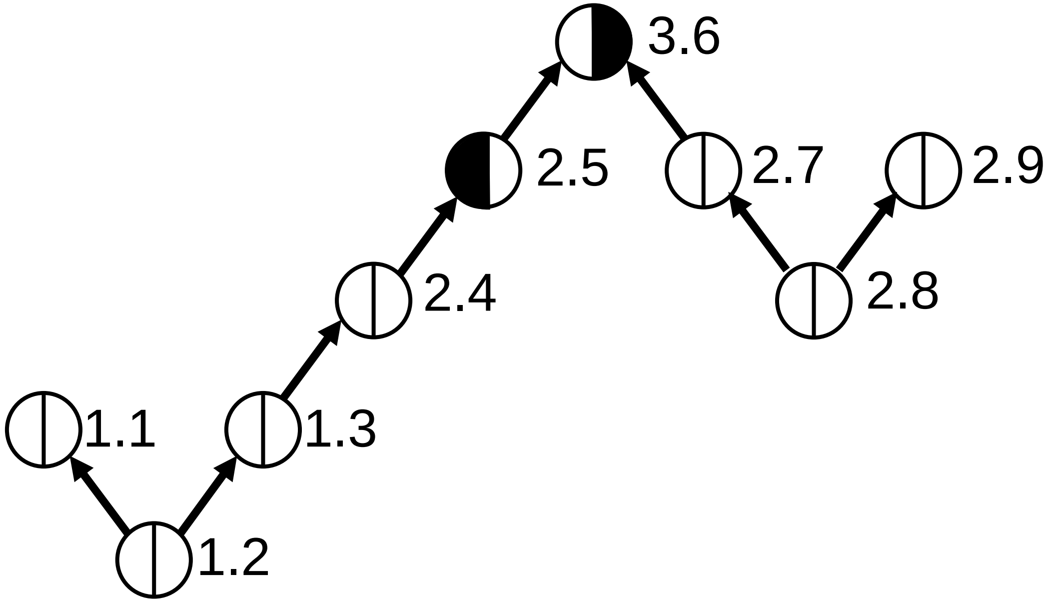

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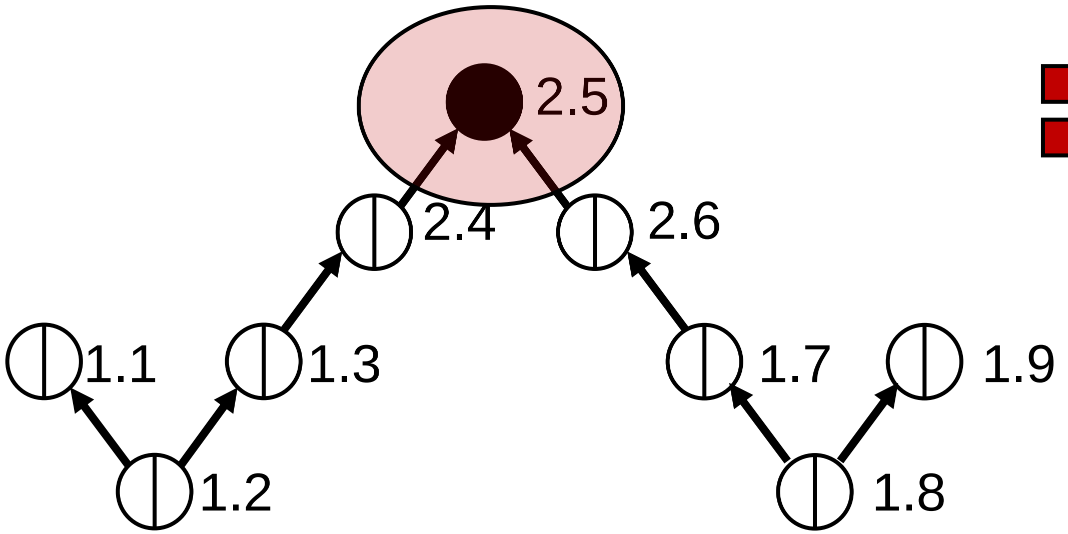

$\neq$

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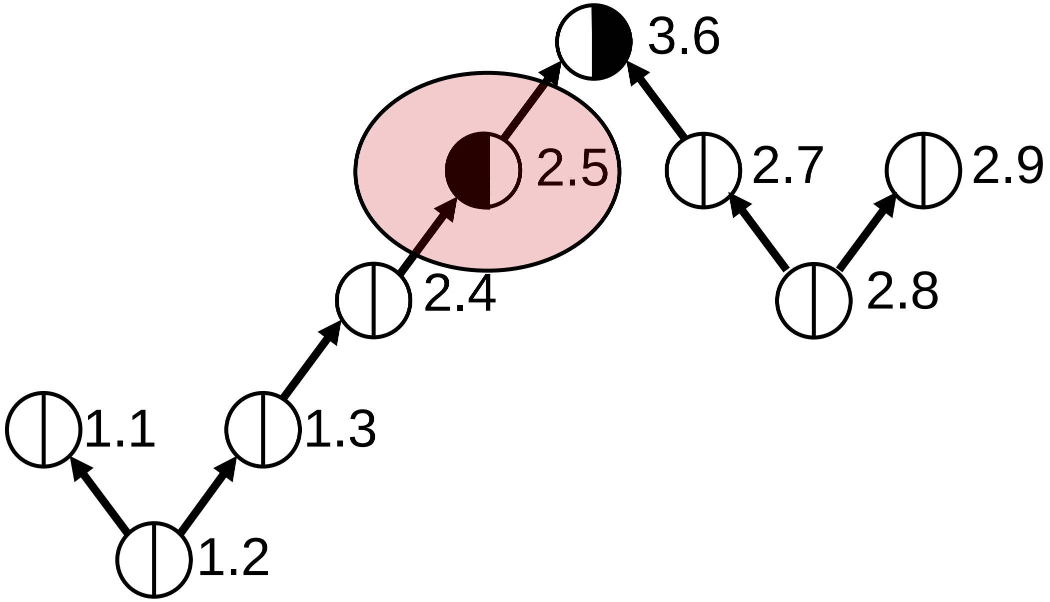
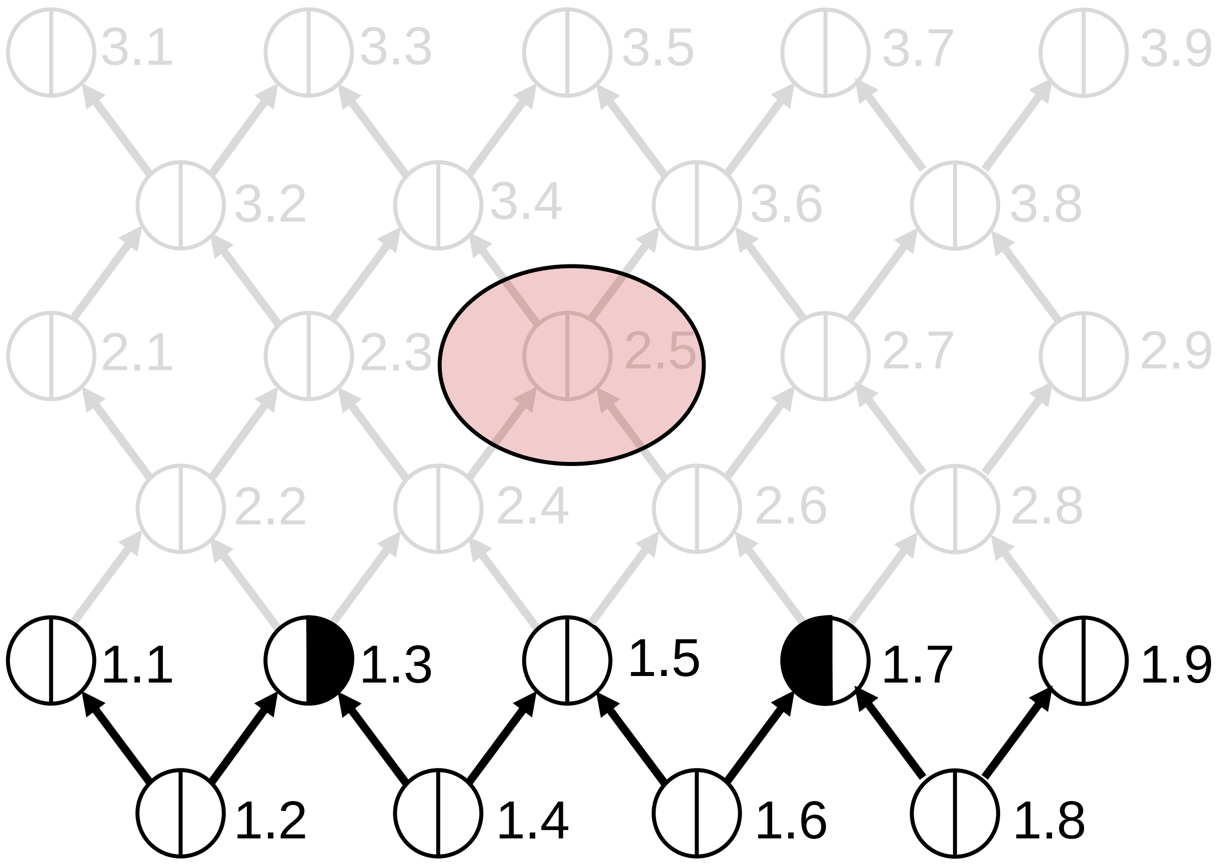


Space-time Determinism



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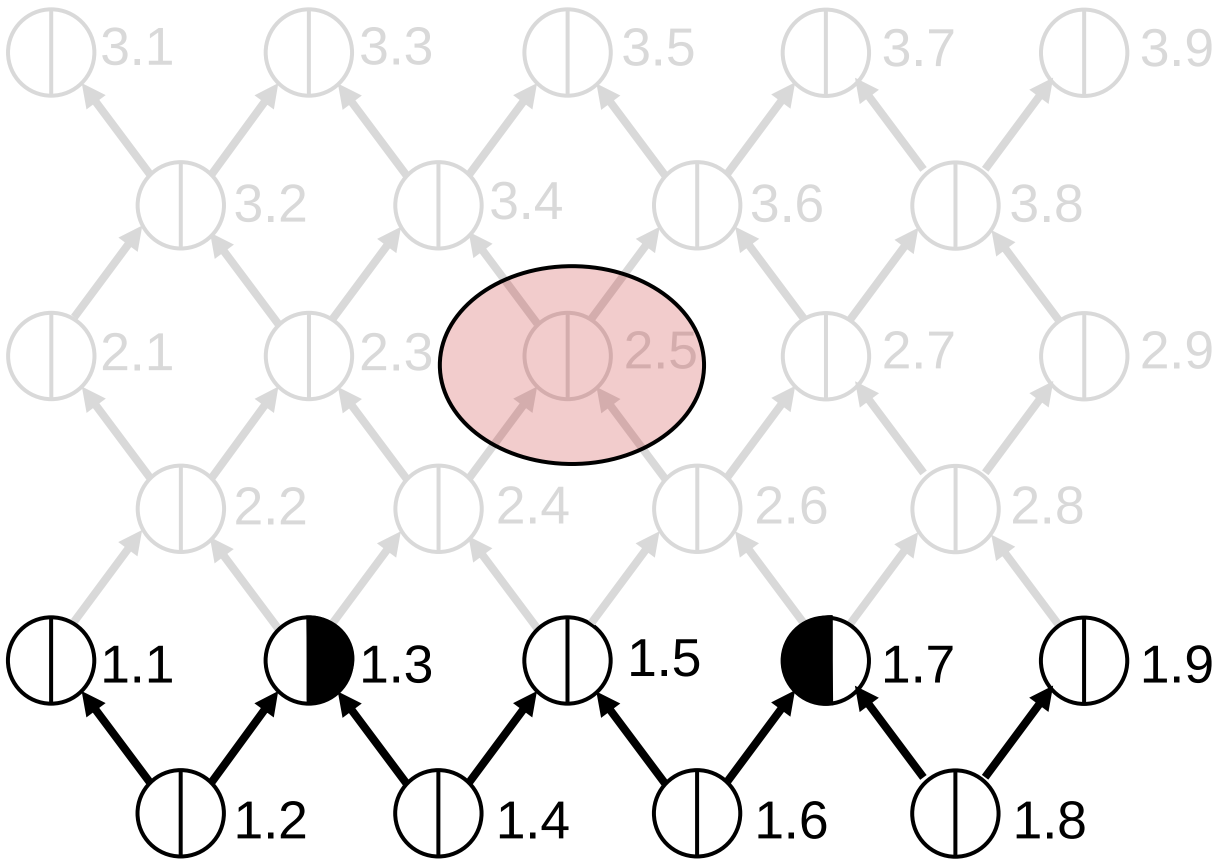

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Same incoming edges ⇒ Same internal state and outgoing edges

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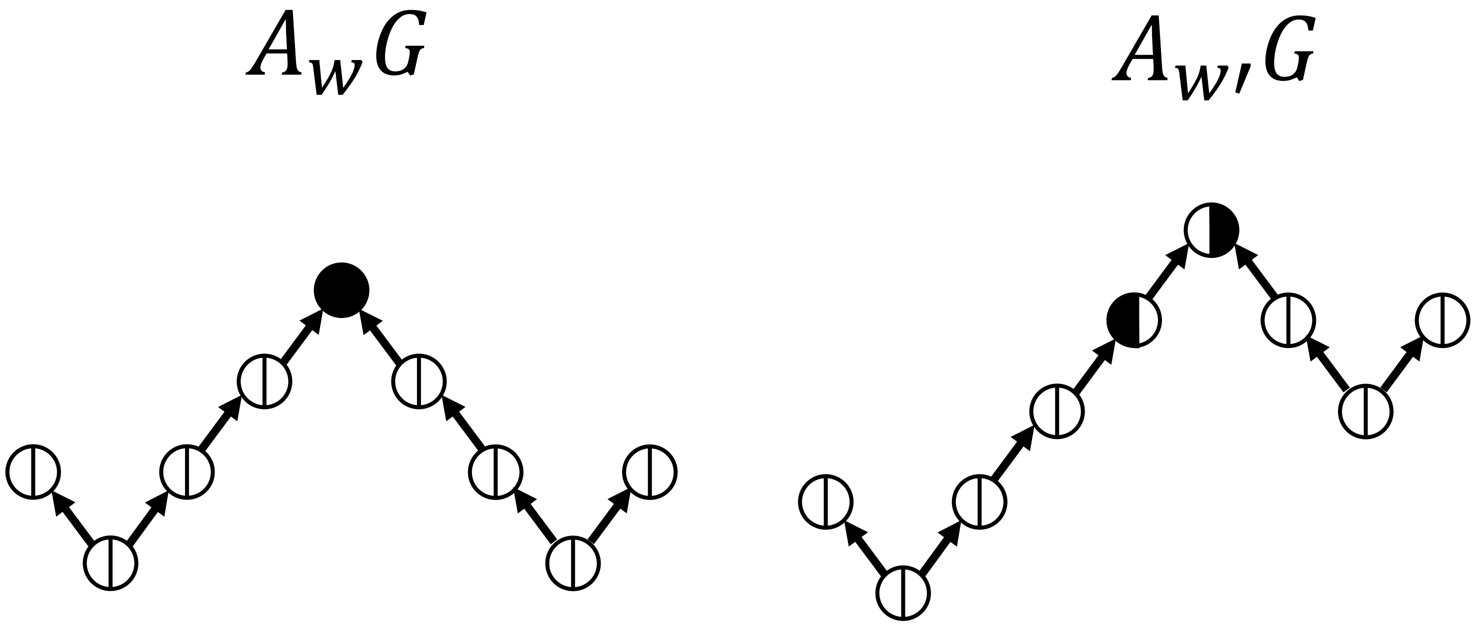

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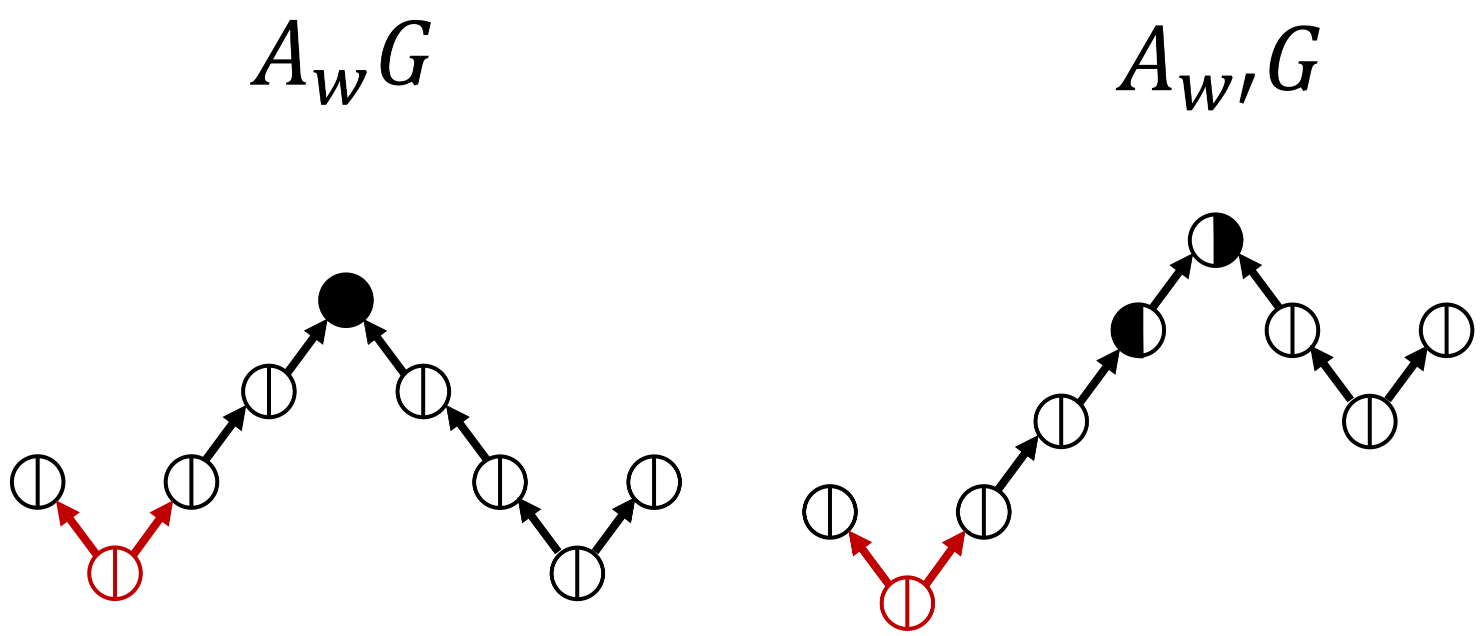

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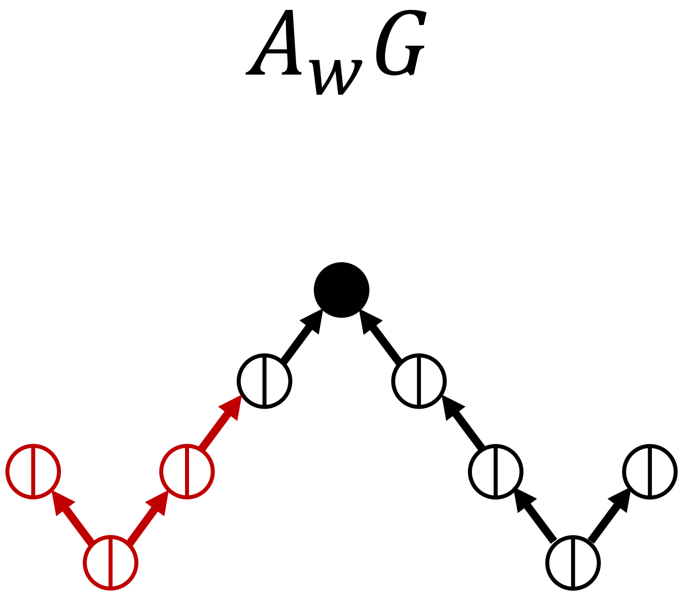

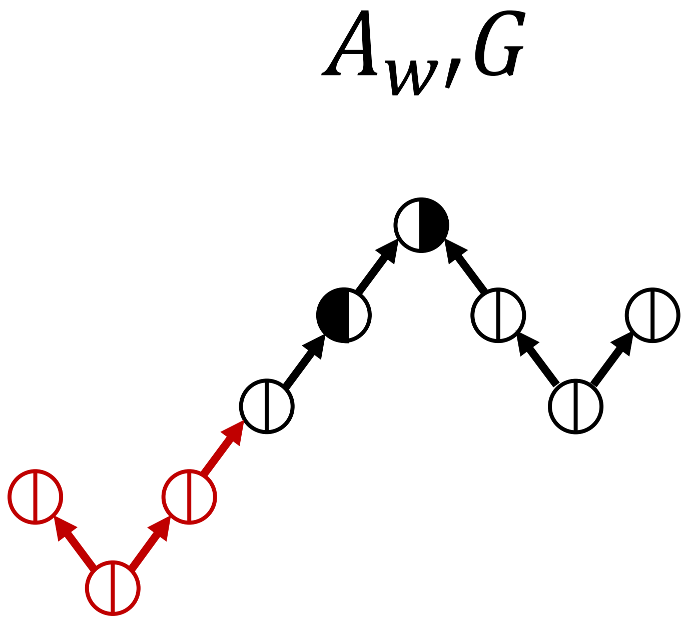

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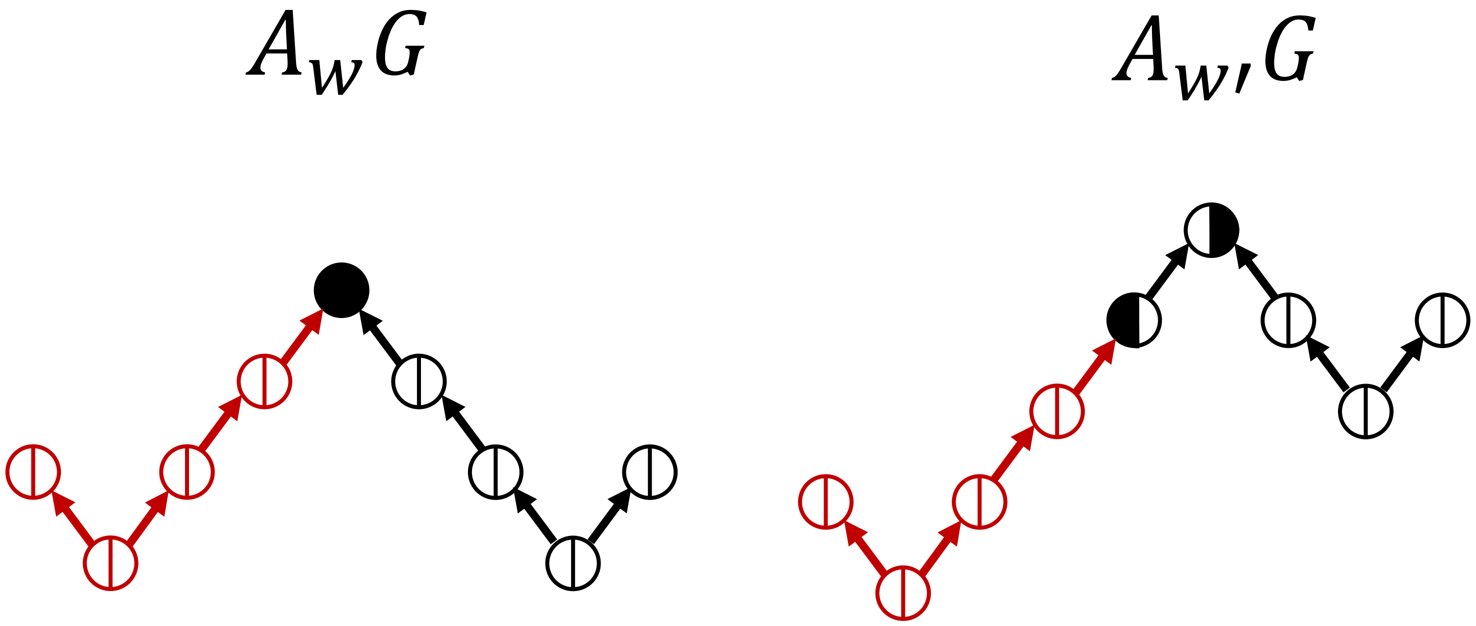

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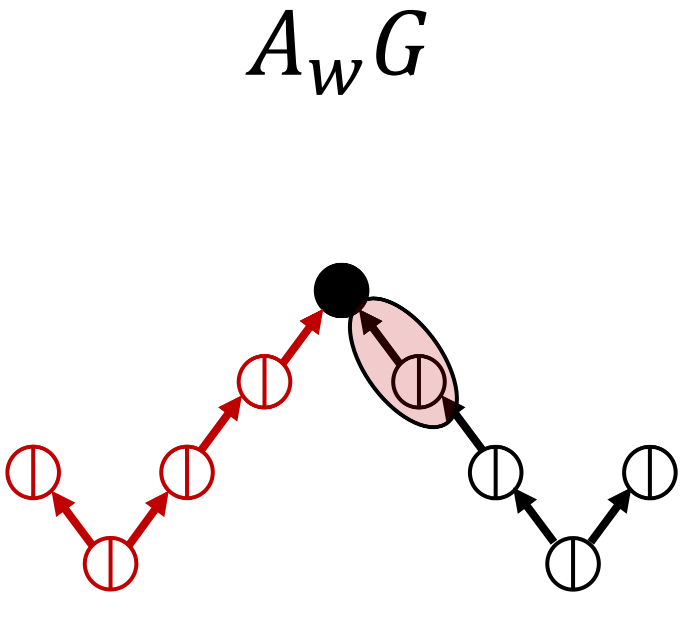

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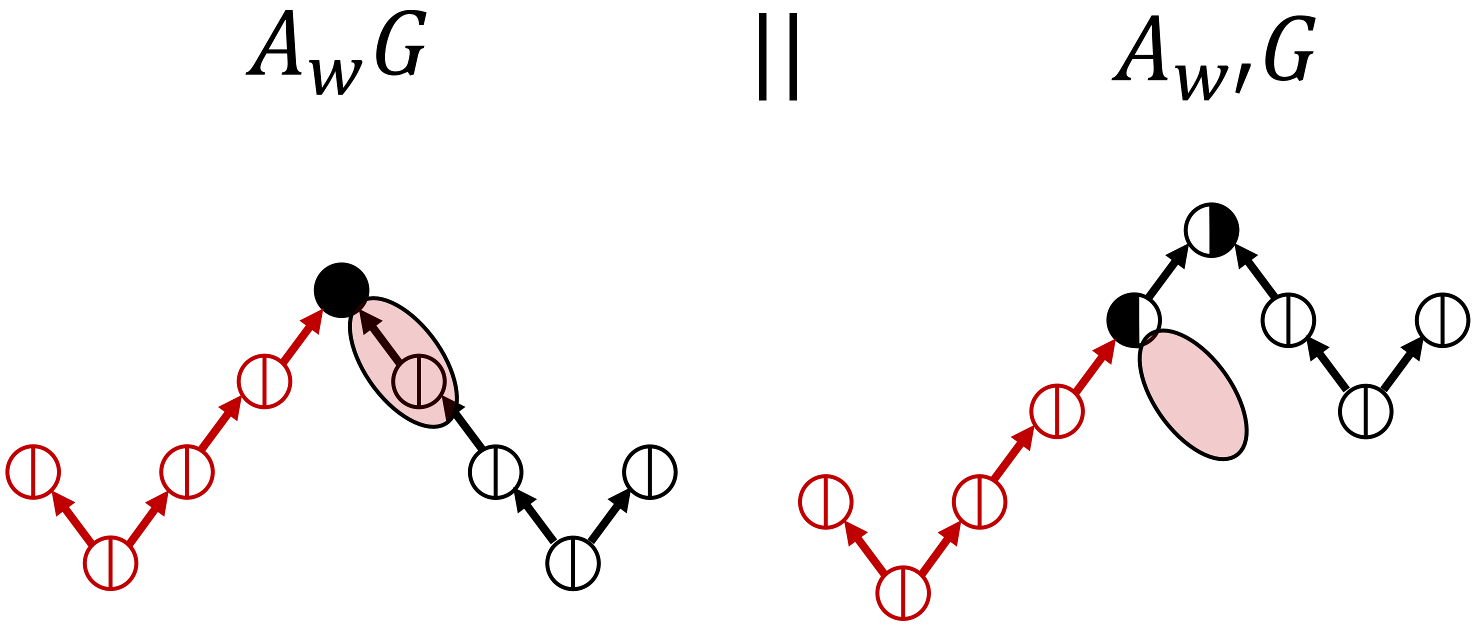

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Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_w$, which means :

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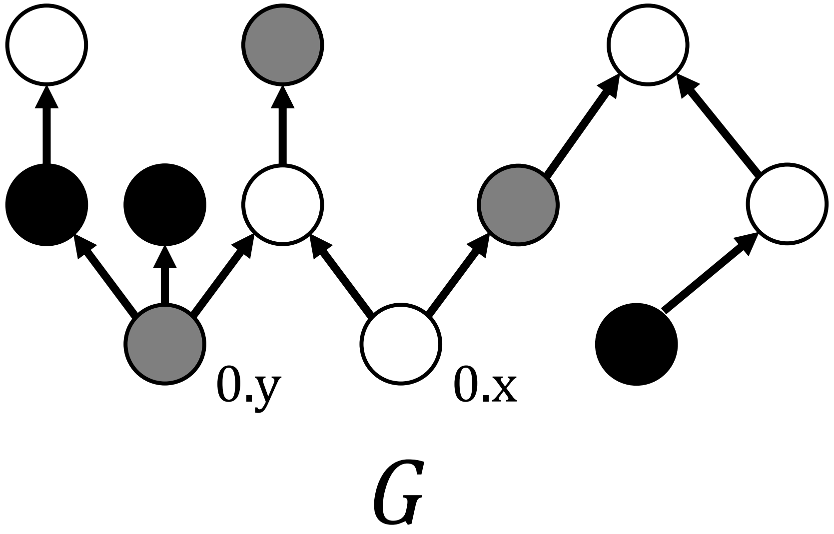
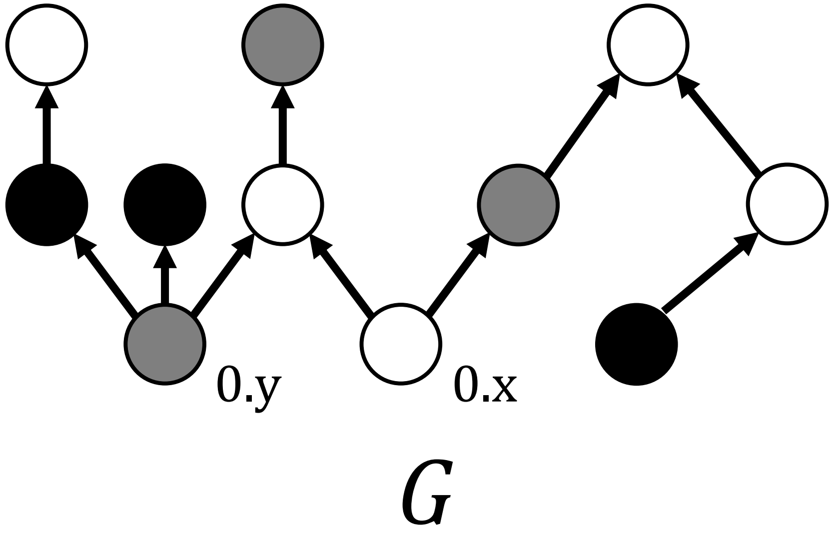
Goal : prove A deterministic, i.e. we always have $A_w||A_w$, which means :

Same incomming edges ⇒ Same internal state and outgoing edges





Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$

Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_w$, which means :

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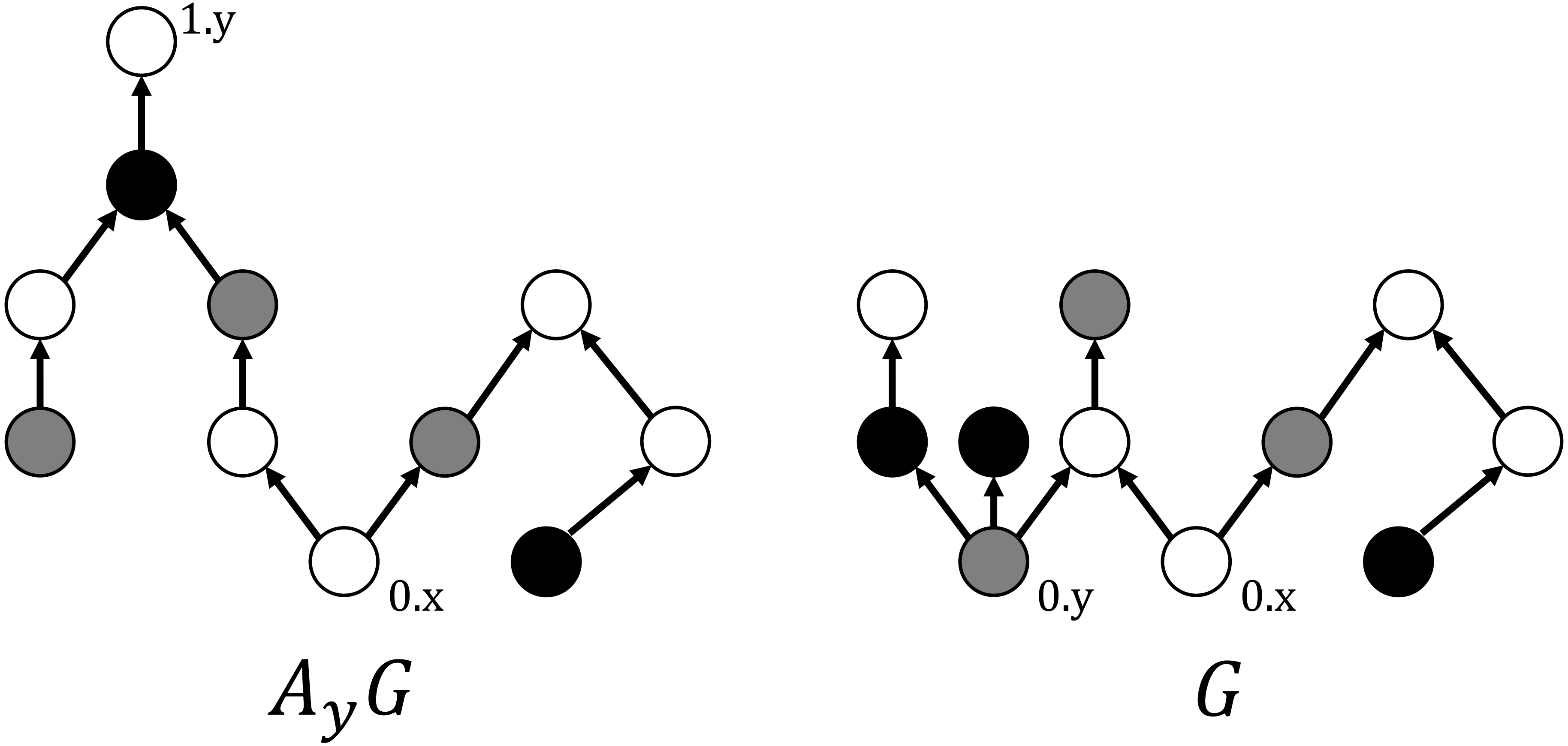

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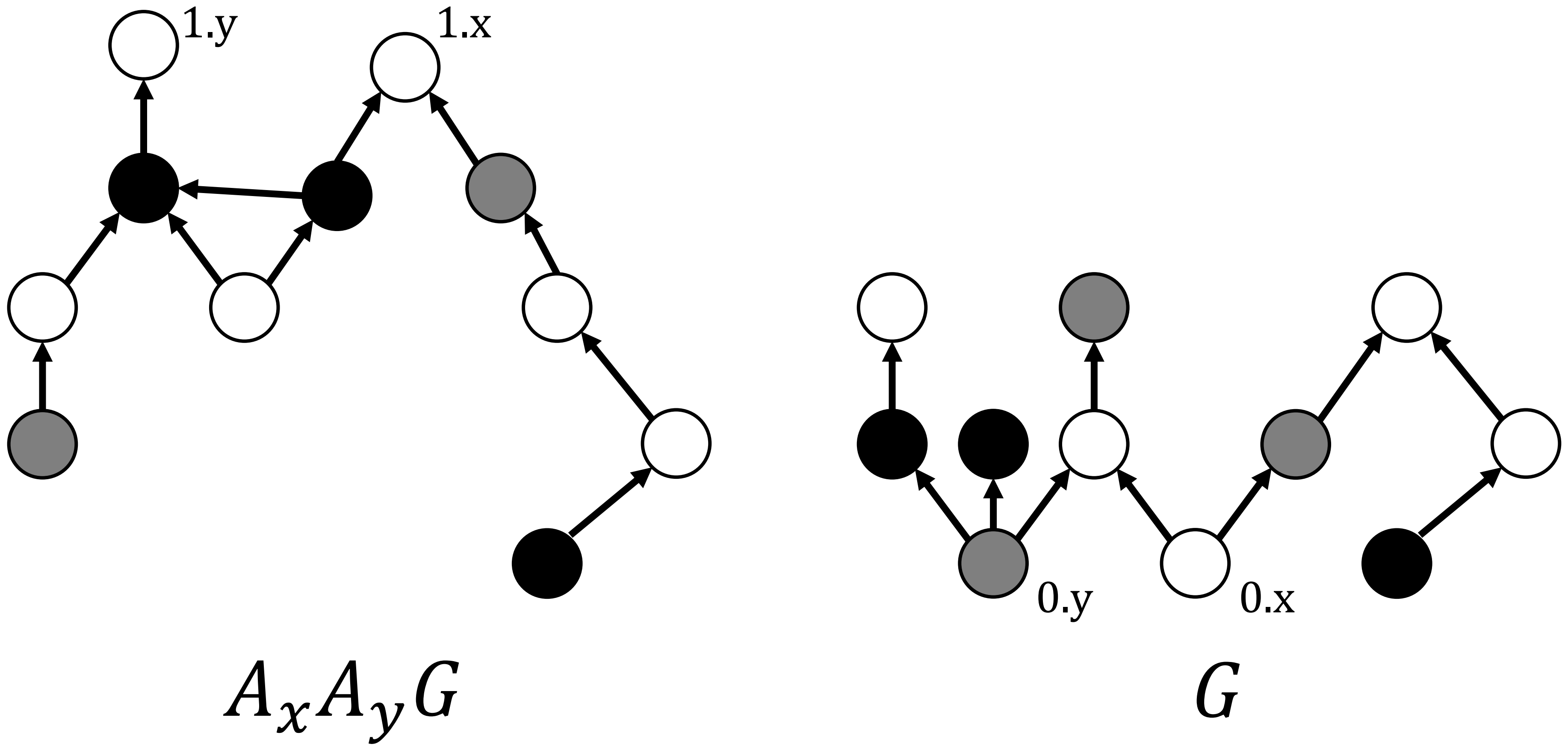

Hypothesis

1. Commutative
   $A_xA_yG = A_yA_xG$

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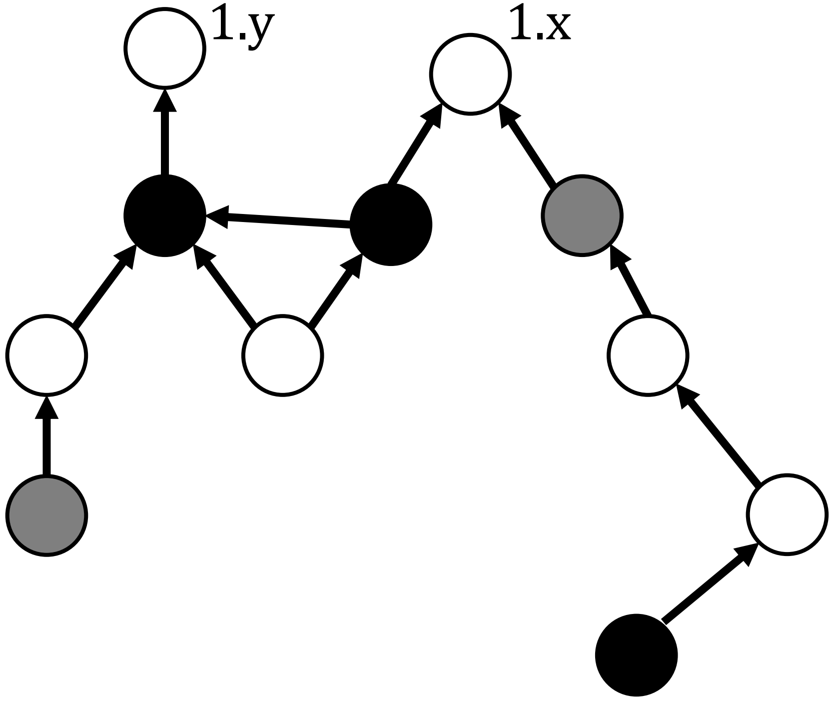
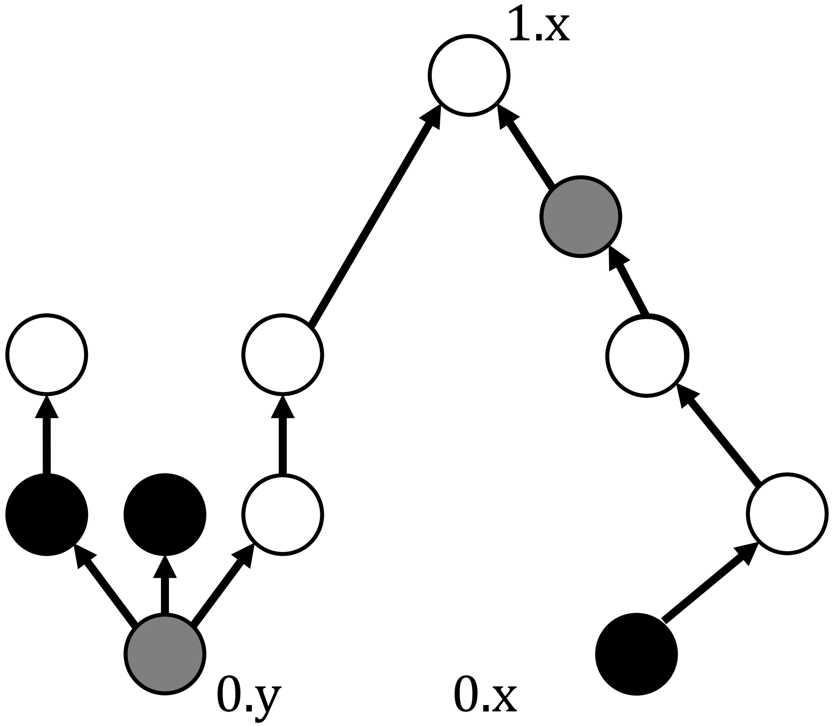


$A_x A_y G$



$A_x G$

Hypothesis

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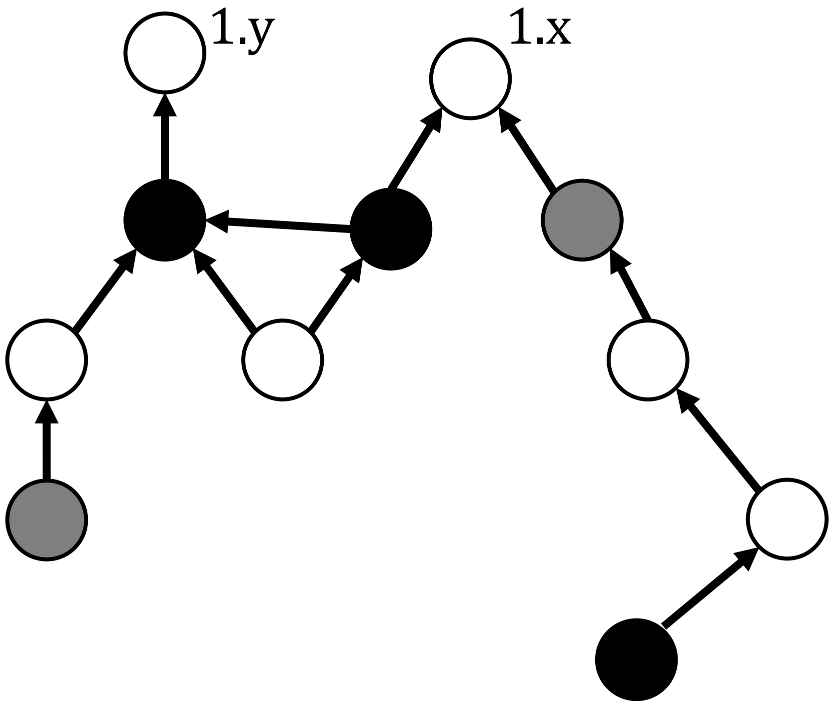
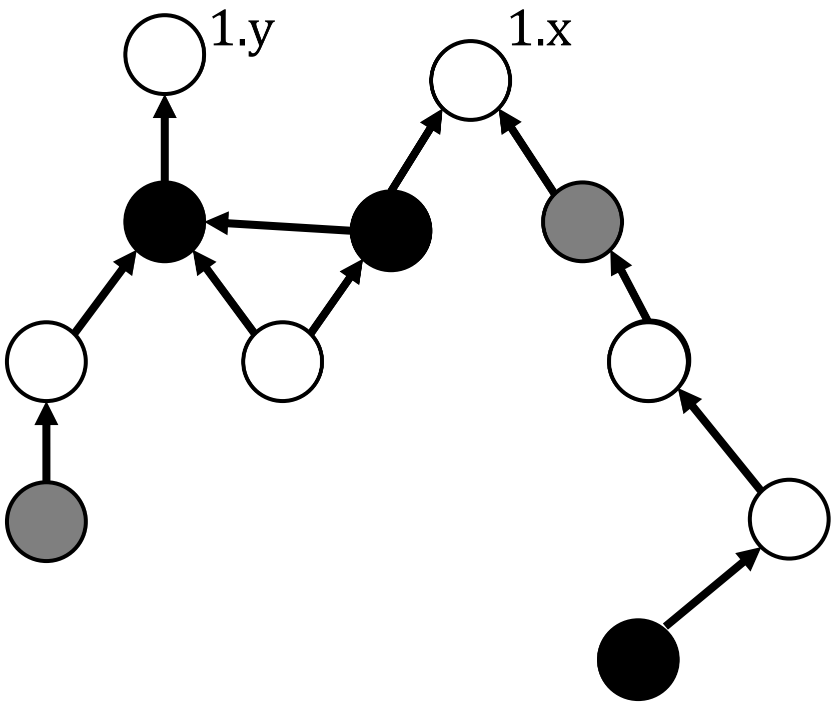


$A_x A_y G$ = $A_y A_x G$



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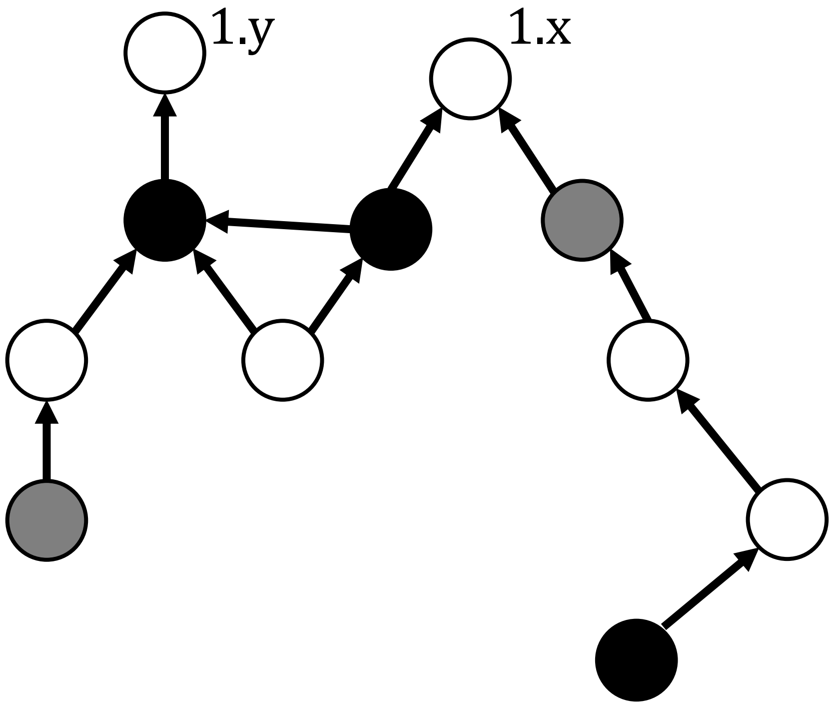
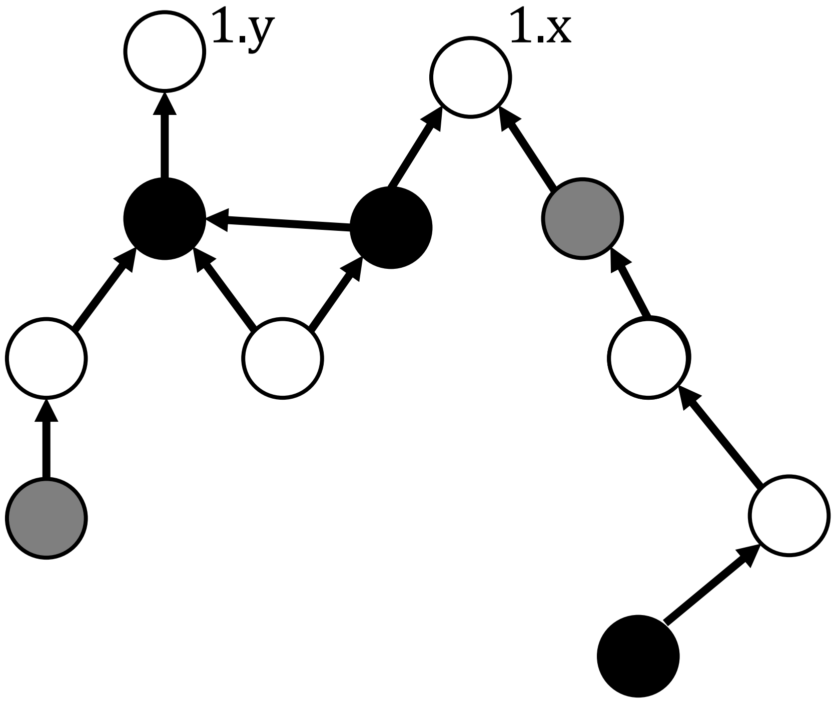


$A_x A_y G$

=



$A_y A_x G$

Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$

$\Downarrow$

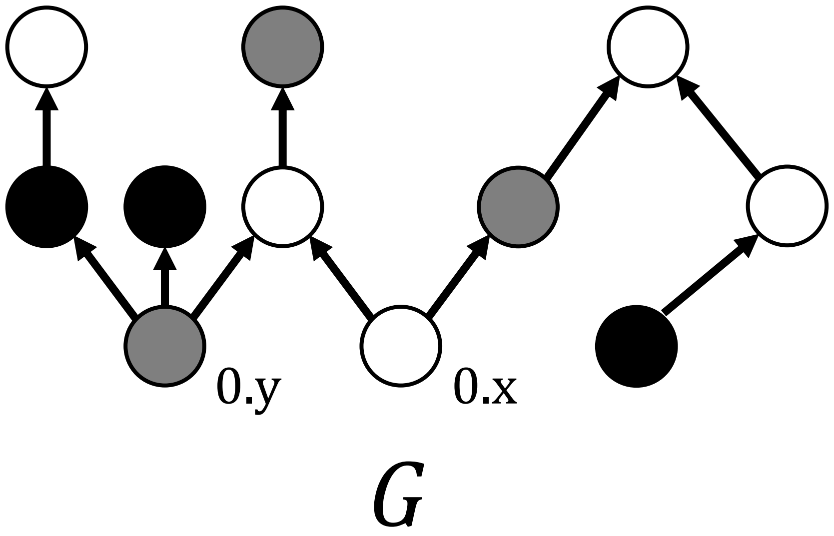
Conclusion

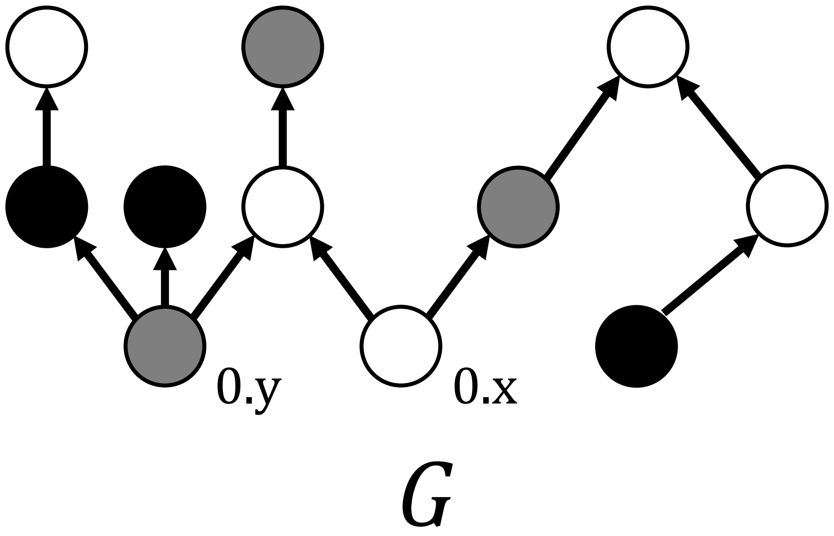
1. $A_x A_y G \,||\, A_y A_x G$

Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_w$, which means :

Same incomming edges $\Rightarrow$ Same internal state and outgoing edges





Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$

$\Downarrow$

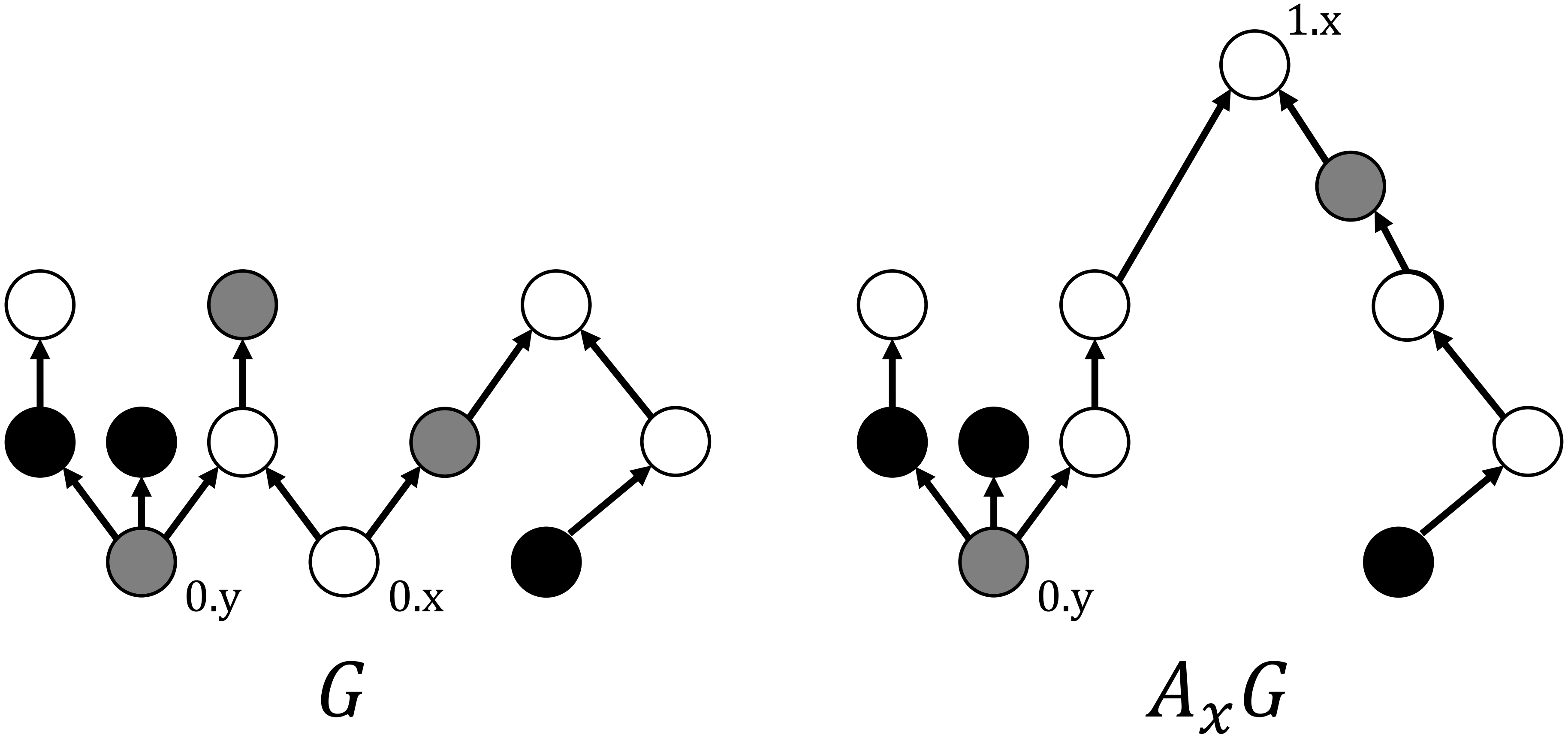
Conclusion

1. $A_x A_y G \mid\mid A_y A_x G$

Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_w$, which means :

Same incomming edges ⇒ Same internal state and outgoing edges



Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$

⇓

Conclusion

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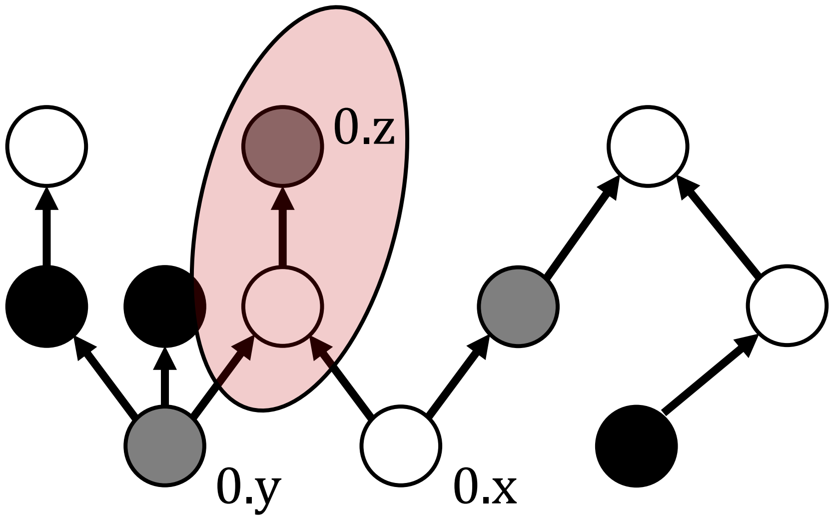
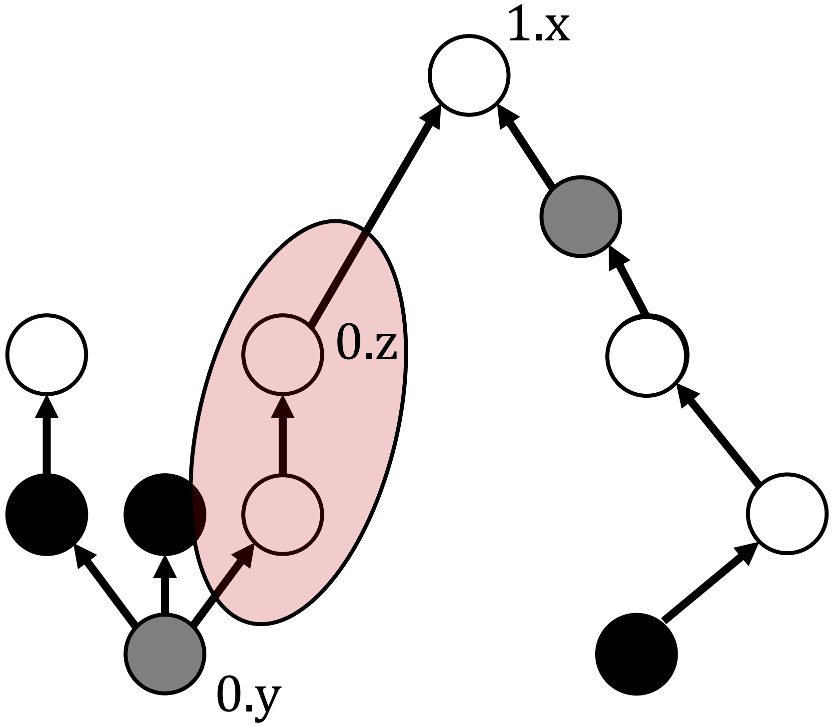


G



$A_x G$

Hypothesis

1. Commutative $A_x A_y G = A_y A_x G$

$\Downarrow$

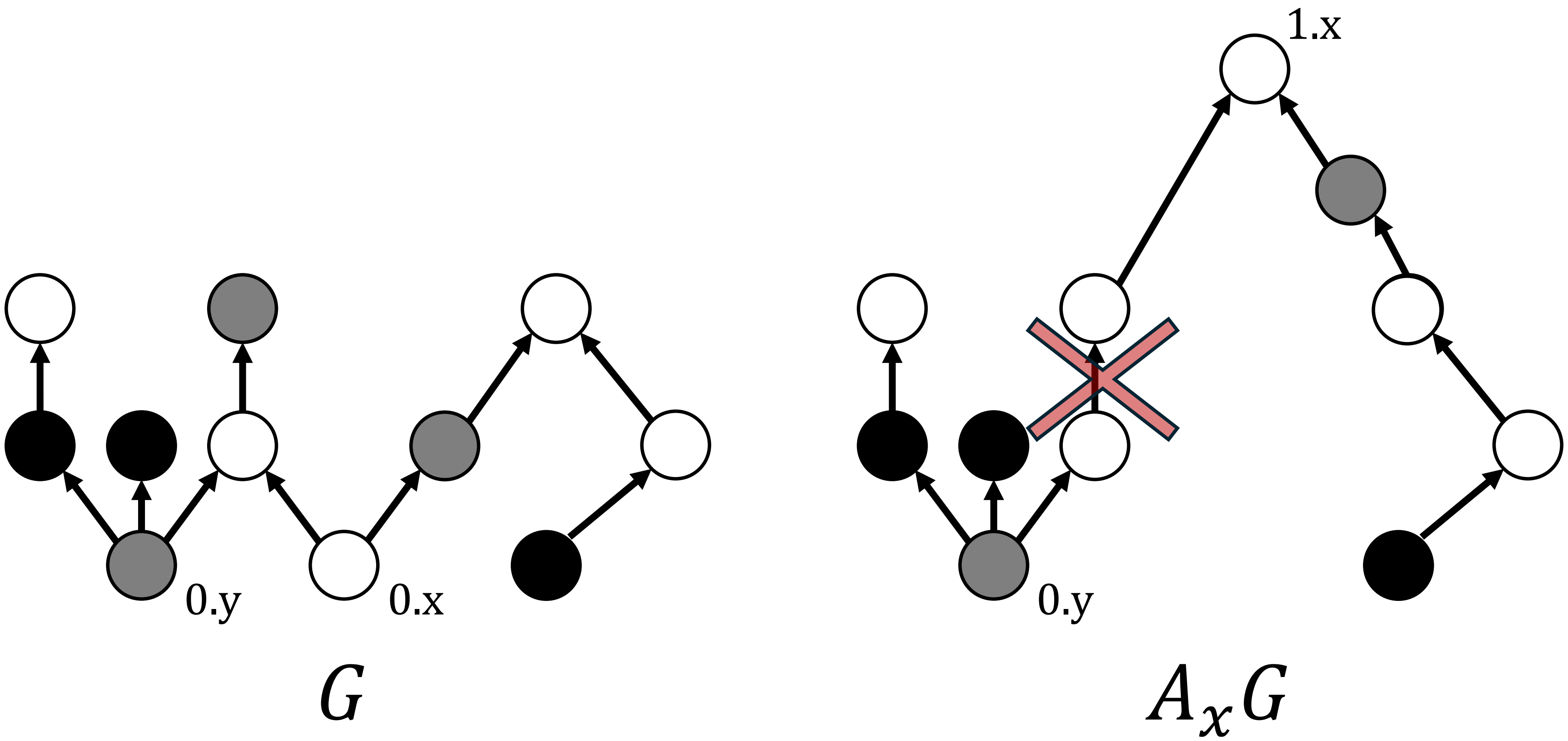
Conclusion

1. $A_x A_y G \mid\mid A_y A_x G$

Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_w$, which means :

Same incomming edges $\Rightarrow$ Same internal state and outgoing edges



Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$
2. Edge decreasing

$\Downarrow$

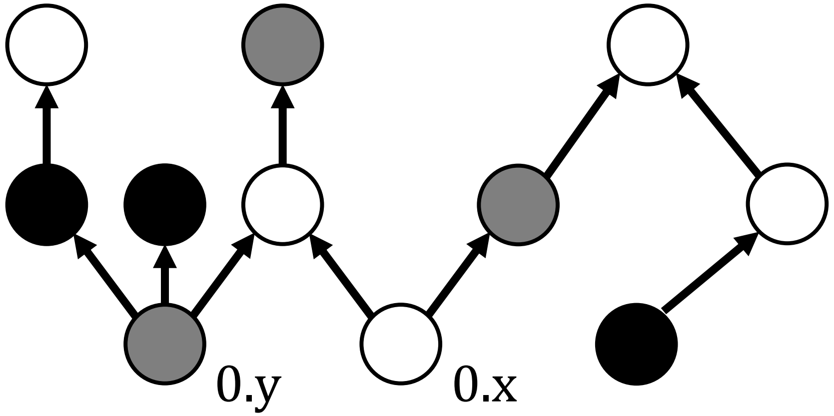
Conclusion

1. $A_x A_y G \,||\, A_y A_x G$

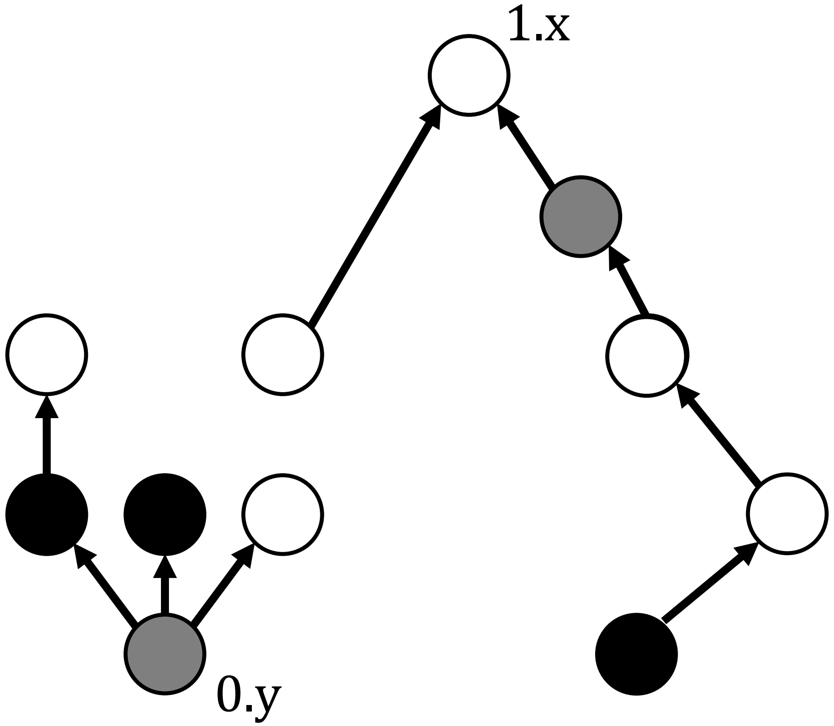
Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_{w'}$ which means :

~~Same incomming edges~~ $\Rightarrow$ Same internal state and outgoing edges



G



$A_x G$

Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$
2. Edge decreasing

$\Downarrow$

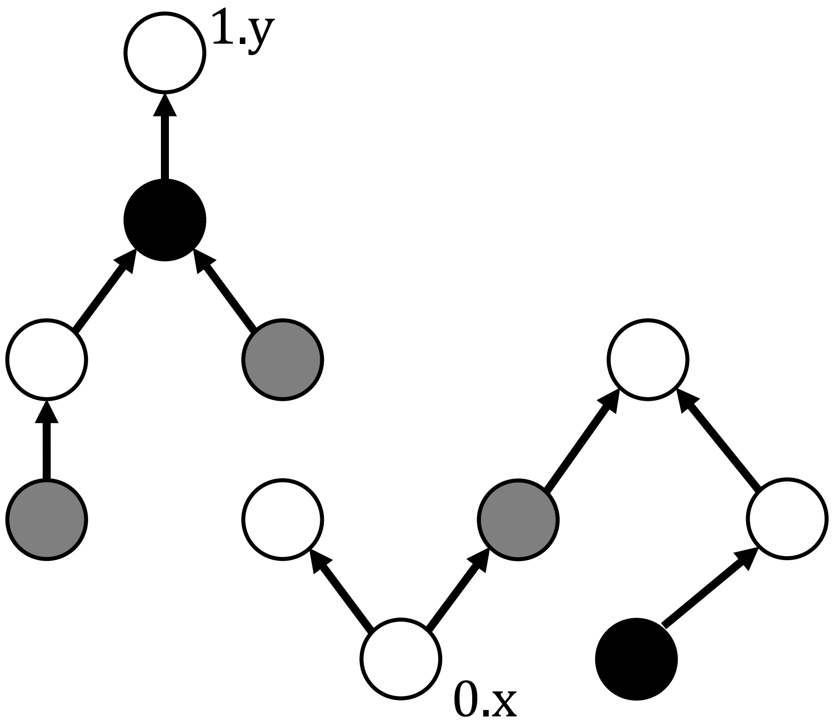
Conclusion

1. $A_x A_y G \,||\, A_y A_x G$
2. $A_x G \,||\, G$

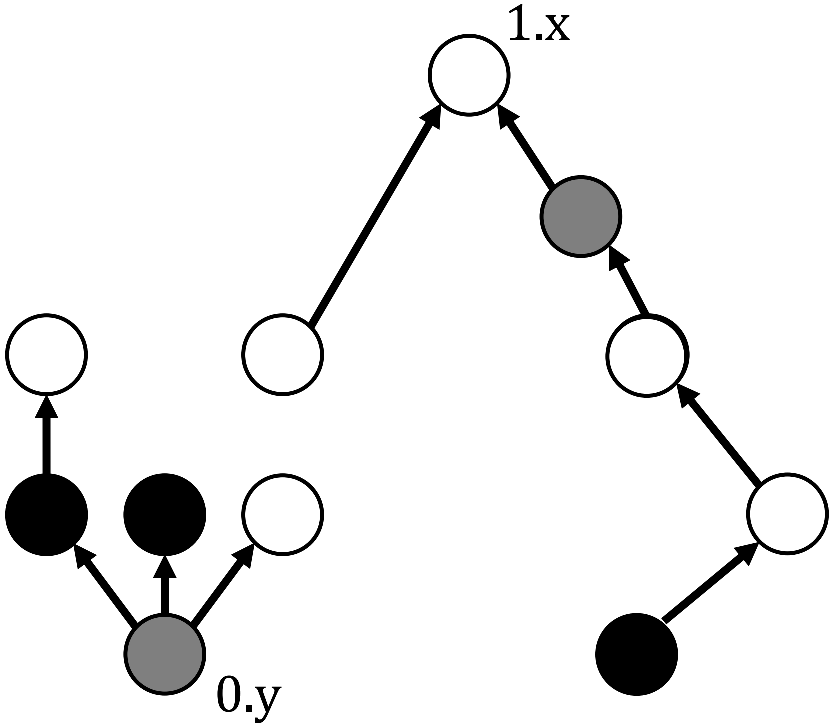
Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_w$, which means :

Same incomming edges ⇒ Same internal state and outgoing edges



$A_y G$



$A_x G$

Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$
2. Edge decreasing

⇓

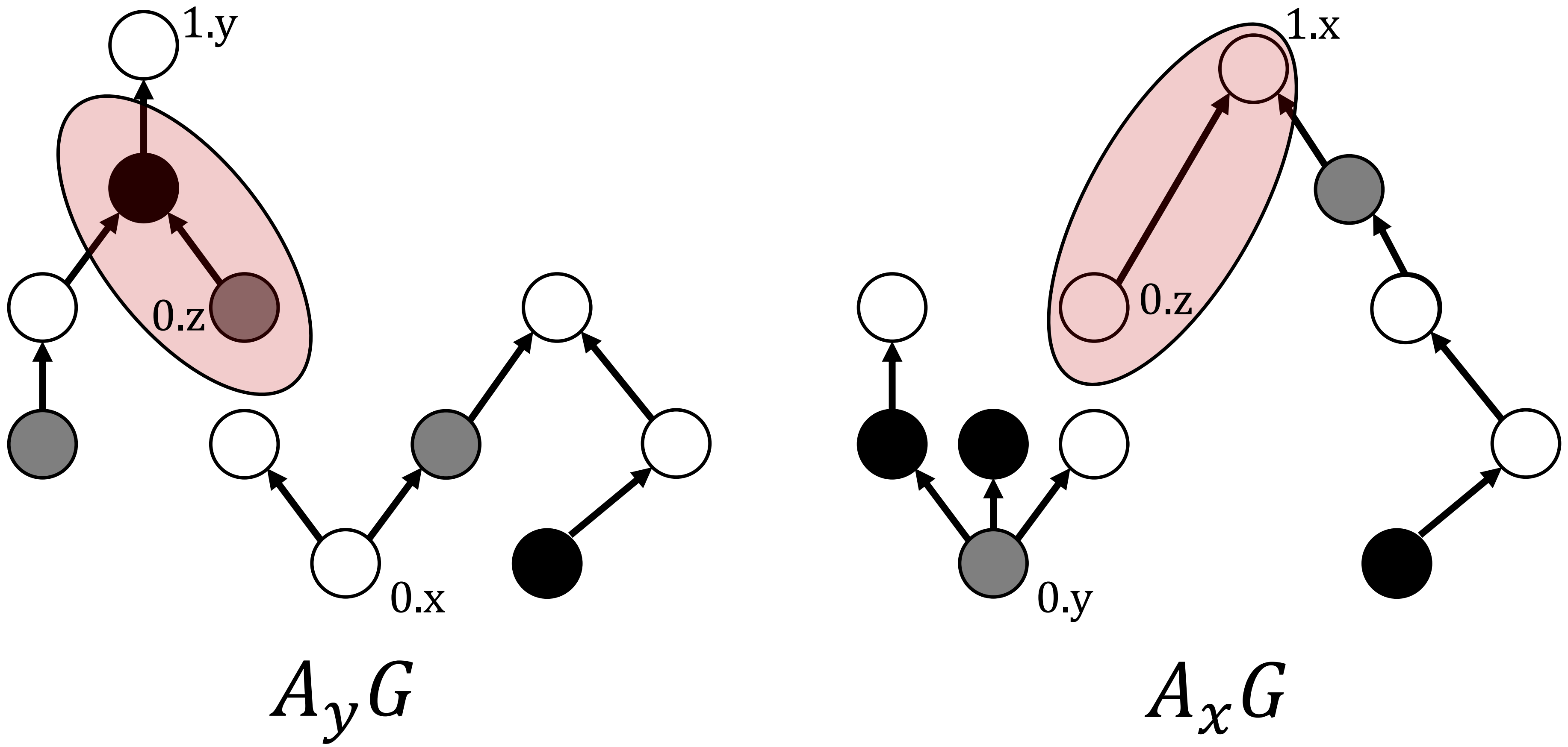
Conclusion

1. $A_x A_y G \; || \; A_y A_x G$
2. $A_x G \; || \; G$

Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_w$, which means :

Same incomming edges $\Rightarrow$ Same internal state and outgoing edges



Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$
2. Edge decreasing

$\Downarrow$

Conclusion

1. $A_x A_y G \,||\, A_y A_x G$
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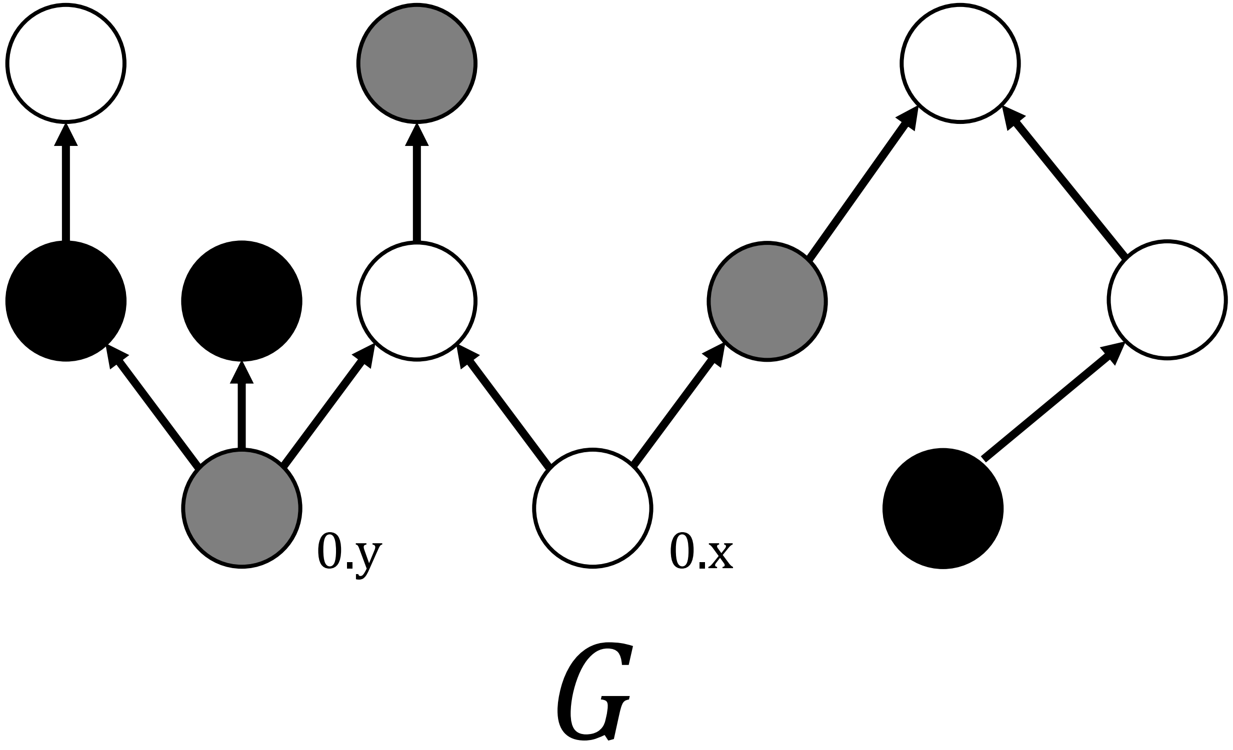

Hypothesis

1. Commutative $A_x A_y G = A_y A_x G$
2. Edge decreasing

⇓

Conclusion

1. $A_x A_y G \,||\, A_y A_x G$
2. $A_x G \,||\, G$

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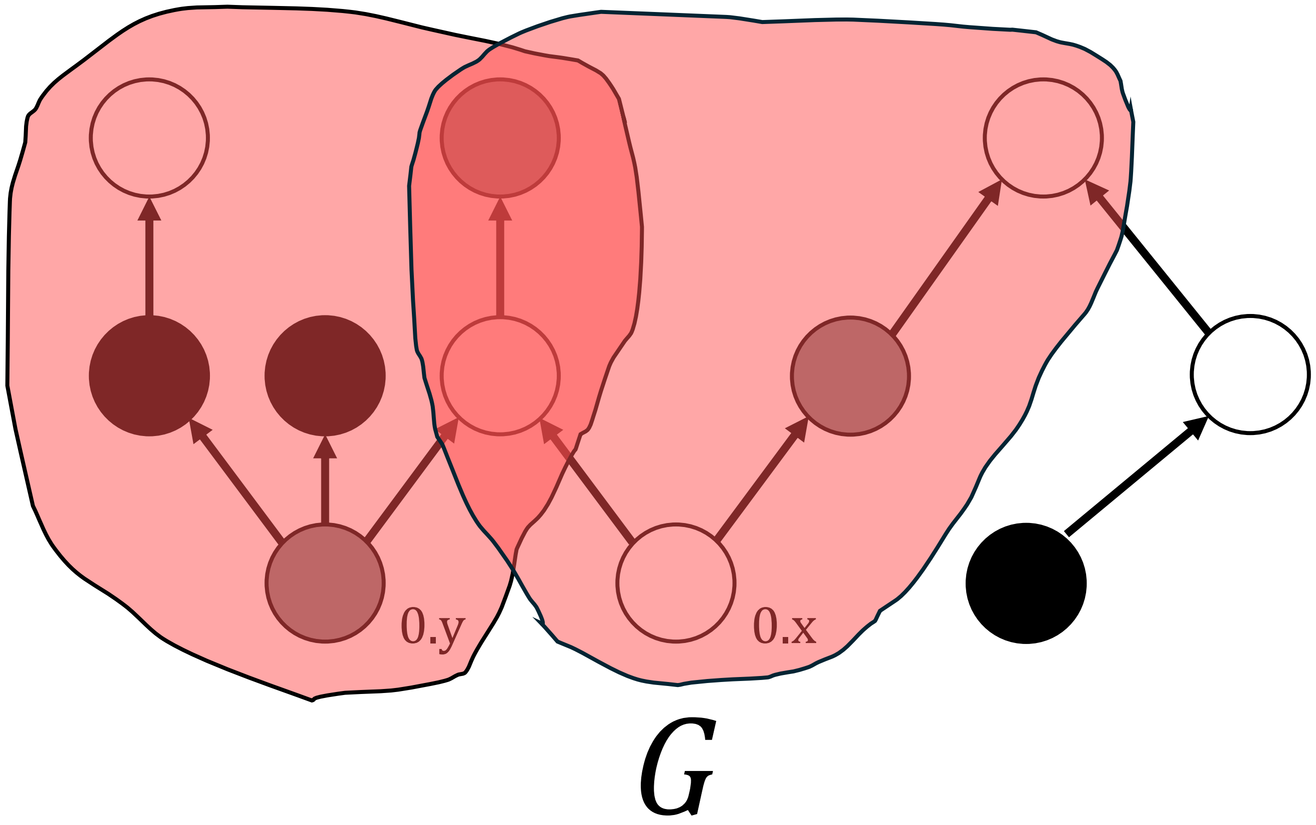

Hypothesis

1. Commutative $A_x A_y G = A_y A_x G$
2. Edge decreasing

⇓

Conclusion

1. $A_x A_y G \,||\, A_y A_x G$
2. $A_x G \,||\, G$

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Goal : prove A deterministic, i.e. we always have $A_w || A_w$, which means :

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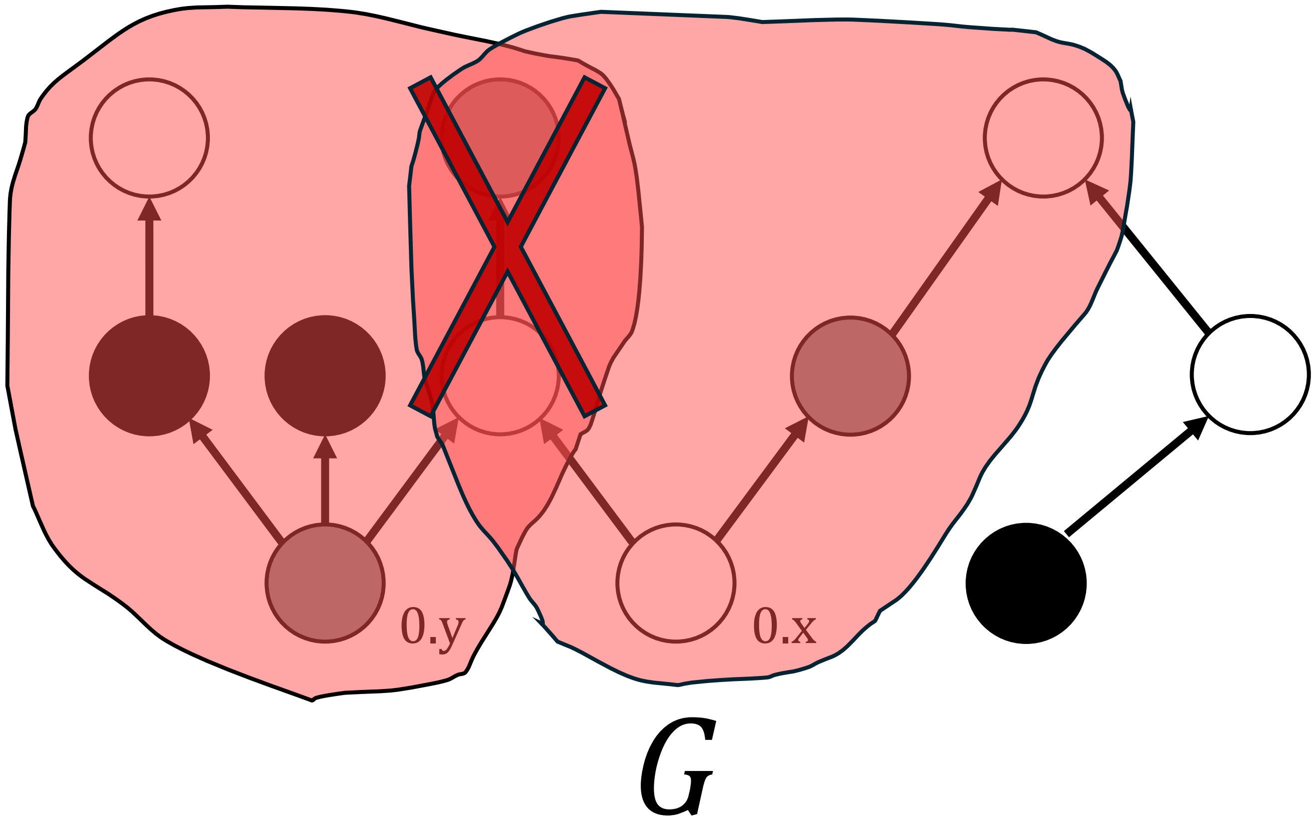


Hypothesis

1. Commutative $A_x A_y G = A_y A_x G$
2. Edge decreasing
3. Private

⇓

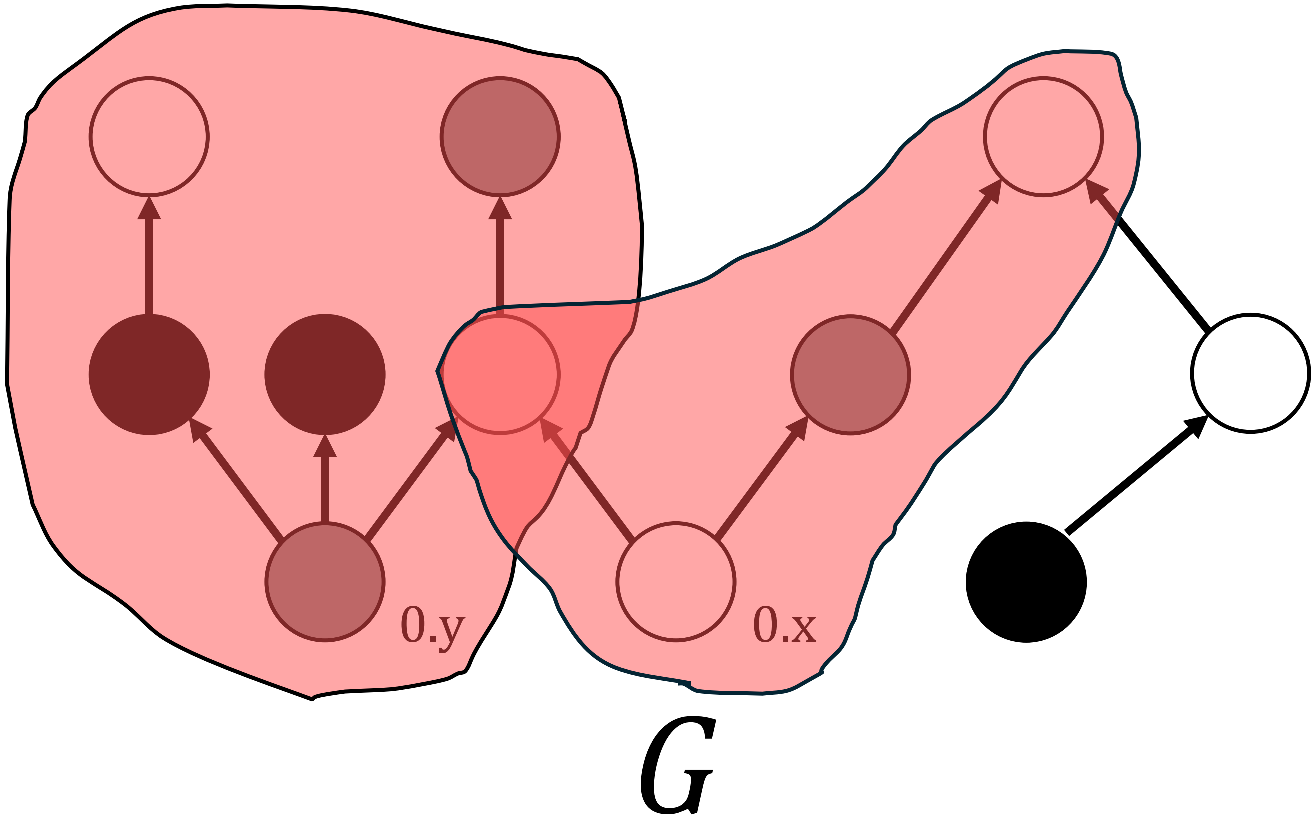
Conclusion

1. $A_x A_y G \,||\, A_y A_x G$
2. $A_x G \,||\, G$

Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_w$, which means :

Same incomming edges ⇒ Same internal state and outgoing edges



Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$
2. Edge decreasing
3. Private

⇓

Conclusion

1. $A_x A_y G \, || \, A_y A_x G$
2. $A_x G \, || \, G$
3. $A_x G \, || \, A_y G$

Ensuring determinism

Goal : prove A deterministic, i.e. we always have $A_w || A_{w'}$ which means :

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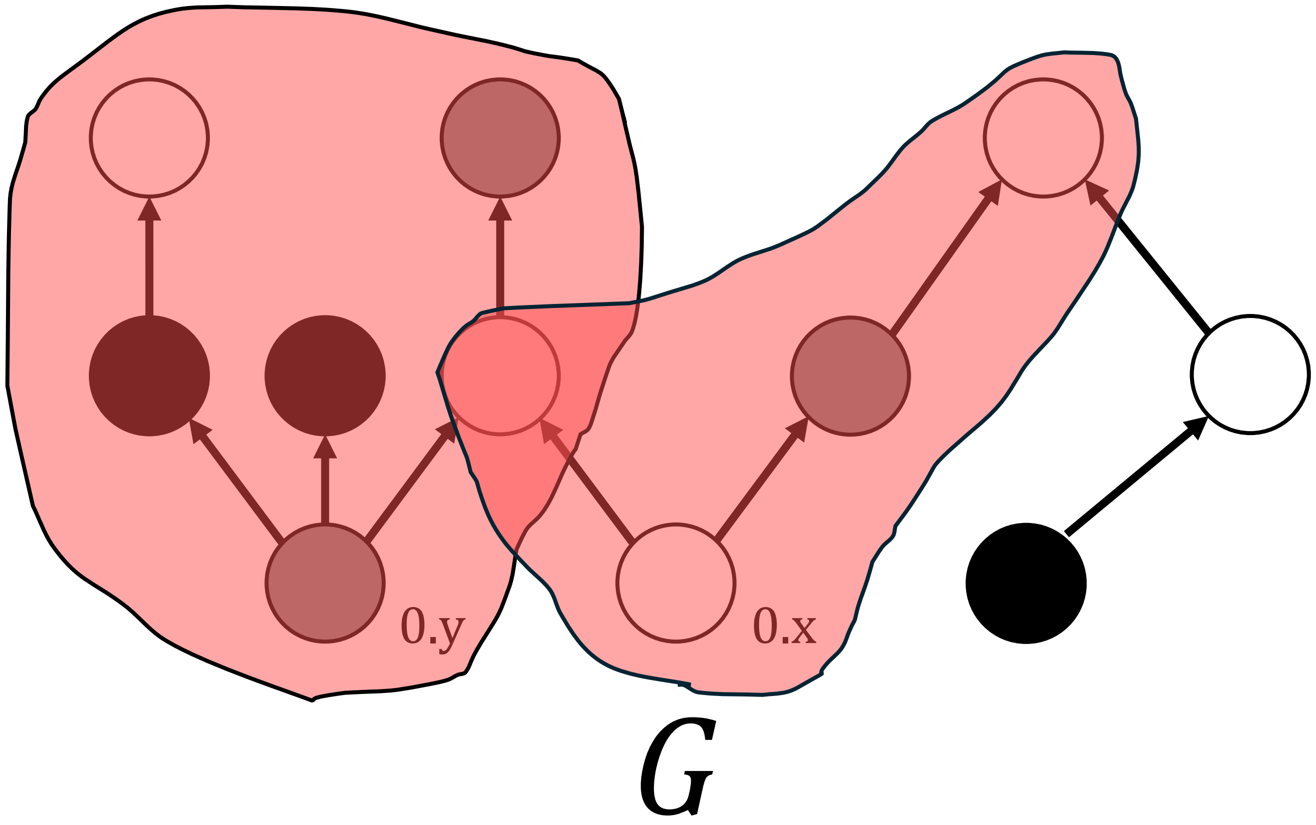


Hypothesis

1. Commutative
   $A_x A_y G = A_y A_x G$
2. Edge decreasing
3. Private

⇓

Conclusion

1. $A_x A_y G \; || \; A_y A_x G$
2. $A_x G \; || \; G$
3. $A_x G \; || \; A_y G$
4. $A_w G \; || \; A_{w'} G$

Ensuring determinism

Theorem 1

Any commutative, edge decreasing and private local rule is deterministic.

Ensuring determinism

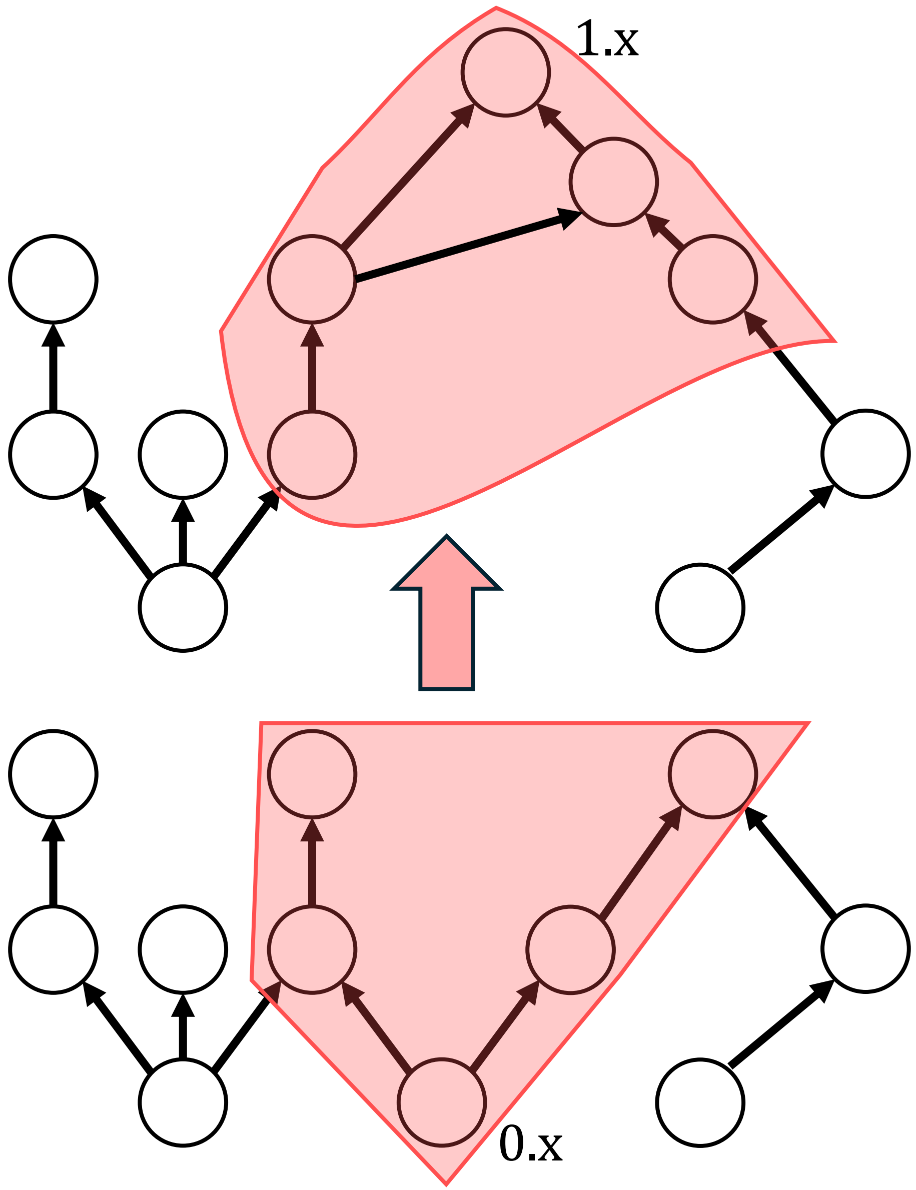
Theorem 1

Any commutative, edge decreasing and private local rule is deterministic.

In general which local rewriting rules are physical ?

- **Determinism** **OK**
- **Reversibility** 

Defining Reversibility

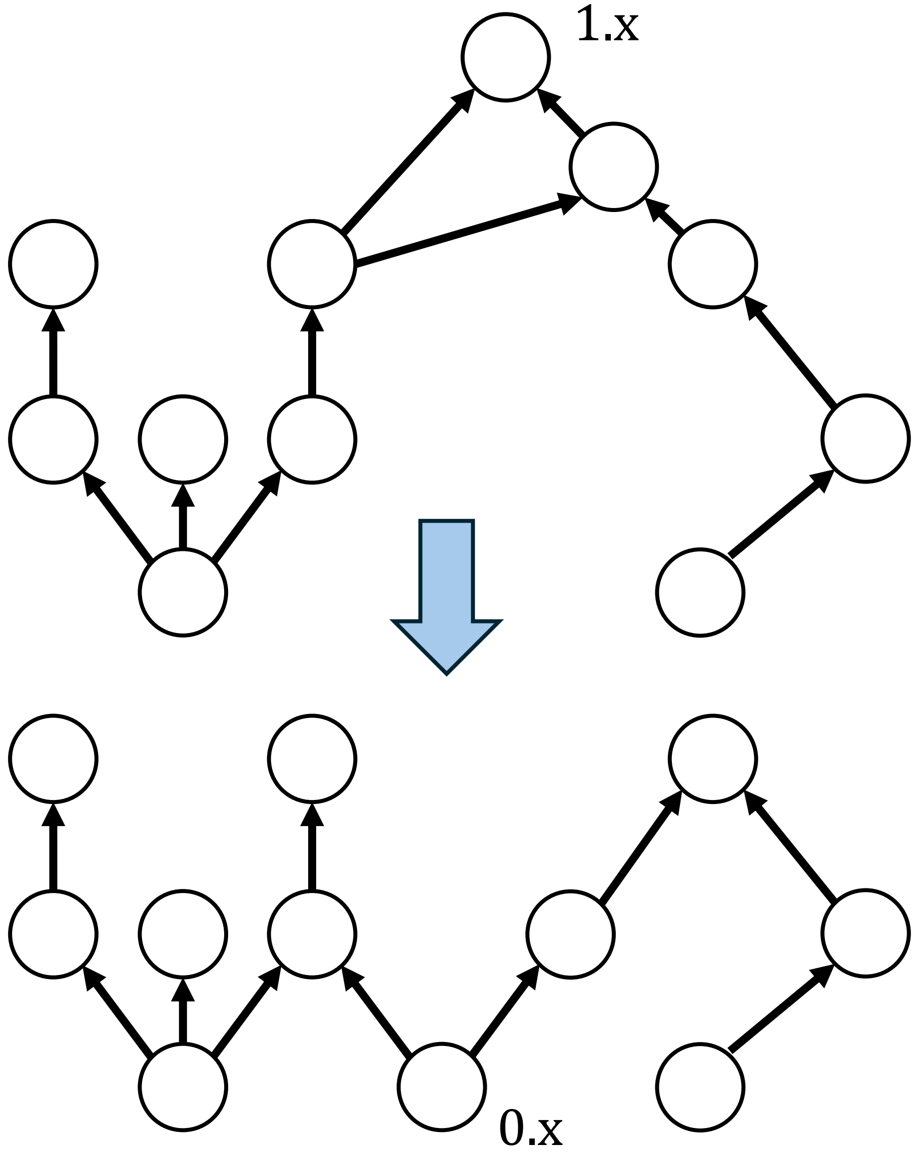


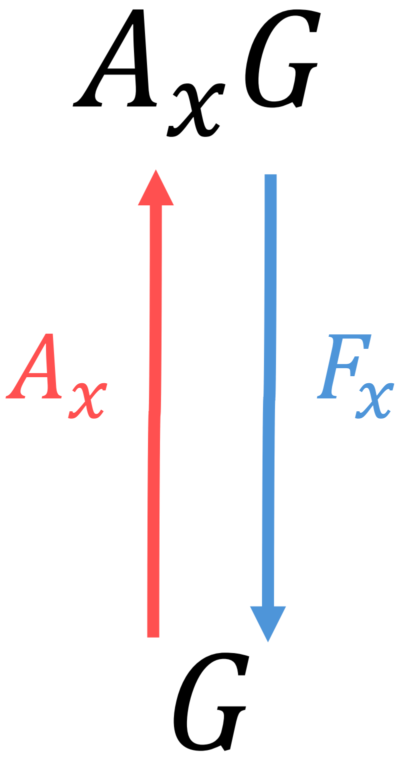
$$A_x G$$

A_x

$$G$$

Defining Reversibility

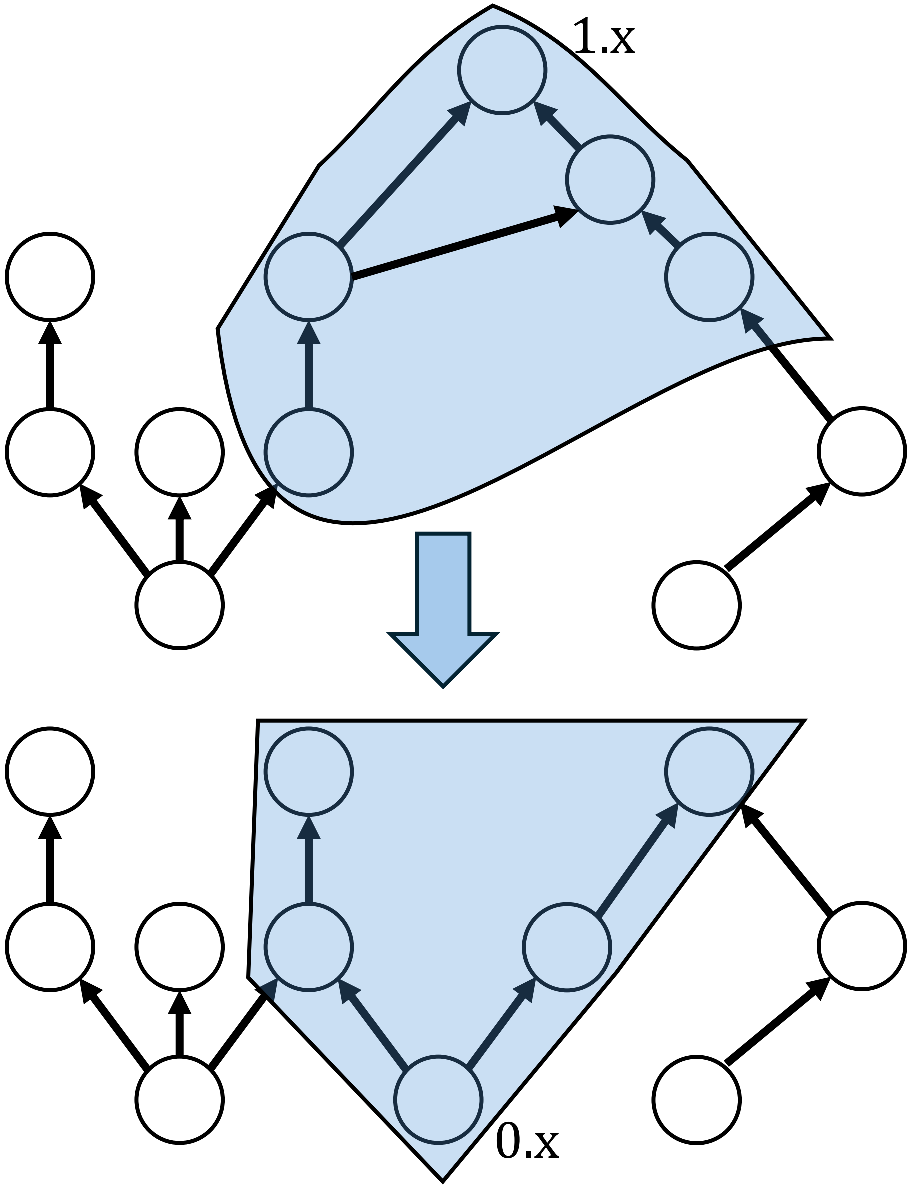


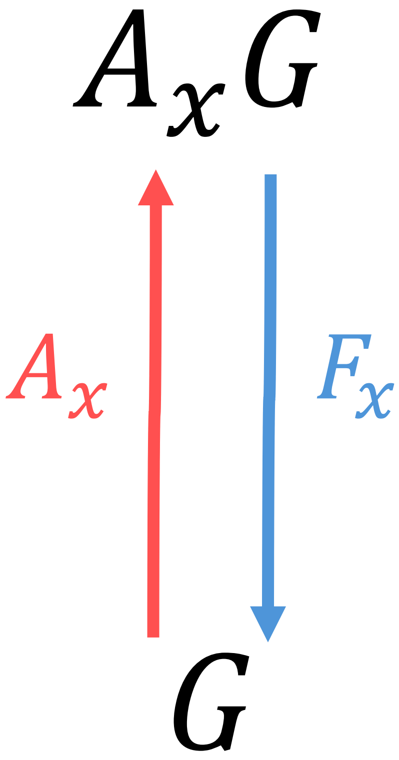


Option 1

- F is a **function s.t.** $F_x A_x G = G$
- **Problem :** non physical, does not match reversibility in CA, …

Defining Reversibility





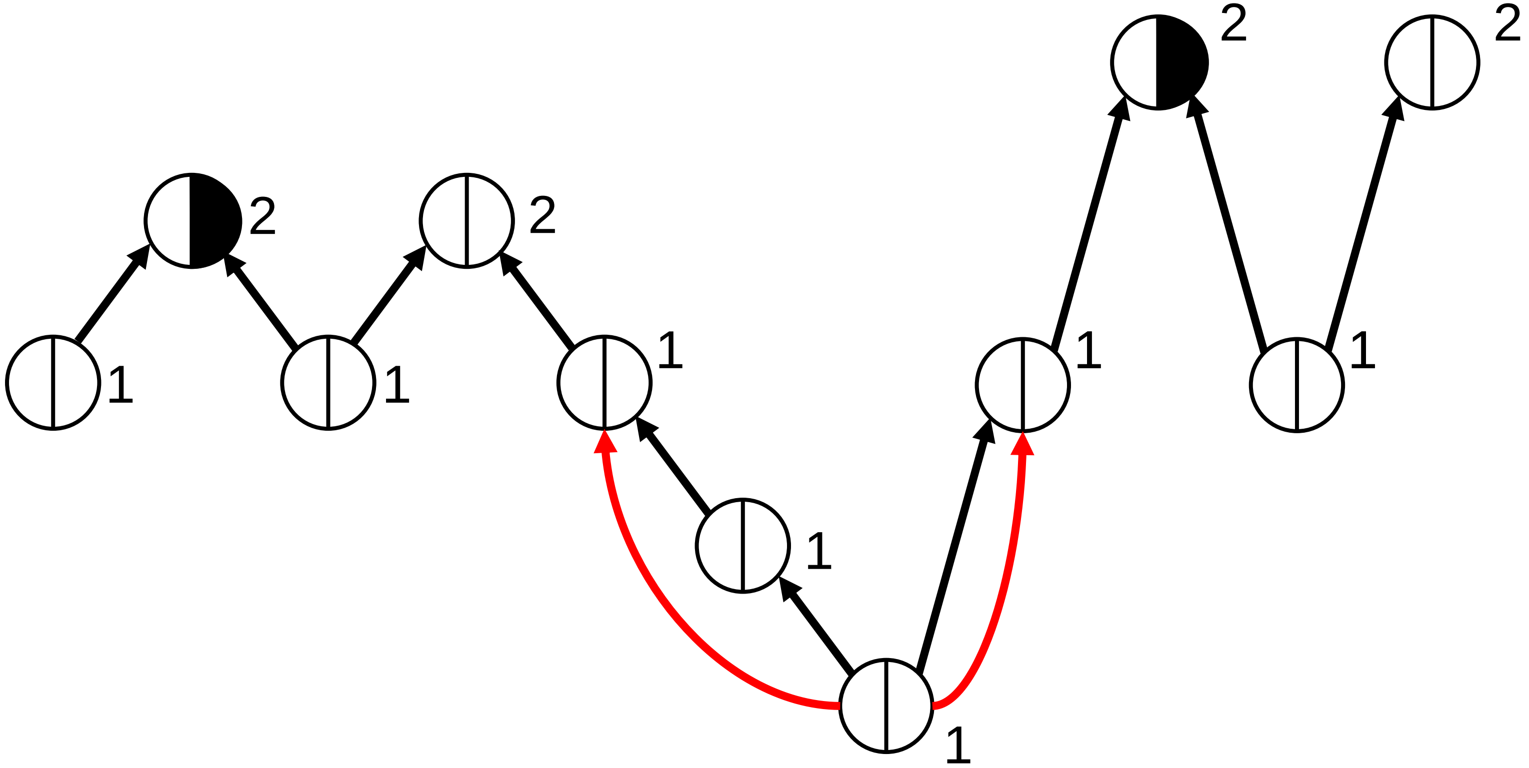
Option 1

- F is a **function s.t.** $F_x A_x G = G$
- **Problem :** non physical, does not match reversibility in CA, …

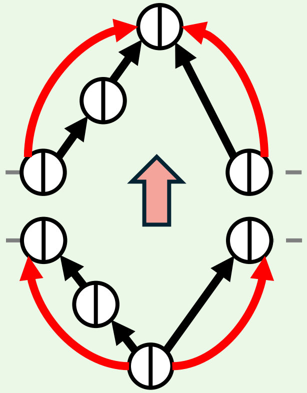
Option 2

- F is a **local rule** s.t. $F_x A_x G = G$
- Then it must be s.t. $A_x F_x G' = G'$

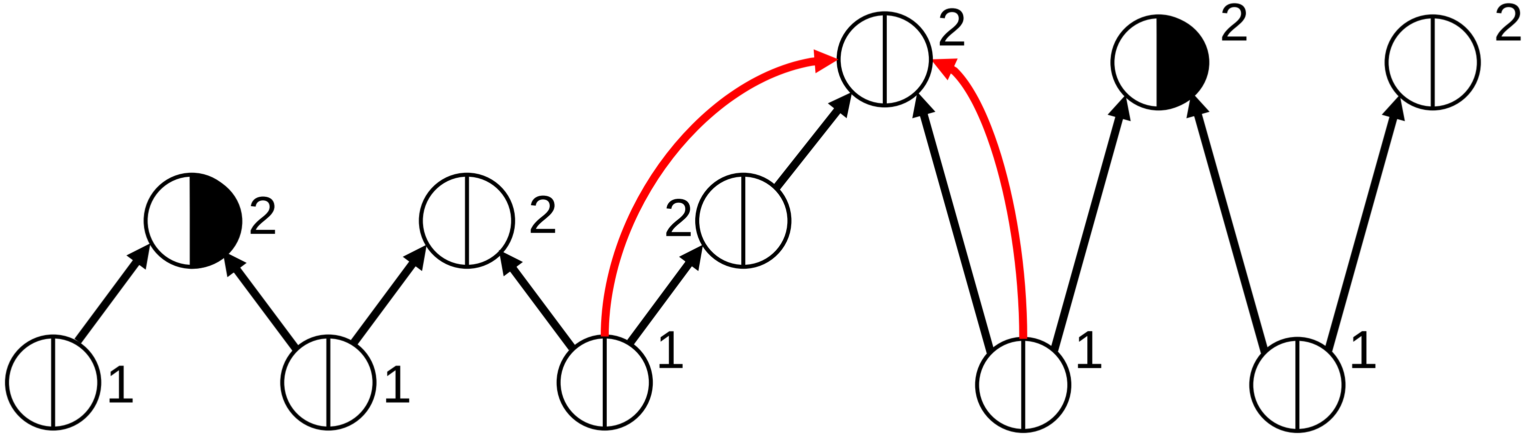
Reversible time dilation



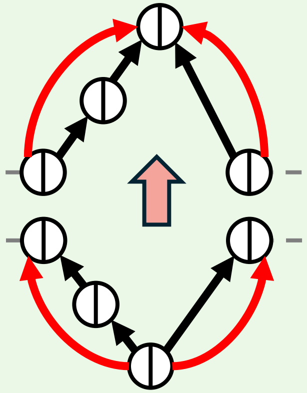
Local rule



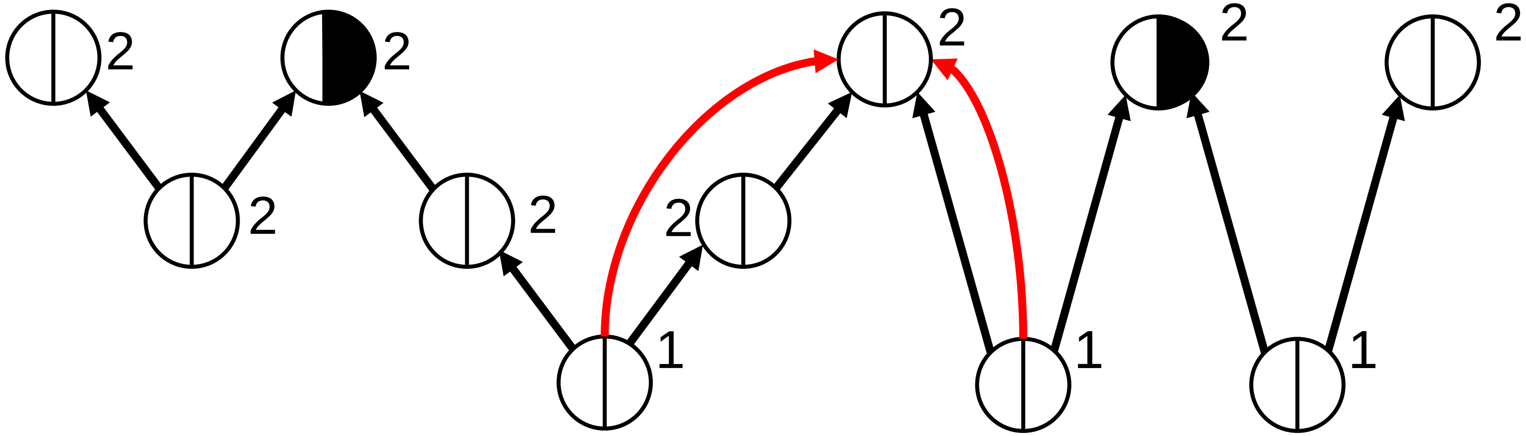
Reversible time dilation



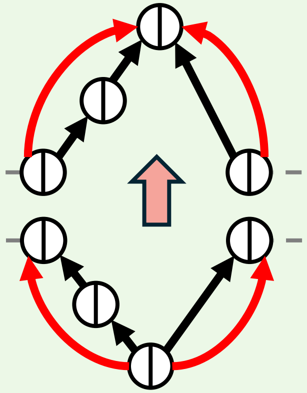
Local rule



Reversible time dilation



Local rule



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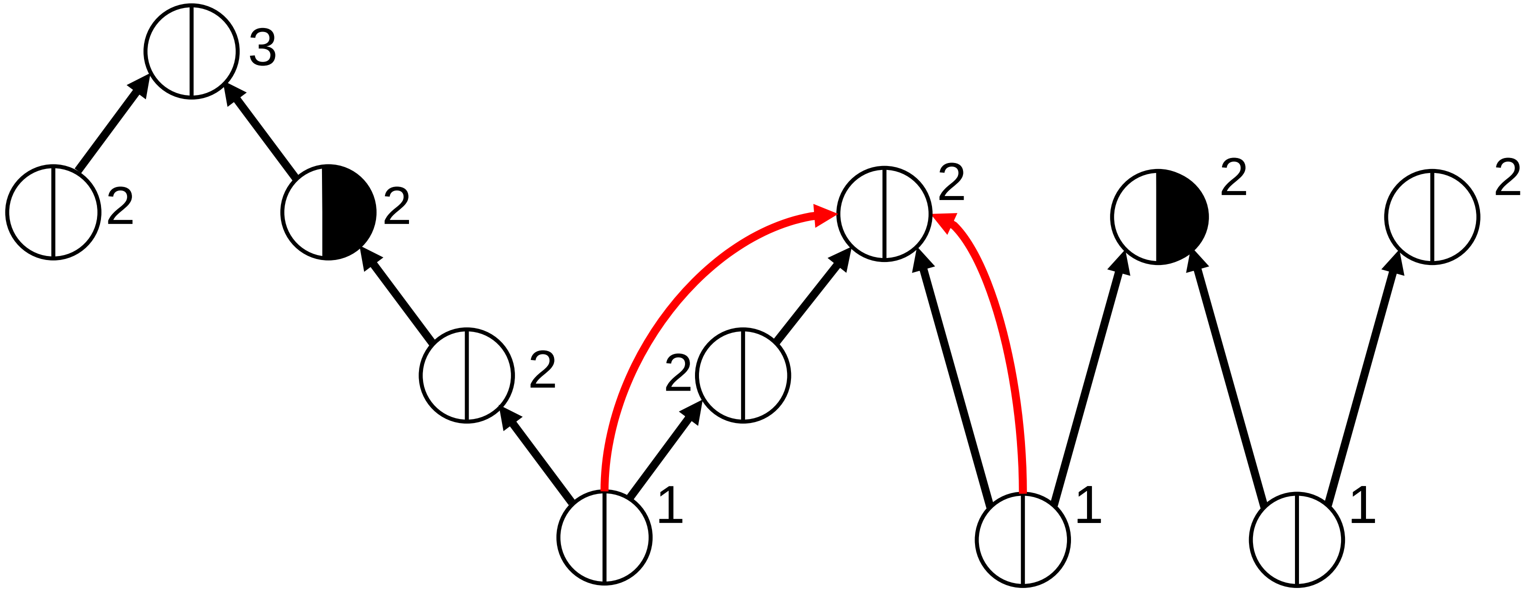
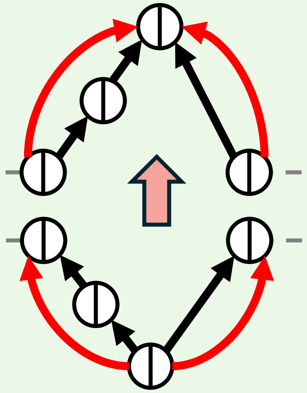


Local rule



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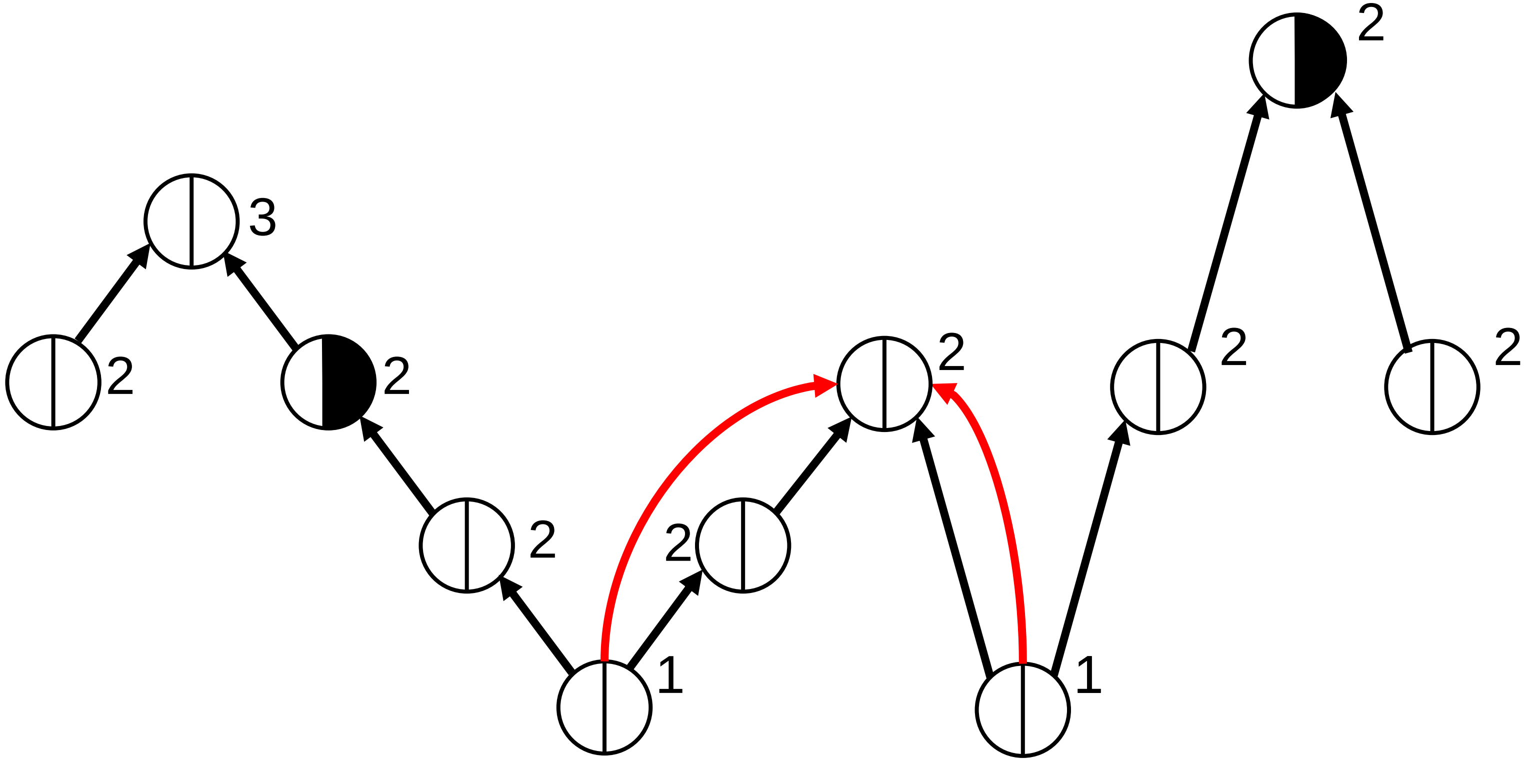
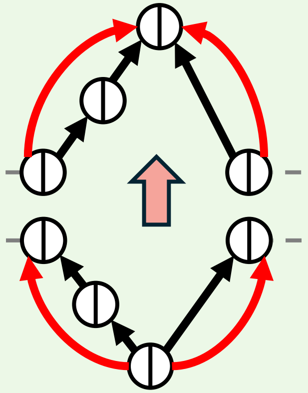
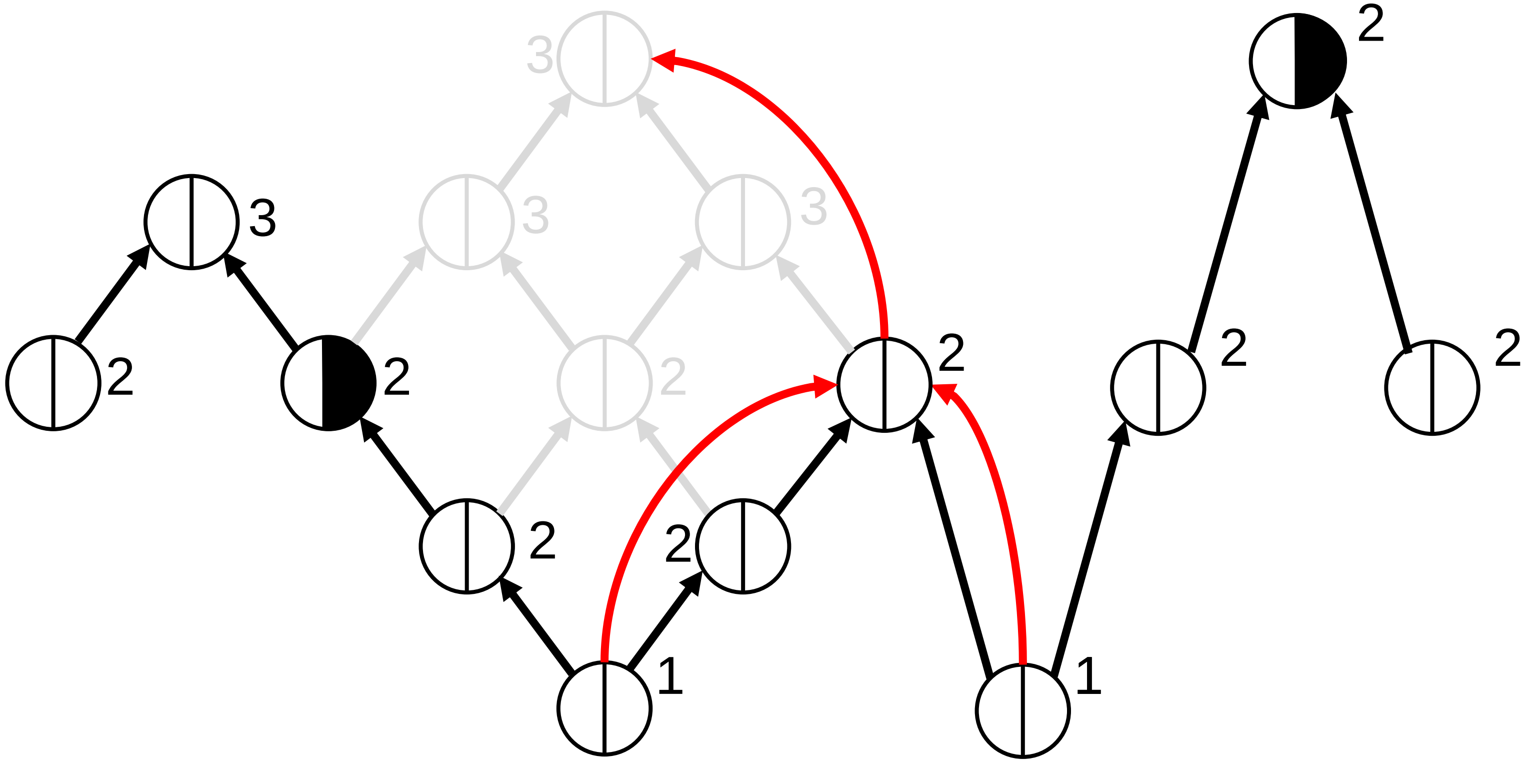


Local rule



Reversible time dilation



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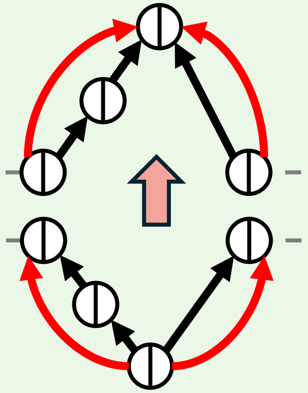
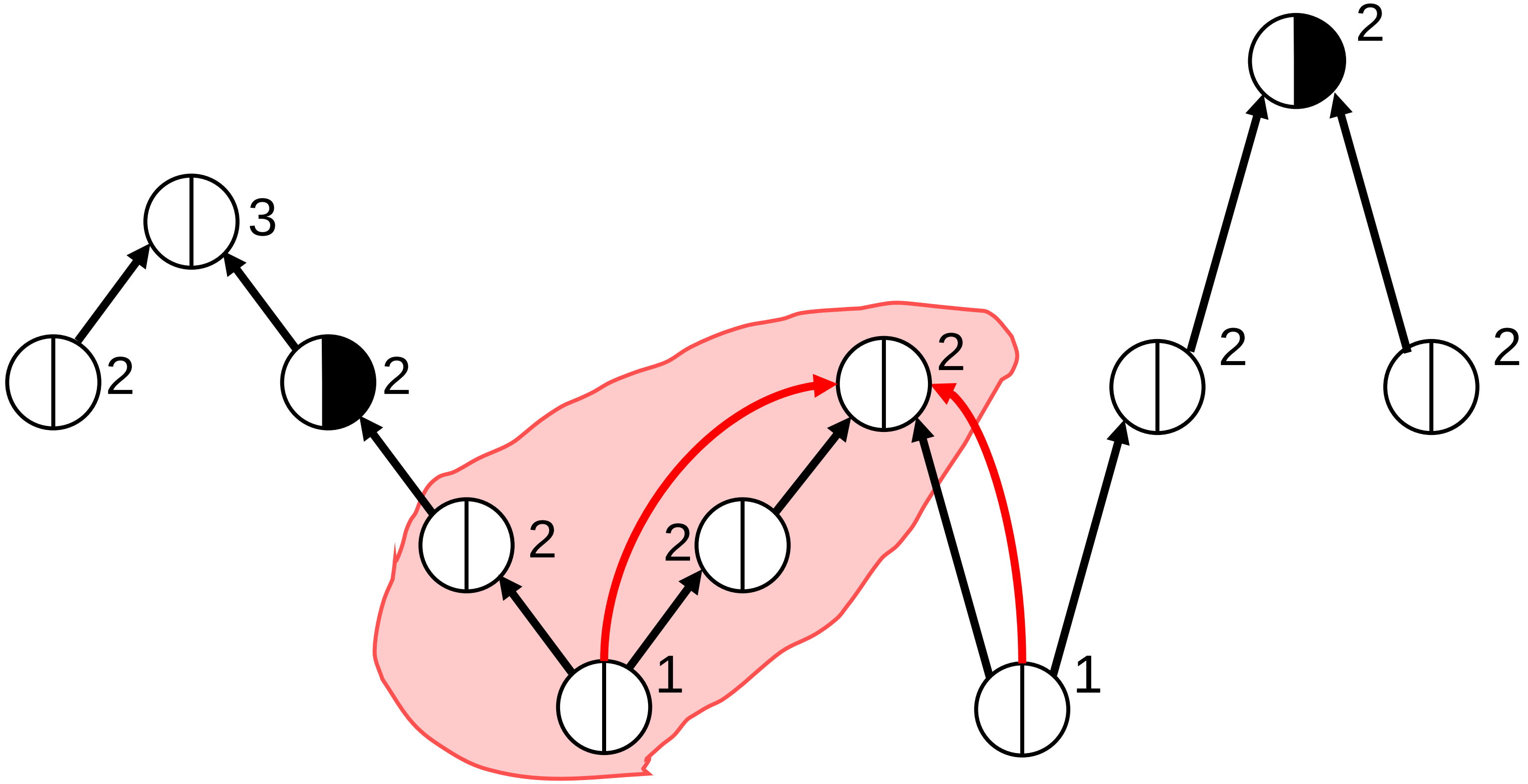
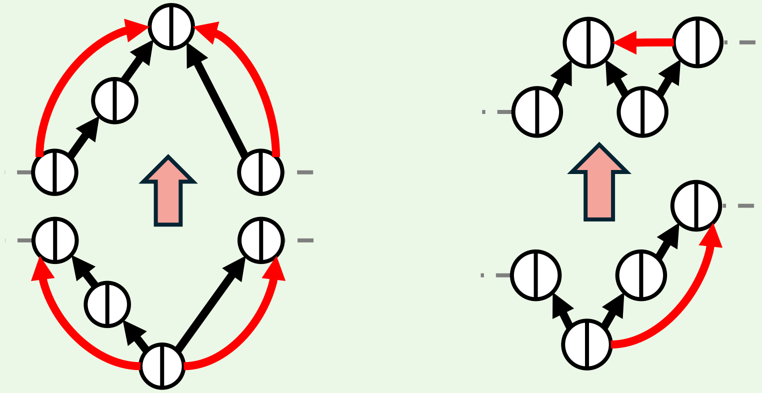


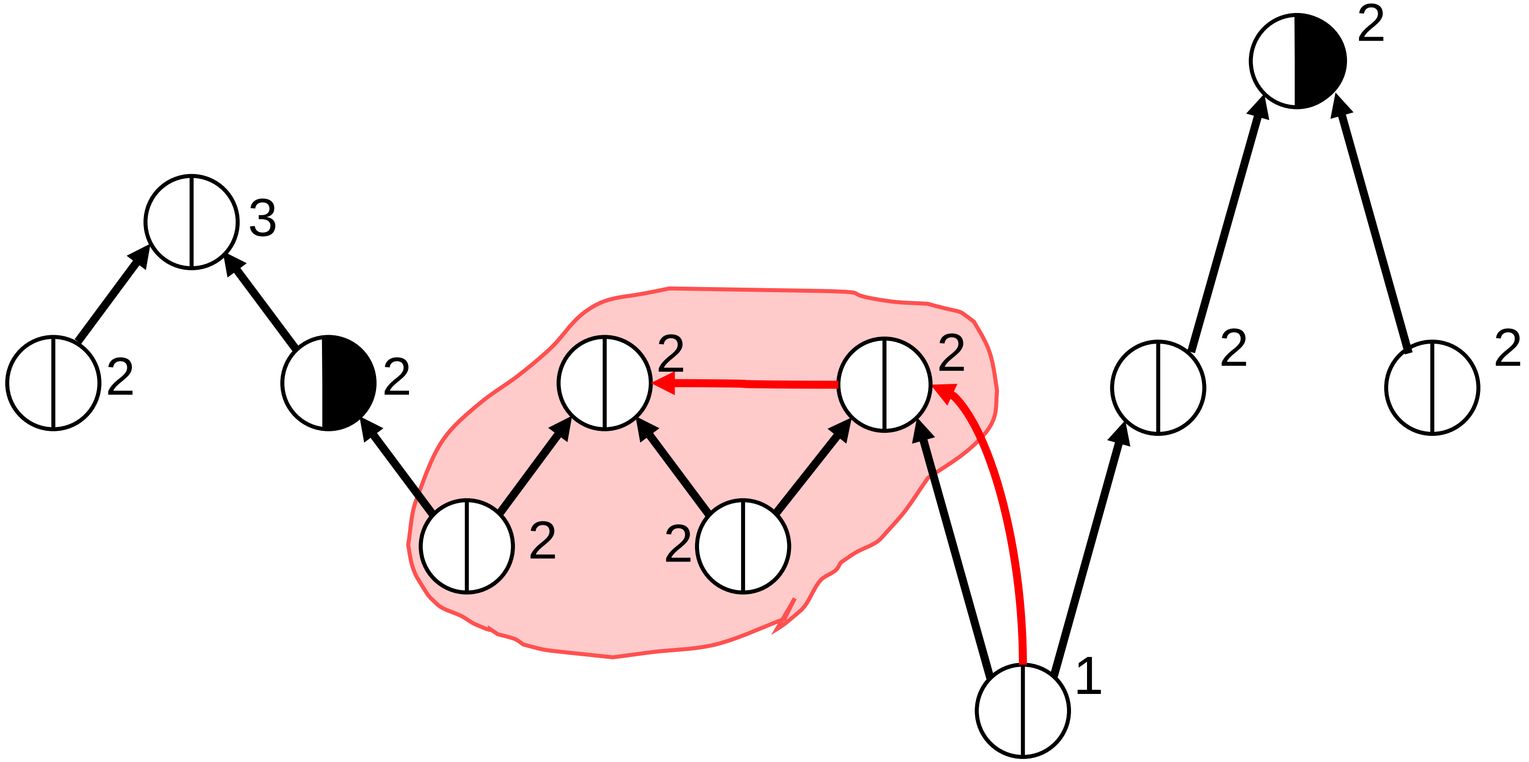
Reversible time dilation



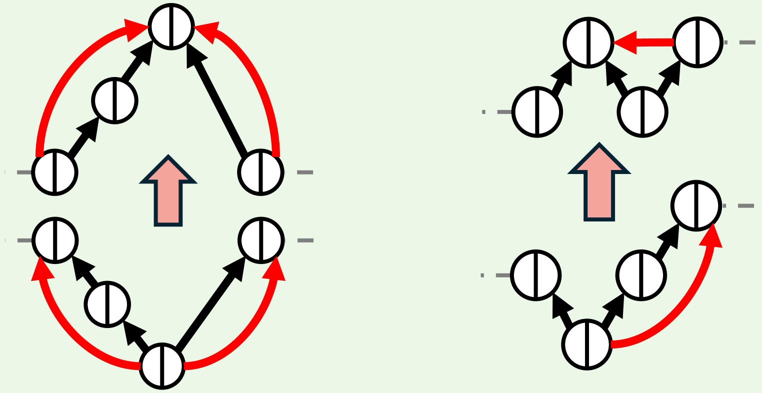
Local rule



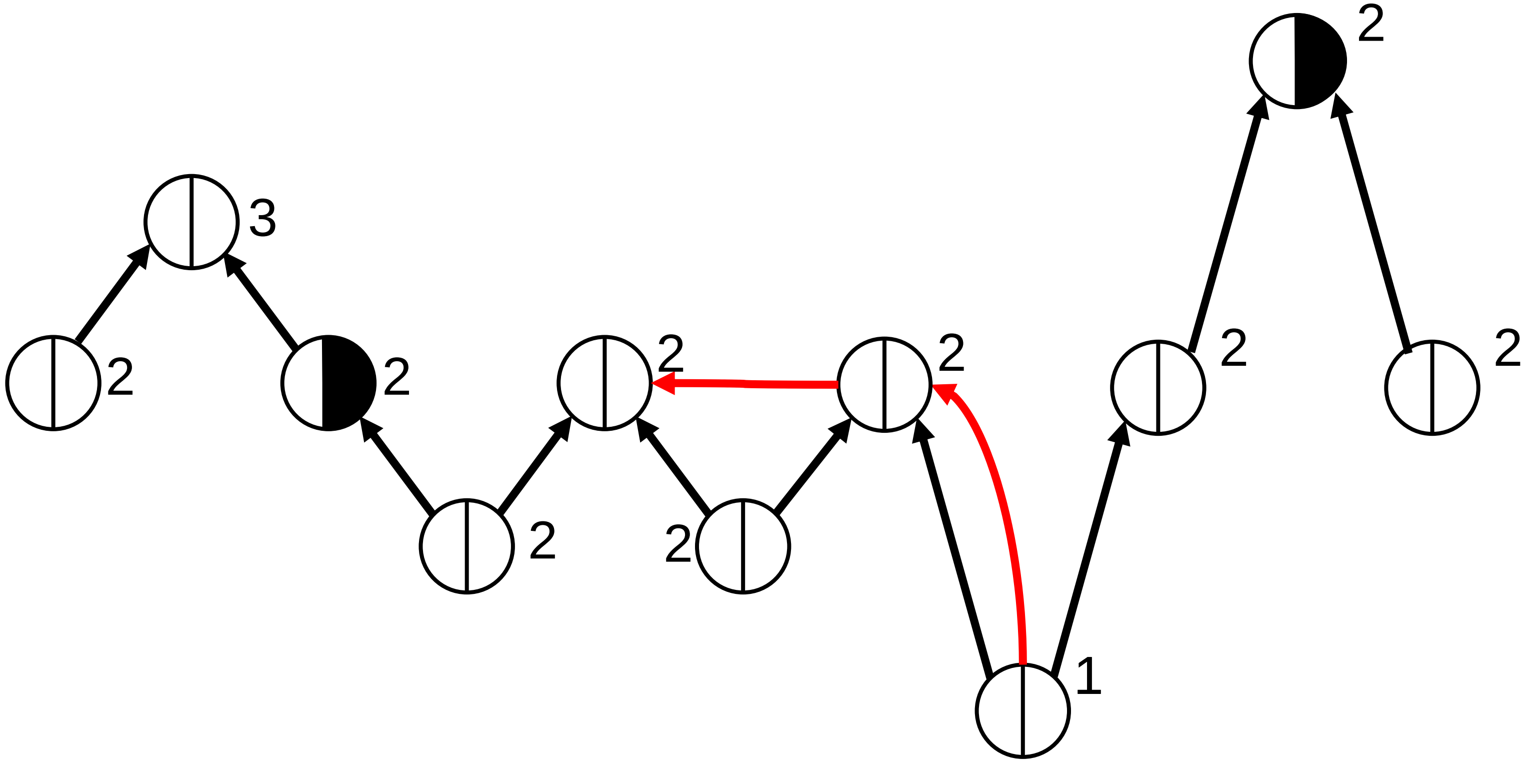
Reversible time dilation



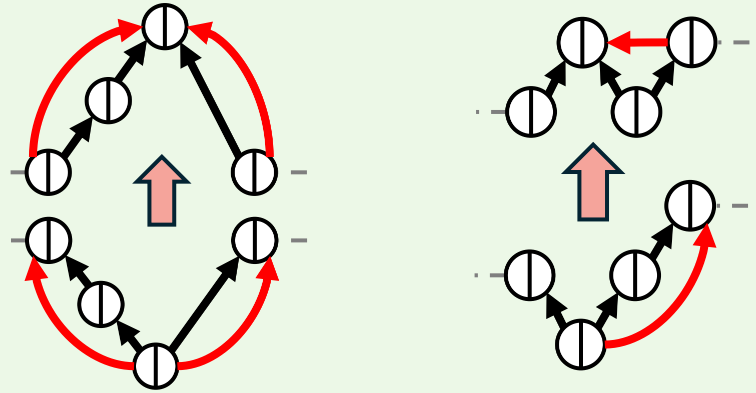
Local rule



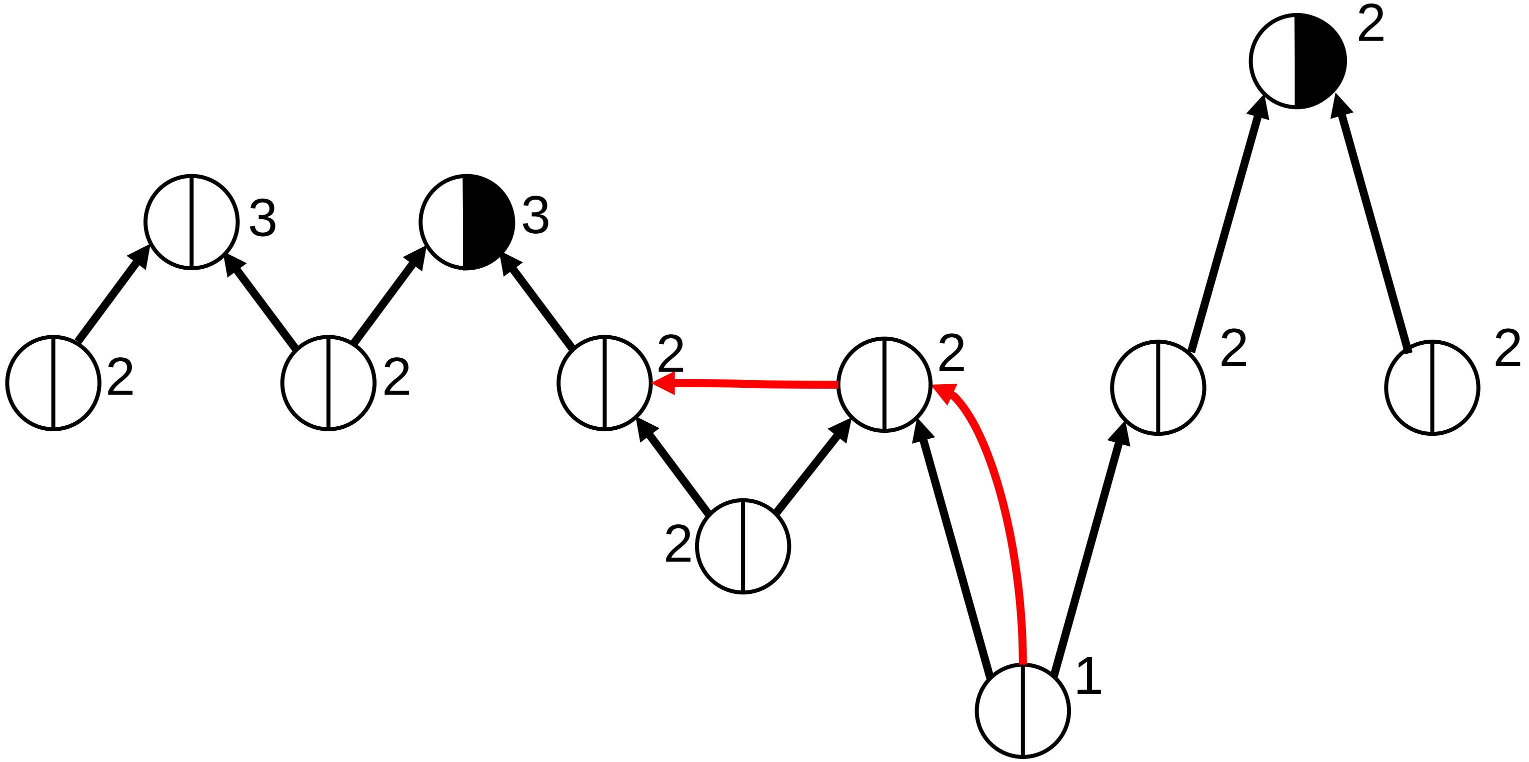
Reversible time dilation



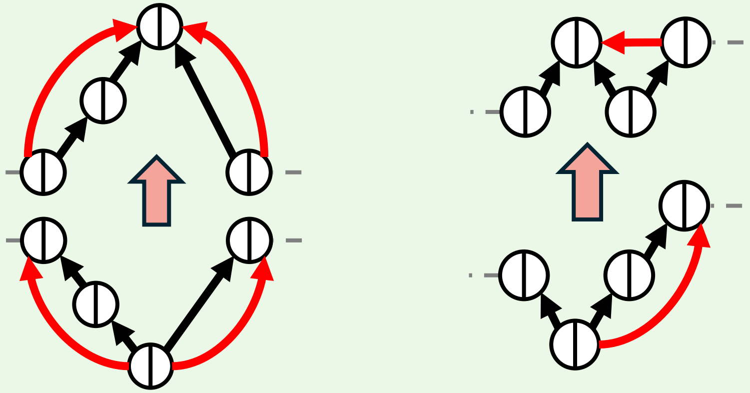
Local rule



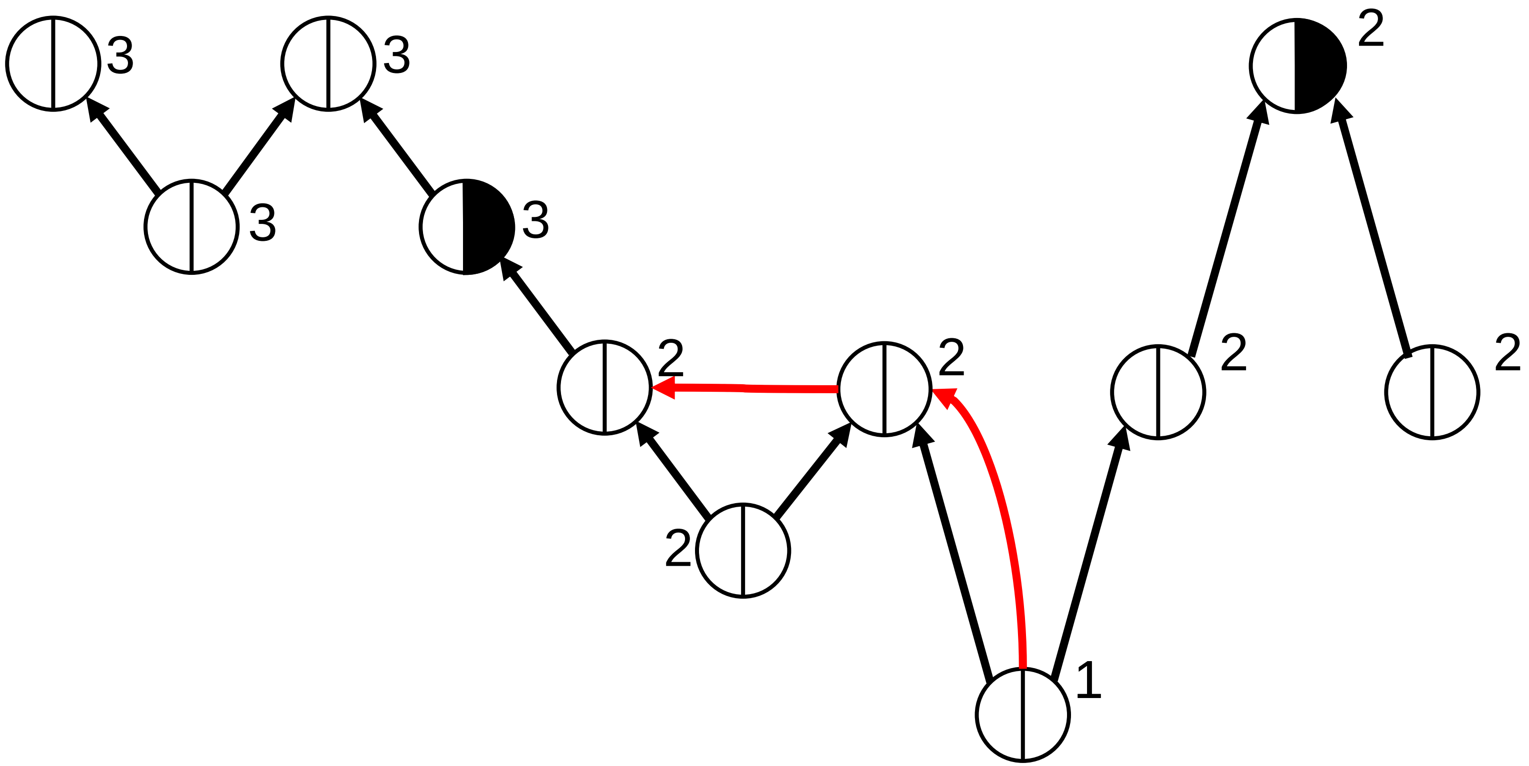
Reversible time dilation



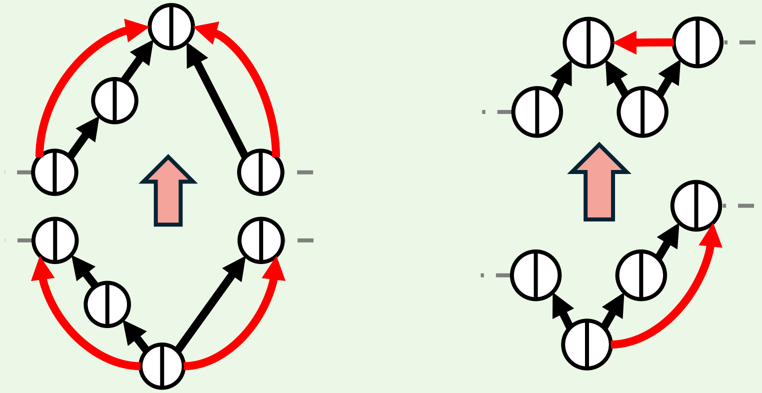
Local rule



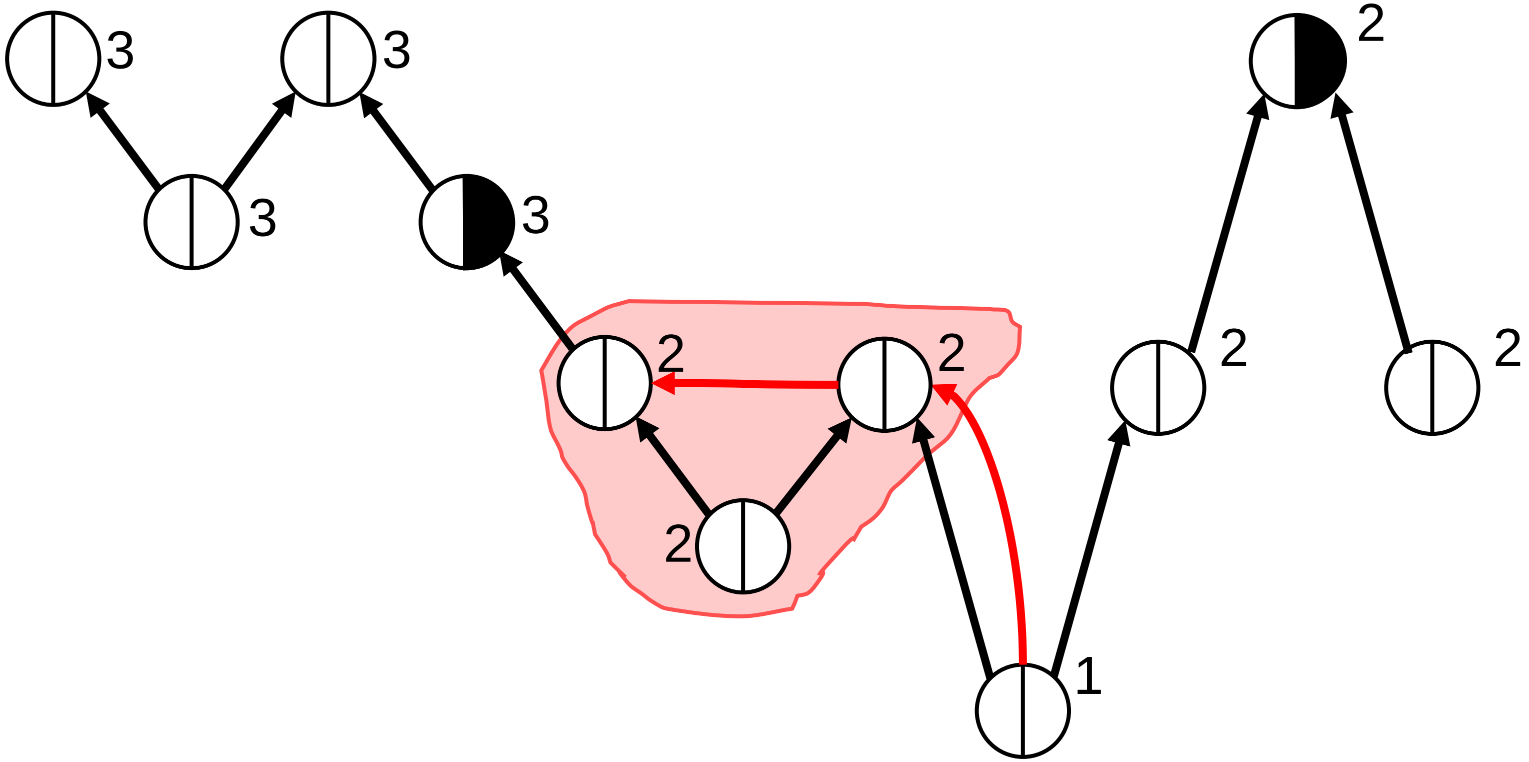
Reversible time dilation



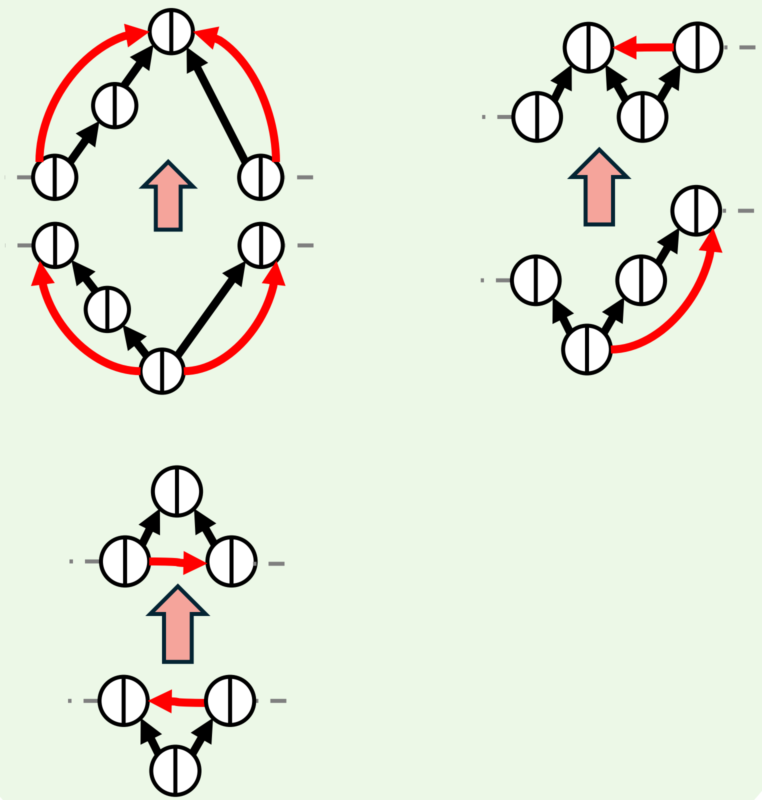
Local rule



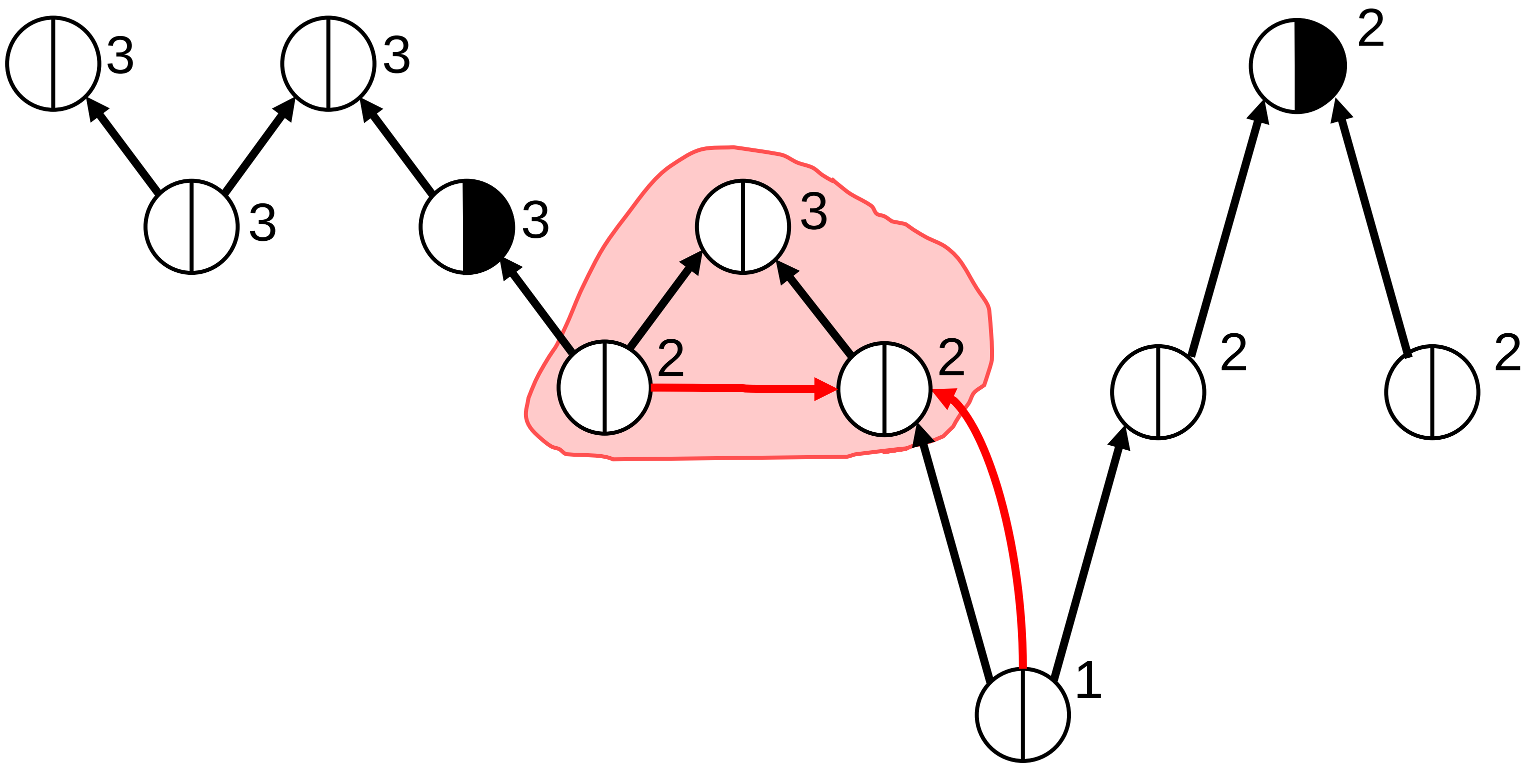
Reversible time dilation



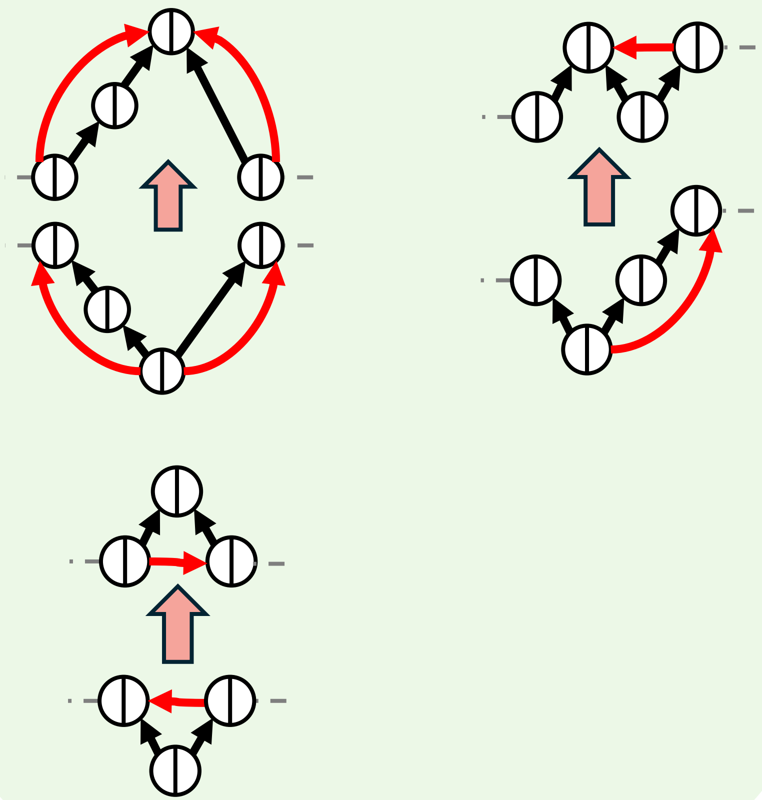
Local rule



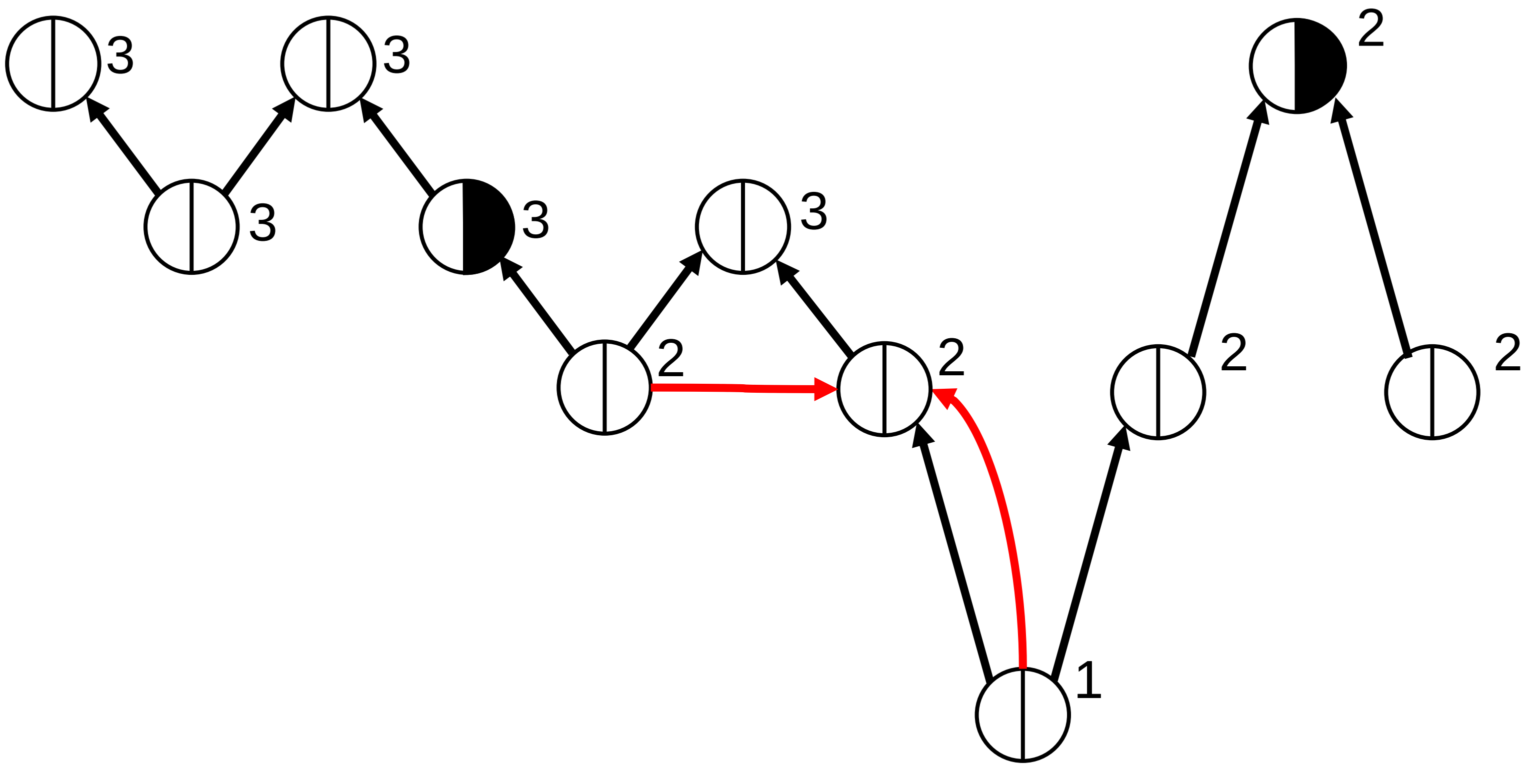
Reversible time dilation



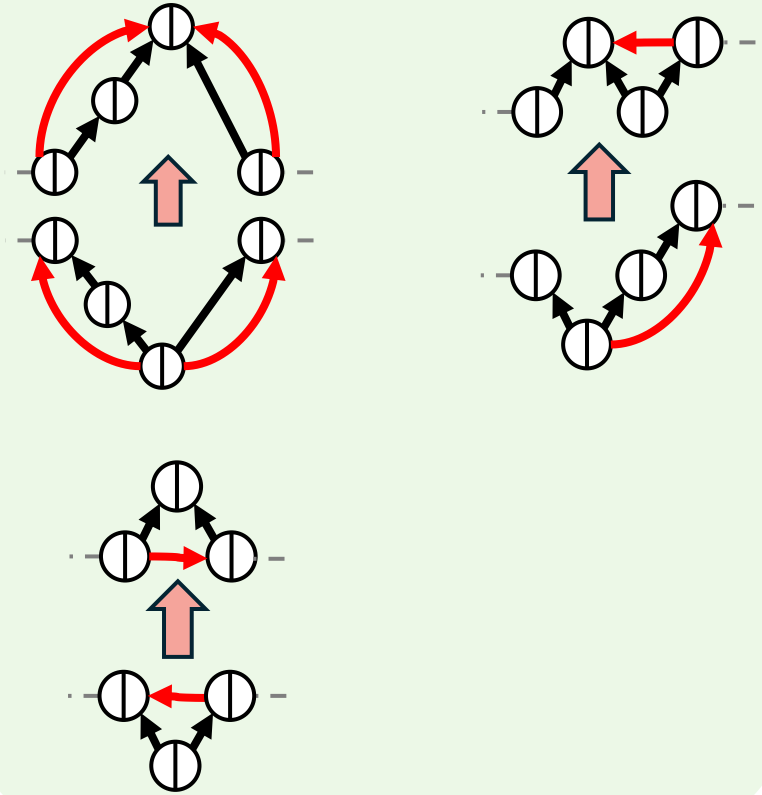
Local rule



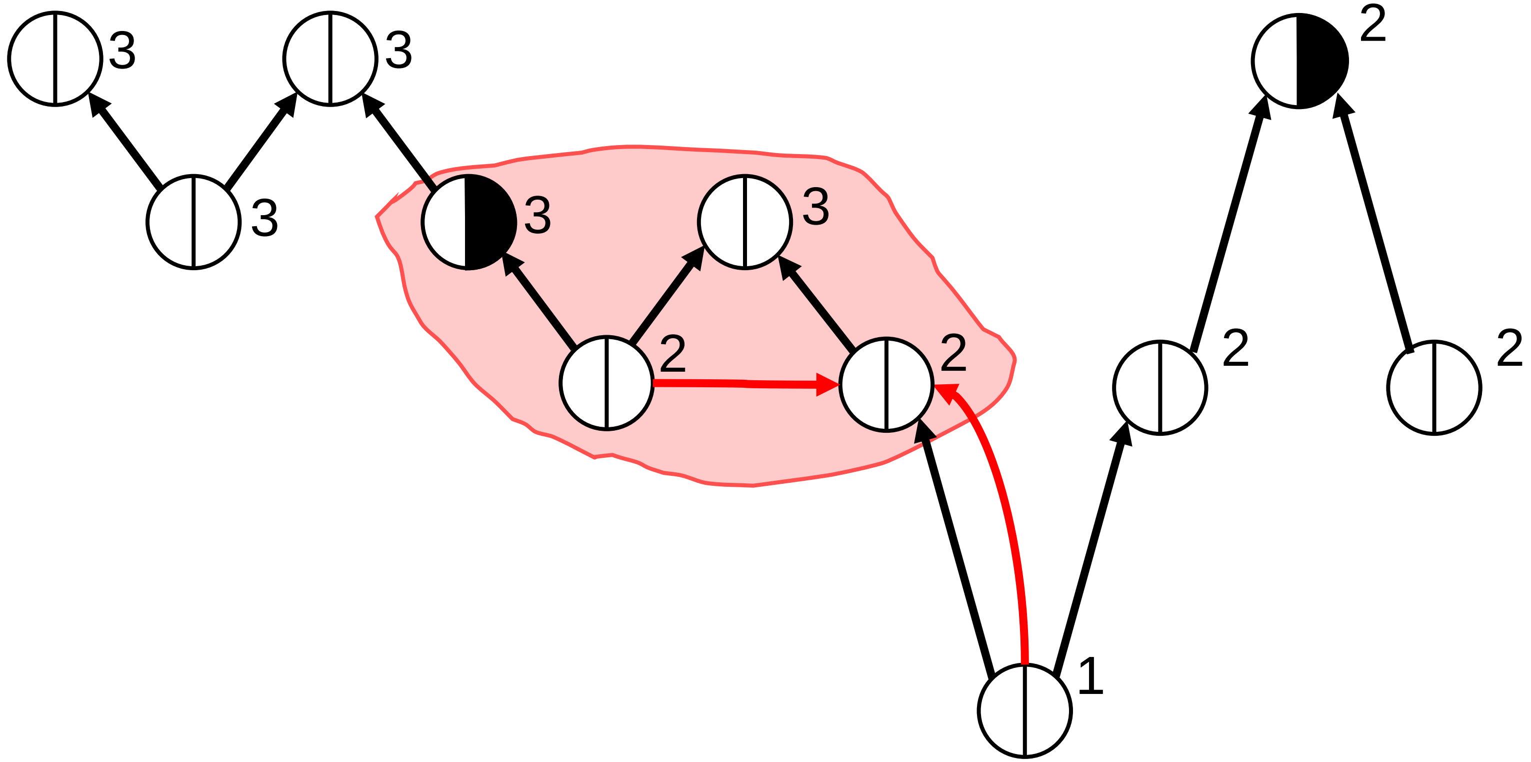
Reversible time dilation

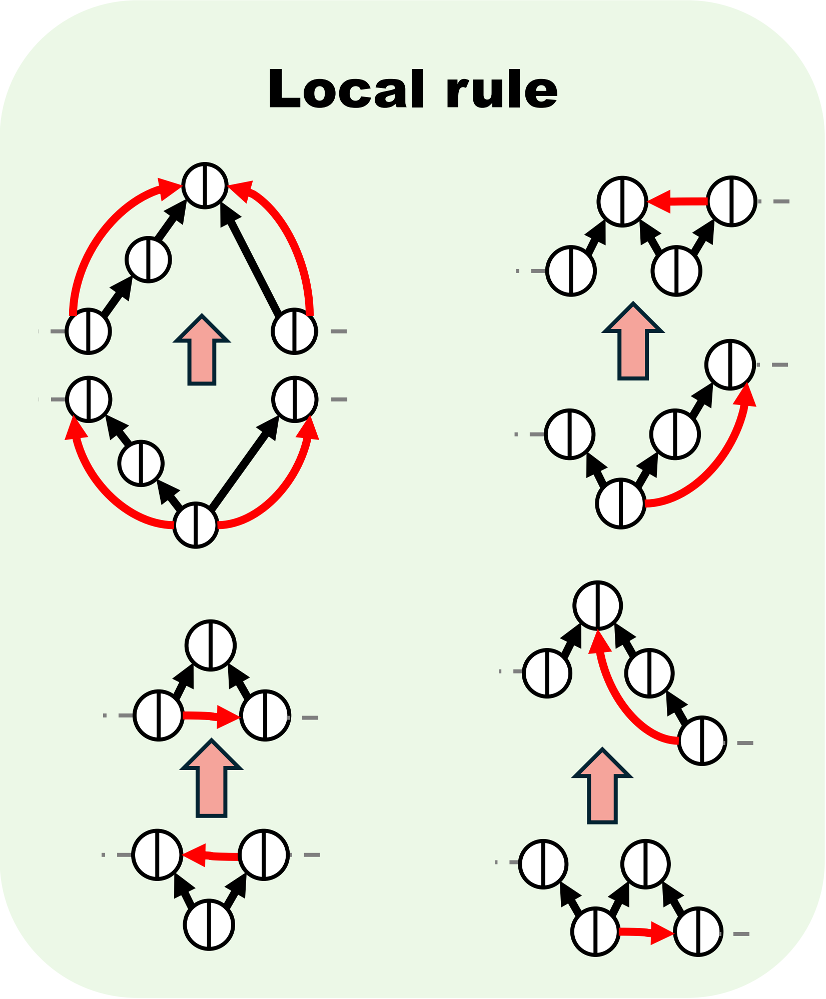


Local rule

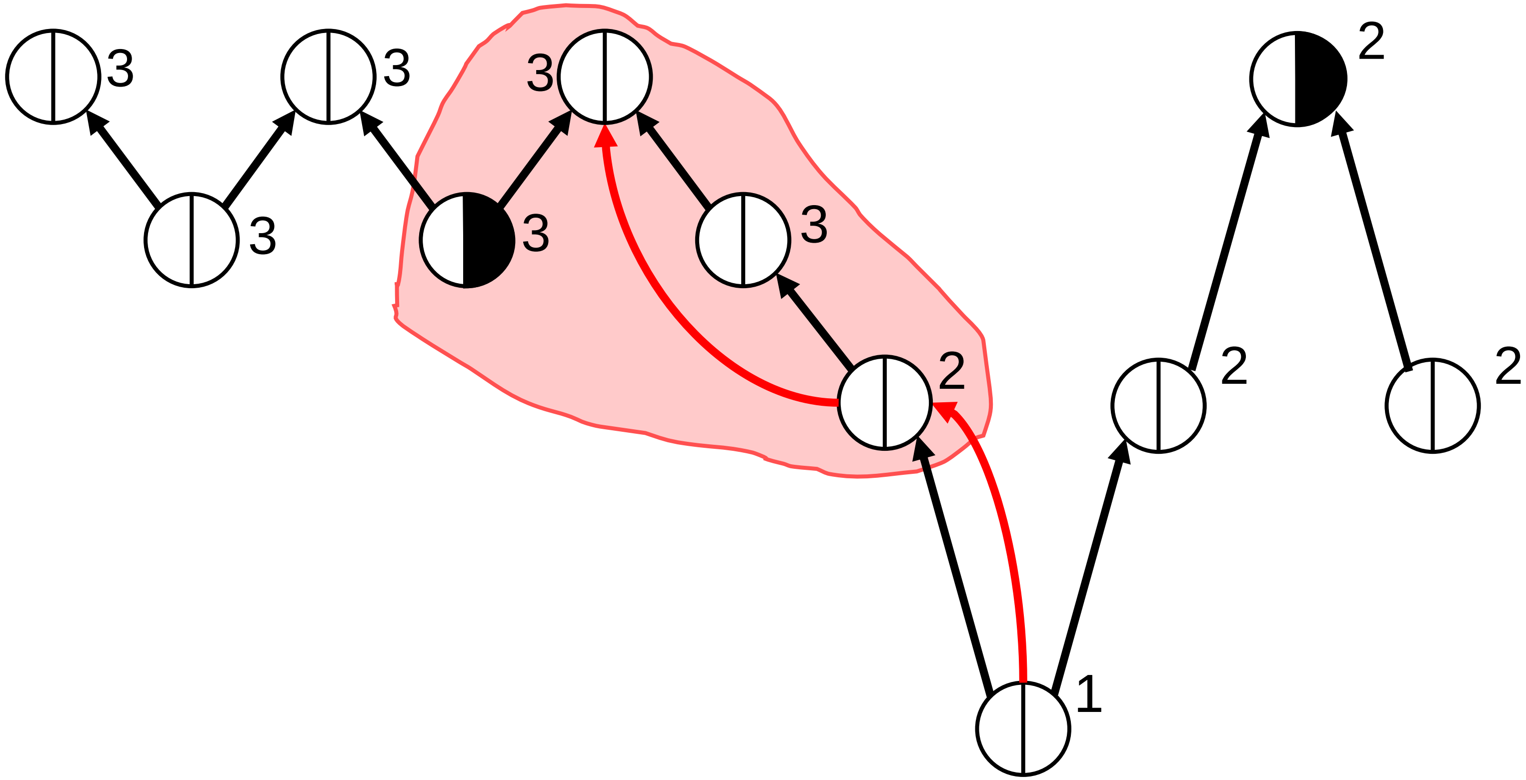


Reversible time dilation

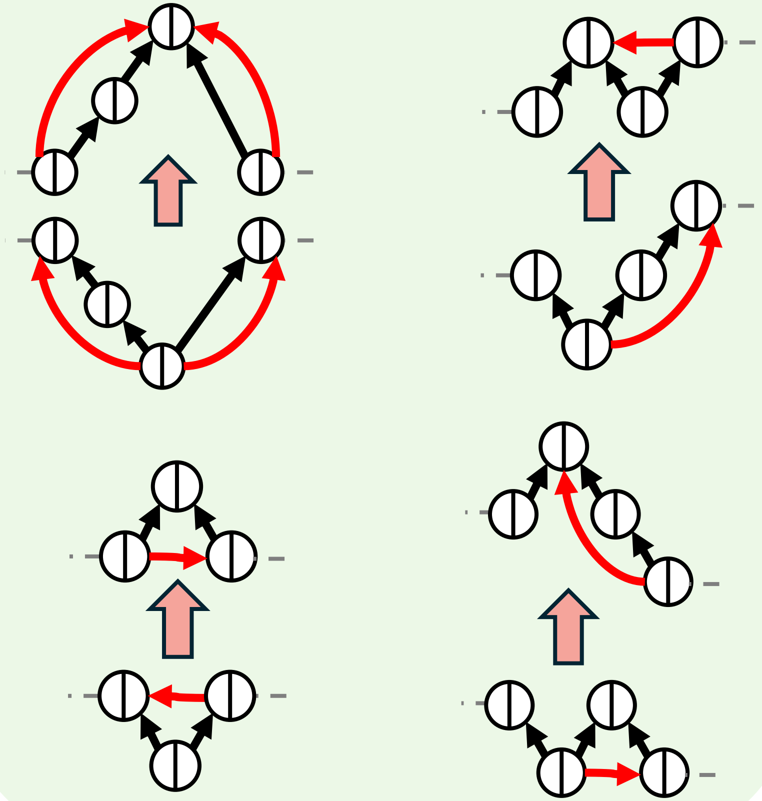




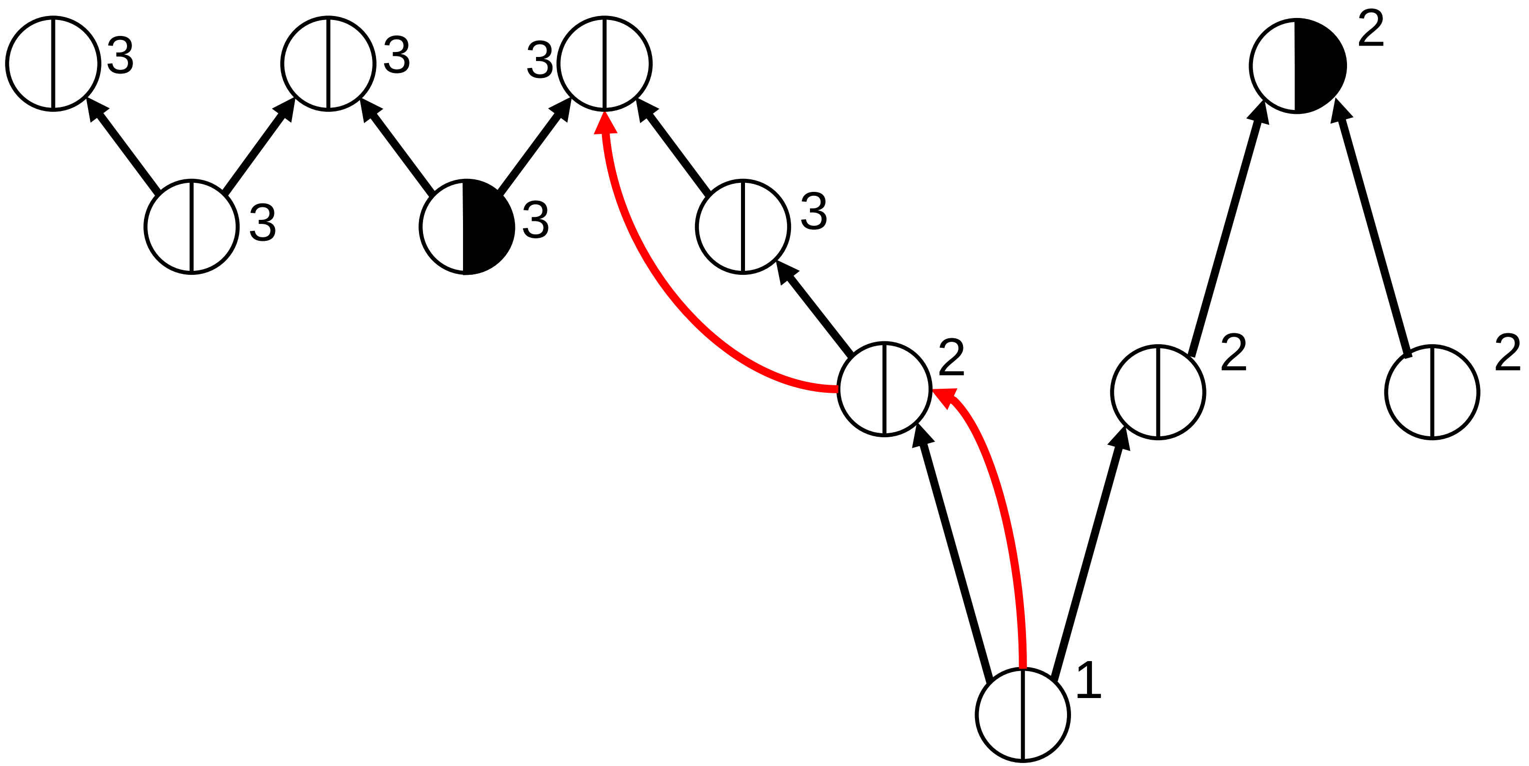
Reversible time dilation

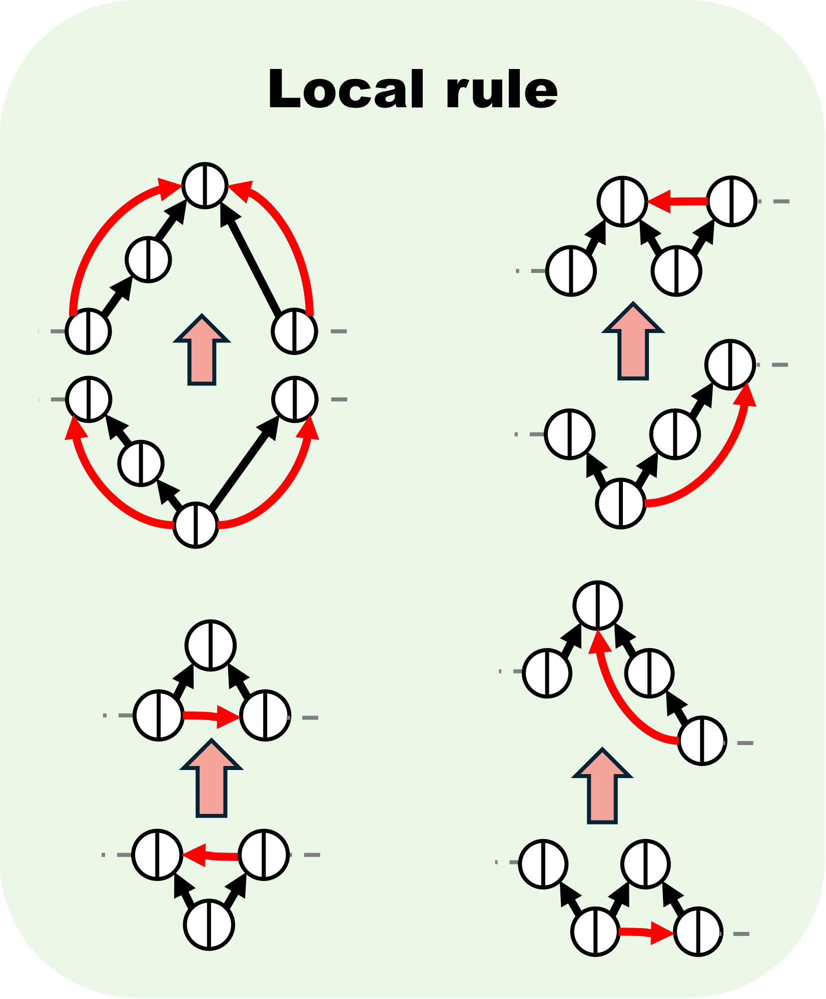


Local rule

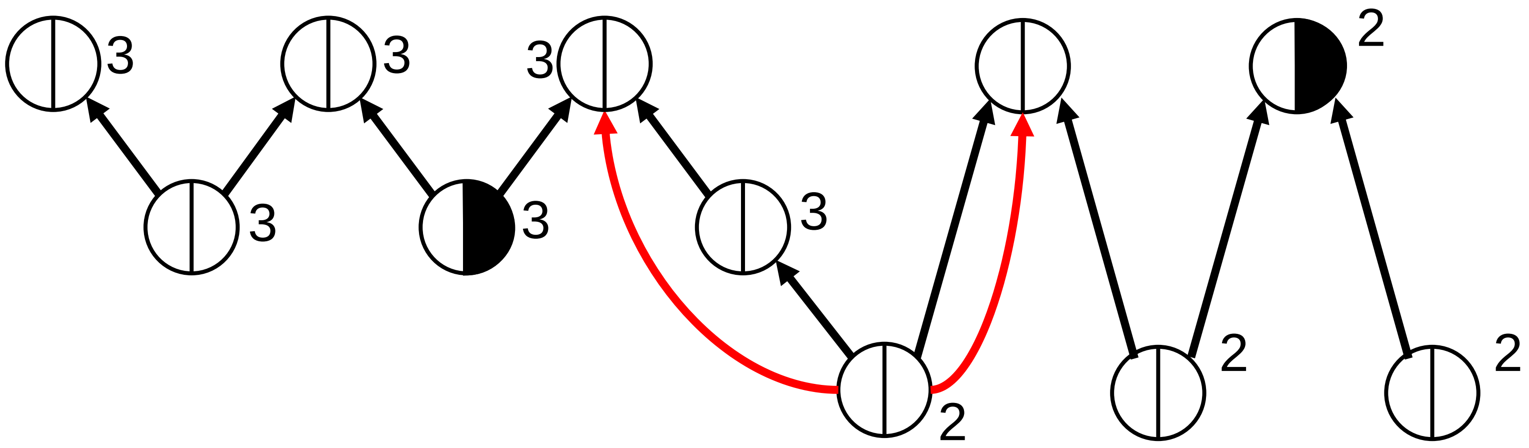


Reversible time dilation

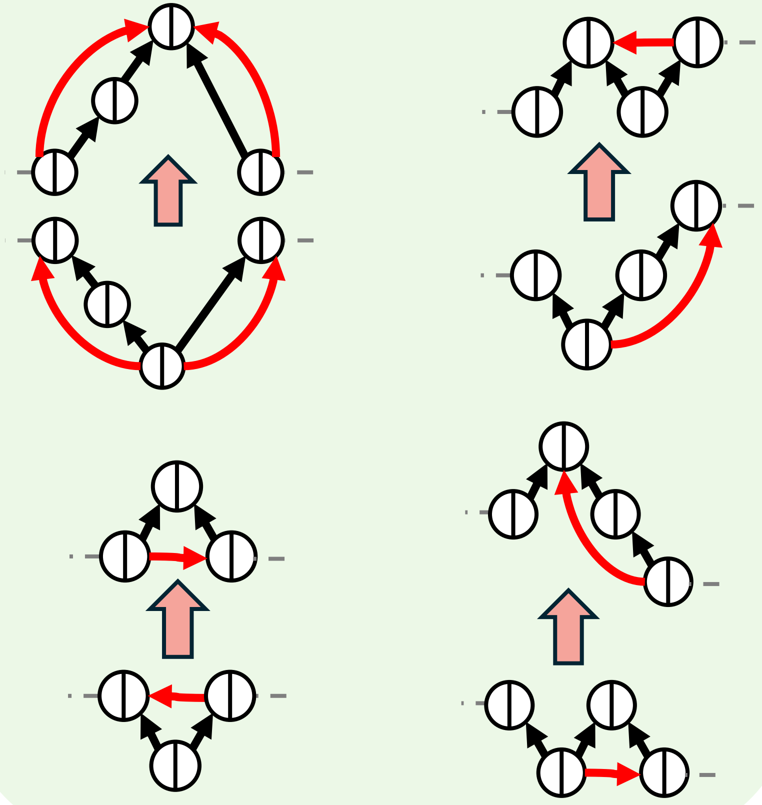




Reversible time dilation

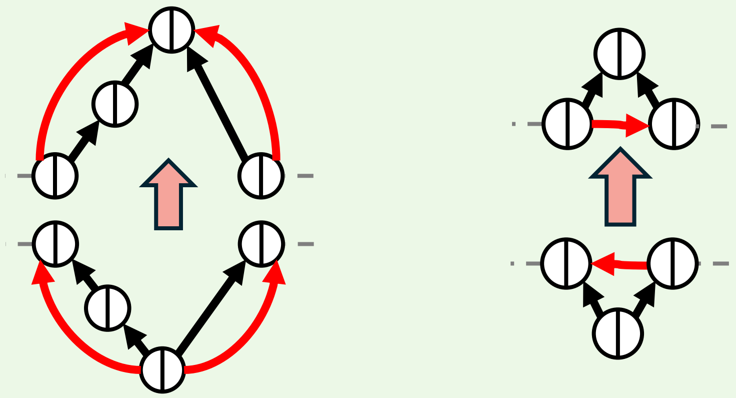


Local rule

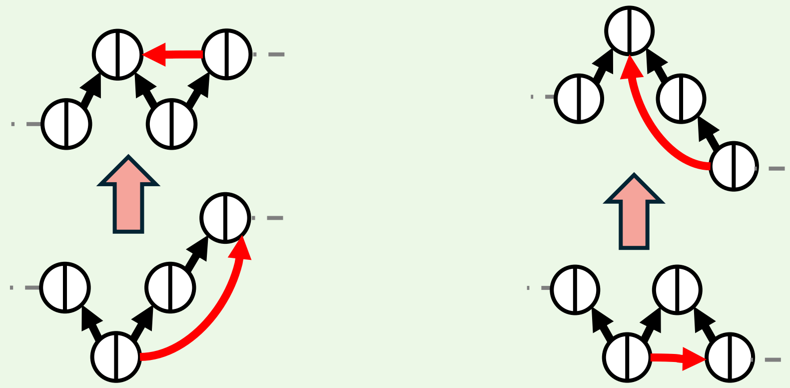


Is this really reversible ?

Is this really reversible ?



Local rule

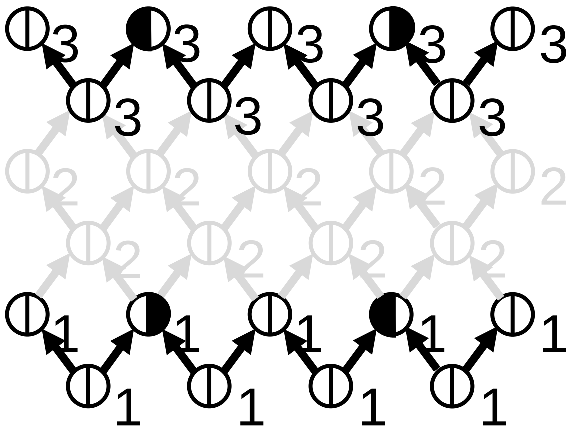


Theorem 2

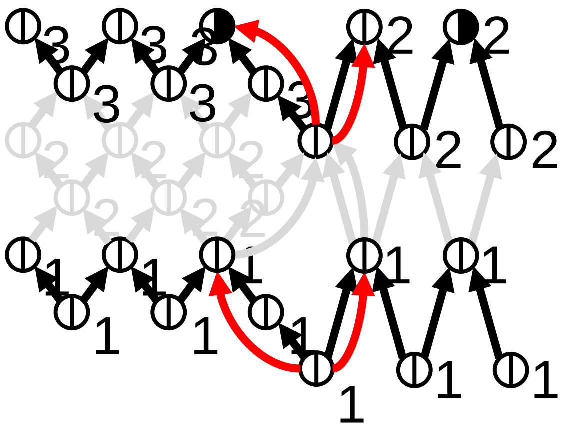
Any disk symmetric local rule is reversible.

Conclusion

Using graph rewriting we **can simulate synchronous** dynamical system …

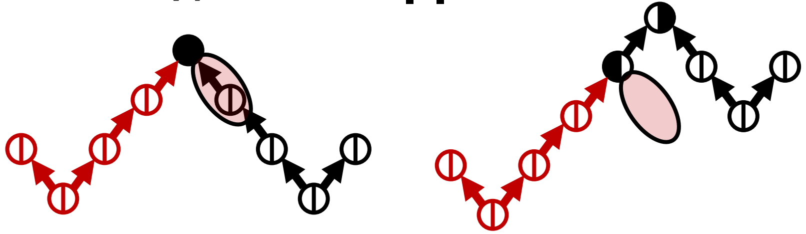


… but also represent some **intrinsically asynchronous** evolution.



While preserving important physical properties such as **determinism**…

$$A_w G \quad || \quad A_{w'} G$$



… and **reversibility.**